International Business Cycles and Exchange Rates

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Backus, Kehoe & Kydland (1994),
Zimmermann (1994):

Too little volatility of trade aggregates.

Exchanges rates have increased in volatility.

So did the trade aggregates.

A link?

Introduce exchange rate shocks into an international real business cycle model

RBC model with three countries

Steps:

1. Establish stylized facts from the data
2. Build a model with exchange rate fluctuations
3. Calibrate the model
4. Simulate the model economies
5. Compare the artificial moments with the data
6. Consequences for future research

Stylized facts:

- vol(output) $>$ vol(consumption)
- vol(investment) $>$ vol(output)
- vol(exports) $\approx$ vol(imports) $\approx$ vol(investment)
- vol(trade balance) high, more in Japan (as exports and imports)
- vol(tot) $\approx$ vol(investment)
- corr(output, investment) $>$ corr(output, cons)
- corr(output, imports) $>$ corr(output, exports)
- corr(output, trade balance) < 0
- corr(output, tot) < 0
- crosscorr(output) $>$ crosscorr(consumption)

The model

Consumer side

In each country, infinitely lived consumer, with intertemporal preferences over consumption and leisure:

$$\max_{\{c_{it}, n_{it}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_{it}, 1 - n_{it}) \right]$$

S.T.

$$E_0 \left[\sum_{t=0}^{\infty} \frac{w_{it} n_{it} + (r_{it} + \delta) k_{it}}{(1 + r_{it})^t} \right] = E_0 \left[\sum_{t=0}^{\infty} \frac{c_{it} + x_{it}}{(1 + r_{it})^t} \right]$$

with

$$U(c, 1 - n) = \frac{1}{\gamma} \left[c^{\mu} (1 - n)^{1 - \mu} \right]^{\gamma}$$

For each agent a firm:

$$\max_{\{n_{it},k_{it}\}} z_{it}F(k_{it},n_{it}) - (r_{it}+\delta)k_{it} - w_{it}n_{it}$$

with

$$F(k,n) = k^{1-\theta}n^{\theta}$$

Use of production:

$$\alpha_i y_{it} = \alpha_i y_{iit} + \alpha_j y_{ijt} + \alpha_k y_{ikt}$$

Use of imports:

$$c_{it} + x_{it} = G(y_{iit}, y_{jit}, y_{kit})$$

where

$$G(y_1, y_2, y_3) = (\omega_1 y_1^{-\rho} + \omega_2 y_2^{-\rho} + \omega_3 y_3^{-\rho})^{-\frac{1}{\rho}}$$

Allows to introduce elasticity of substitution,
$y_{ijt} > 0$ and $y_{jit} > 0$

Exchange rates:

Share π_i of imports billed in foreign currency

$$E_0\left[\sum_{t=0}^{\infty}\left(\frac{\alpha_j\left(\pi_j p_{it}+\left(1-\pi_j\right)p_{jt}e_{ijt}\right)y_{ijt}}{\left(1+r_{it}\right)^t}+\right.\right.$$

$$\left.\left.+\frac{\alpha_k\left(\pi_k p_{it}+\left(1-\pi_k\right)p_{kt}e_{ikt}\right)y_{ikt}}{\left(1+r_{it}\right)^t}\right)\right]$$

$$=\quad \alpha_i E_0\left[\sum_{t=0}^{\infty}\left(\frac{\left(\pi_i p_{jt}e_{ijt}+\left(1-\pi_i\right)p_{it}\right)y_{jit}}{\left(1+r_{it}\right)^t}+\right.\right.$$

$$\left.\left.+\frac{\left(\pi_i p_{kt}e_{ikt}+\left(1-\pi_i\right)p_{it}\right)y_{kit}}{\left(1+r_{it}\right)^t}\right)\right]$$

Laws of motion:

Capital

$$k_{i,t+1} = (1 - \delta)k_{i,t} + x_{it}$$

Investment projects

$$s_{j,t+1} = s_{j+1,t} \qquad\qquad j = 1, J - 1$$

Technology innovations

$$\begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \\ z_{3,t+1} \end{pmatrix} = A_z(L) \begin{pmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix}$$

Exchange rates

$$\begin{pmatrix} e_{21,t+1} \\ e_{31,t+1} \end{pmatrix} = A_e(L) \begin{pmatrix} e_{21t} \\ e_{31t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix}$$

Identities:

Investment

$$x_t = \sum_{j=1}^{J} \phi_j s_{jt}$$

Terms of trade

$$p_{jit} = \beta \frac{\partial G(y_{iit}, y_{jit}, y_{kit})}{\partial y_{jit}} \left[\frac{\partial G(y_{iit}, y_{jit}, y_{kit})}{\partial y_{iit}} \right]^{-1}$$

$$p^*_{it} = \frac{y_{jit} p_{jit} + y_{kit} p_{kit}}{y_{jit} + y_{kit}} \times$$

$$\frac{\alpha_j y_{ijt} + \alpha_k y_{ikt}}{\left((1-\pi_j) e_{ijt} \frac{p_{jt}}{p_{it}} + \pi_j\right) \alpha_j y_{ijt} + \left((1-\pi_k) e_{ikt} \frac{p_{kt}}{p_{it}} + \pi_k\right) \alpha_k y_{ikt}}$$

Trade balance

$$\left(\pi_i + (1-\pi_i) \frac{p_{jt}}{p_{it}} e_{ijt}\right) \frac{\alpha_j}{\alpha_i} y_{ijt} +$$
$$\left(\pi_i + (1-\pi_i) \frac{p_{kt}}{p_{it}} e_{ikt}\right) \frac{\alpha_k}{\alpha_i} y_{ikt} - p_{jit} y_{jit} - p_{kit} y_{kit}$$

The business cycle in this economy:

$$\varepsilon_{zt} \longrightarrow z_t \longrightarrow \text{productivity}$$
$$\varepsilon_{et} \longrightarrow e_t \overset{\nearrow}{\longrightarrow} p_t^*$$

$$w_t \longrightarrow n_t$$

if shock is persistent:

$$w_t \longrightarrow c_t$$

$$r_t \longrightarrow c_t, x_t$$

$$p_t^*, x_t, c_t \longrightarrow y_{1t}, y_{2t}$$

$$z_t, n_t, x_{t-J} \longrightarrow y_t$$

Calibration:

Take some parameter values from the literature:

$$\beta, \delta, \rho, \theta, n, c, \gamma, \pi$$

Estimate some:

$$\frac{y_{ji}}{y_i}$$

$$\begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \\ z_{3,t+1} \end{pmatrix} = \begin{pmatrix} \overline{z}_1 \\ \overline{z}_2 \\ \overline{z}_3 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix} \rightsquigarrow \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_{z1})^2 & r_{z12}\sigma_{z1}\sigma_{z2} & r_{z13}\sigma_{z1}\sigma_{z3} \\ r_{z12}\sigma_{z1}\sigma_{z2} & (\sigma_{z2})^2 & r_{z23}\sigma_{z2}\sigma_{z3} \\ r_{z13}\sigma_{z1}\sigma_{z3} & r_{z23}\sigma_{z2}\sigma_{z3} & (\sigma_{z3})^2 \end{bmatrix} \right)$$

$$\begin{pmatrix} e_{21,t+1} \\ e_{31,t+1} \end{pmatrix} = \begin{pmatrix} \overline{e}_{21} \\ \overline{e}_{31} \end{pmatrix} + \begin{pmatrix} a_{e21} & 0 \\ 0 & a_{e31} \end{pmatrix} \begin{pmatrix} e_{21t} \\ e_{31t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix} \rightsquigarrow \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_{e21})^2 & r_e \sigma_{e21} \sigma_{e31} \\ r_e \sigma_{e21} \sigma_{e31} & (\sigma_{e31})^2 \end{bmatrix} \right)$$

Determine the others using the first order conditions.

Solution procedure:

Complex problem

Pareto Optimum = Market equilibrium

Quadratic approximation of the value fonction

Linear decision rules

Simulation with random numbers

Replication of stylized facts:

Y vol(output) > vol(consumption)

Y vol(investment) > vol(output)

y vol(exports) ≈ vol(imports) ≈ vol(investment)

Y vol(trade balance) high, more in Japan (as exports and imports)

y vol(tot) ≈ vol(investment)

Y corr(output, investment) > corr(output, cons)

Y corr(output, imports) > corr(output, exports)

y corr(output, trade balance) < 0

n corr(output, tot) < 0

N crosscorr(output) > crosscorr(consumption)

What did the exchange rate fluctuations bring?

Y $\rightarrow$ **Y** vol(output) $>$ vol(consumption)

Y $\rightarrow$ **Y** vol(investment) $>$ vol(output)

N $\rightarrow$ **y** vol(exports) $\approx$ vol(imports) $\approx$ vol(investment)

N $\rightarrow$ **Y** vol(trade balance) high, more in Japan (as exports and imports)

N $\rightarrow$ **y** vol(tot) $\approx$ vol(investment)

y $\rightarrow$ **Y** corr(output, investment) $>$ corr(output, consumption)

Y $\rightarrow$ **Y** corr(output, imports) $>$ corr(output, exports)

Y $\rightarrow$ **y** corr(output, trade balance) < 0

N $\rightarrow$ **n** corr(output, tot) < 0

N $\rightarrow$ **N** crosscorr(output) $>$ crosscorr(cons)

Are these results stable?

- σ and π are critical for vol(trade): higher σ, lower π
- σ influences corr(output, invest) > corr(output, cons): lower σ
- idem for corr(output, imports) > corr(output, exports), also higher π
- corr(output, trade balance): lower σ
- corr(output, tot): lower σ, same π
- crosscorr(output) > crosscorr(cons): Hopeless?

Time-to-ship: prevents to much consumption smoothing

What now?

Find better estimates of σ and π

Endogenize the exchange rate movements

Cost of exchange rate movements