Competing for Time: A Study of Mobile Applications*

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Abstract

A smartphone user allocates her time to multiple mobile applications. To study the competitive relationship among apps, I develop a discrete-continuous model of time allocation with a binding time constraint and estimate it with a weekly panel of app usage in China. If two apps are often used together, it is because either they are complements or the preferences of the two apps are positively correlated. To disentangle complementarity (substitutability) from correlation in preferences, I use the exclusion restriction that updates of an app should affect the utility of this app but not those of other apps. I estimate the model on three pairs of apps (substitutes, complements, and independent apps). I recover a reasonable competition pattern and simulate mergers of the three pairs of apps. I find that a seemingly innocuous merger of independent apps can hurt consumers due to the

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binding time constraint. My results confirm that users and firms can both benefit from a merger of complements. I also find that usage-based pricing leads to higher profits and total surplus compared with subscription pricing because it enables price discrimination based on usage.

1 Introduction

Smartphones and mobile applications (apps) have become an integral part of everyday life. According to eMarketer, an average U.S. adult spends two hours and 43 minutes per day on smartphones in 2019.[1] This number is projected to increase in the next three years. Mobile devices also prove to be conducive to entrepreneurship and innovations. According to Statista, as of the first quarter of 2020, there are 2.5 million apps available on the Google Play store and 1.8 million apps available on the Apple App Store. Many of the "unicorns"—private companies valued at more than $1 billion—build their business around one single app or a portfolio of apps. ByteDance, the parent company of Toutiao, a news aggregator app, and Tik Tok, a short video app, was valued at $75 billion in August 2018.[2] To put the number into perspective, the market capitalization of Ford Motor was about $40 billion in August 2018.

Despite the importance of the mobile Internet industry, we lack a tool to analyze the competition relationship among apps. Understanding the competition relationship among apps is crucial to the design of an optimal portfolio of apps. Tech giants employ different strategies in developing (or acquiring) apps. For example, Tencent developed WeChat Contacts, WeChat Read and other complementary apps around the popular app WeChat. In contrast, Facebook acquired nascent competitors like Instagram and Whatsapp.[3] The effectiveness of different strategies should be evaluated with a model of competition among apps.

Antitrust authorities struggled to deal with mergers of free apps (Wu, 2017; Prat & Valletti, 2018; Scott Morton *et al.*, 2019; Cabral, 2020). None

[1] See the report by eMarketer at https://www.emarketer.com/content/us-mobile-time-spent-2020

[2] See the report by the Wall Street Journal at https://www.wsj.com/articles/beijing-bytedance-technology-seeks-to-raise-3-billion-privately-1533661087

[3] See "These Confidential Charts Show Why Facebook Bought WhatsApp" at https://www.buzzfeednews.com/article/charliewarzel/why-facebook-bought-whatsapp

of the major acquisitions by tech giants were blocked in recent years in the United States (Cabral, 2020). And the same is true for China. This suggests that anti-trust authorities lack the tools to analyze mergers of apps. The UK Office of Fair Trading (OFT) approved the acquisition of Instagram by Facebook in 2012 partly on the ground that in the market for camera apps Facebook would still face competition from other photo apps after the merger.[4] This market definition is certainly debatable and some have called for the anti-trust authorities to reverse this and other similar decisions (Wu, 2018; Hughes, 2019).

The difficulty in understanding the competitive relationship among apps is twofold. First, there are no price variations and therefore we cannot estimate price elasticities. Functional definitions do not work. WeChat, the flagship app of Tencent, is classified as "Social Networking" by Apple App Store and "Communication" by Google Play Store as of 2019. However, users know that WeChat is more than the two definitions: it is also a mobile payment app, a publishing platform, a platform of mini programs, and so on. Second, the competition landscape is further complicated by the binding time constraint: users have at most 24 hours a day. Every minute spent on Tik Tok is a minute not spent on Facebook. This is the wealth effect in classical demand theory. To evaluate the magnitude of the wealth effect of one app on another, we need a structural model of consumer demand for apps.

In this paper, I propose and estimate a model of time allocation to apps and then simulate mergers with the estimated parameters. The model features a quadratic utility function *a la* Thomassen *et al.* (2017)[5]. In this model, an app is described by a taste parameter, a satiation parameter, and interac-

[4]See Anticipated acquisition by Facebook Inc of Instagram Inc, ME/5525/12 (Office of Fair Trading, 22 August 2012) at https://assets.publishing.service.gov.uk/media/555de2e5ed915d7ae200003b/facebook.pdf. The Federal Trade Commission (FTC) did not disclose why it approved the acquisition.

[5]Lewbel & Nesheim (2019) also use a quadratic utility model.

tion parameters with other apps. The taste parameter is the marginal utility at zero usage, and the satiation parameter determines how fast the marginal utility depreciates as an user spend more time on this app. The interaction parameters measure the interactions between apps: if the interaction parameter between a pair of apps is positive then they are complements; otherwise they are substitutes. The parameters have random components that can be correlated across apps. Users allocate their time to apps and offline activities subject to a time constraint. I use GMM estimation *a la* Berry *et al.* (1995) so that I can use instrument variables (IVs). Quadratic utility functions are second-order approximations to any reasonable utility functions. Therefore my model nests the random coefficient discrete choice model of Berry *et al.* (1995) as a special case with only taste parameters.

I estimate the model with weekly market-level app usage data from China in the first quarter of 2017. This is the first academic paper to use this type of data. Markets in the data set are demographic groups in China defined by age, gender, and province. I observe the number of active users, users who use an app (or an Android smartphone) at least once during a week, and usage (time spent) of popular apps (or Android smartphones). The active user data help me identify taste parameters of apps and the usage data help me identify satiation parameters. In addition, for each pair of apps, I observe the number of users who use both apps in a week. The common user data are informative but not sufficient for the identification of complementarity/substitutability.

The econometric challenge is to separate correlated preference from complementarity/substitutability. A large number of common users can be the result of complementarity between the two apps or the fact that the preferences of the two apps are positively correlated due to unobserved characteristics. My identification strategy is based on an extended definition of complements (substitutes): when there is an exogenous increase in the utility of one app, the usage of its complements (substitutes) would increase (decrease). Updates of an app should affect the utility of this app but not the *utilities* of other apps. However, they could change the *usage* of other apps through complementarity/substitutability. This is similar to the strategy used in Gentzkow (2007).

To ensure my model recovers a reasonable competition pattern, I select three pairs of apps: a pair of substitute a priori (Baidu Map and Amap), a pair of complements a priori (Baidu and Baidu Map), and a pair of apps with independent functions[6] (WeChat and Kwai). In each case, the estimated results conforms to beliefs widely held in the industry. IVs are crucial to my estimation. When I assume away correlated preferences and rely only on the common user data for identification, Baidu Map and Amap are estimated to be independent apps. I also find that within a pair of apps, the effect of one on another is often asymmetric: the larger app[7] has a larger effect on the smaller app and the smaller app has a smaller or negligible effect on the larger app.

To simulate mergers of apps, I borrow an estimate of the marginal value of leisure time from Shiaw (2004) and combine it with the utility model I estimated. On the supply side, I use subscription pricing and usage-based pricing and simulate market outcomes for the two pricing strategies separately.

The simulation results of substitutes are intuitive. The post-merger monopolist internalizes the substitutability between Baidu Map and Amap by increasing the prices of Baidu Map (by 24%) and Amap (by 19.6%). Consumer surplus decreases by 18.9% and profits increase by 2% after the merger. My results confirm that consumers and firms can both benefit from a merger of complements. The post-merger monopolist internalizes the complementarity between Baidu and Baidu Map by lowering the prices of Baidu (by 1%) and Baidu Map (by 2.8%). As a result, consumer surplus increases by 1.5% and profits increase by 0.15%. This finding suggests that developing or

[6]The apps are neither competing with nor complementing each other.

[7]An app is larger if it has a larger number of active users.

acquiring complementary apps to a flagship app is a profitable strategy for tech firms.

I find that a seemingly innocuous merger of independent apps (WeChat and Kwai) can hurt consumers as they are competing with each other for user time. After the merger, the prices of WeChat and Kwai increase by 1.5% and 39.7%. Profits increase by 0.3% whereas consumers surplus decreases by 3.3%. The merger results would help antitrust authorities understand how mergers affect consumers and tech firms make better investment decisions.

My results show that usage-based pricing leads to higher profits and total surplus compared with subscription pricing. Usage-based pricing enables firms to discriminate users based on usage, which is a good proxy of willingness-to-pay (WTP). The two pricing strategies have different distributional effect: users with high WTP will benefit from subscription pricing whereas users with low WTP will benefit from usage-based pricing.

This paper contributes to the emerging literature on mobile applications. Due to data limitations, researchers mostly focus on the supply side of apps (Liu *et al.*, 2013; Yin *et al.*, 2014; Bresnahan *et al.*, 2014b,a; Liu, 2017; Wen & Zhu, 2017; Ershov, 2018; Leyden, 2019). Demand for apps is either absent in the papers or described with aggregate ranking or downloads data from app stores (Carare, 2012; Ghose & Han, 2014; Li *et al.*, 2016; Yi *et al.*, 2017; Le Guel *et al.*, 2020; Deng *et al.*, 2020).[8] An immediate predecessor of this paper is Han *et al.* (2016). They adopt a multi-nominal discrete-continuous extreme value (MDCEV) model developed by Bhat (2005) and allow for correlation in the utility between different apps by adding a factor analytic structure. With individual panel data from Nielsen KoreanClick, they estimate positive or negative correlations in preferences across apps. However, substitutes or complements are not modeled in their paper. As they have

[8]Both Wu *et al.* (2018) and Lee (2018) use a panel of individual usage of smartphone. However, both observe usage of categories rather than apps. Lee (2018) estimates the demand for smartphone. Wu *et al.* (2018) uses a hidden Markov model to analyze what motivates mobile app usage.

noted in their paper, the correlation of preferences of Naver and Daum and that of Kakao Talk and Kakao Story are estimated to be positive and large. However, common sense suggests that the first pair are substitutes and the second pair complements. In contrast, this paper explicitly disentangles substitutability/complementarity from correlated preferences with the common user data and IVs. My model also differs from Han *et al.* (2016) in using market level data. Though they are widely used in the industry, to the best of my knowledge, this is the first paper to use market level data of app usage in academic research.

In terms of methodology, this paper is a second-order extension of Berry *et al.* (1995). My model is the first to combine four appealing features in a model of consumer demand: discrete-continuous decisions, interactions between products, wealth effect, and estimation with instruments. This paper relates to the literature on the demand of differentiated goods in economics and marketing especially when complementarity is of interests (Kim *et al.*, 2002; Nair *et al.*, 2005; Song & Chintagunta, 2006, 2007; Mehta, 2007; Gentzkow, 2007; Thomassen *et al.*, 2017; Ershov *et al.*, 2018; Vélez-Velásquez, 2019; Lewbel & Nesheim, 2019; Wang, 2020). This paper also relates to the study of time allocation in transportation research (Kitamura, 1984; Bhat, 2005; Pawlak *et al.*, 2015, 2017; Bhat, 2018). This model is a flexible second-order approximation to consumer decisions and hence can be adapted to study other topics.

2 Data

There are two types of app usage data available in the industry: individual level data and market level data. The first type resembles traditional surveys. Firms pay individuals to get their permission to install an app or software to monitor the usage of their devices. The data sets used by Han *et al.* (2016) , Lee (2018), Wu *et al.* (2018), and Boik *et al.* (2016) fall into this category. The data set used in this paper are aggregate market data. They are estimated data based on a large amount of observations from different sources. Wireless carriers, app developers are the two major sources. For example, China Unicom provides app usage data based on traffic data from its users.[9] App developers mostly use third-party libraries to analyze the behaviors of their users.[10] Those data are then traded and matched based on unique device identifiers. To sum, market level data are estimated from snapshots of millions of devices whereas individual data are 24*7 information from thousands of users.

Data used in this paper are from iResearch, a consulting firm in China focusing on the mobile Internet industry. There are three parts of our data: the app usage data, the smartphone usage data, and the common user data. I introduce them in the following subsections. All the data are weekly data from the first quarter of 2017 in China.

2.1 The App Usage Data

I have app usage data of the top 300 apps on Android cellphones of 290 demographic groups (which is the definition of market in this data set) for 13 weeks in China. In this data set, a market is a demographic group defined by gender (male and female), age groups (below 24, 25~30, 31~35, 36~40, and above 40), and areas (28 provinces and an "other" category). I do not have all the 300 apps as some have an estimated number of active users that is too small to be reliable. The threshold is 50,000. On average, we observe about 82 apps for each week-market pair. We observe more apps for large demographic groups in the data set. In total, we have 312,724 week-market-app observations. For each unit of observation, we observe the number of

[9] See https://www.cubigdata.cn

[10] For a story about how this works, see the report by the Wall Street Journal: https://www.wsj.com/articles/you-give-apps-sensitive-personal-information-then-they-tell-facebook-11550851636?mod=article_inline

devices (in ten thousands) that used the app at least once during the week (henceforth, active user) and the average number of minutes spent on the app per device during the week (henceforth, average time spent). The summary statistics are in the upper panel of table 1. The zeros in the table result from the technical difficulty of estimating usage of some apps, for example, input methods.

2.2 The Smartphone Usage Data

iResearch also provides the total usage of the device, i.e., the smartphone usage data. Similarly, we have the number of active devices (in ten thousands) that are used at least once during the week (henceforth, active users) and the average number of minutes spent on Android smartphones per device during the week (henceforth, average time spent). With those data, I calculate market shares of apps in each market which is the number of active user of an app divided by the number of active users of Android smartphones in that market. The summary statistics are in the middle panel of table 1.

2.3 The Common User data

An important part of the data set are the common user data. For each pair of apps, we observe the number of Android smartphone users that used both apps at least once during the week (henceforth, common user). Again the 50000 threshold applies. On average, we observe about 110 apps each week. I only have common user data at the national level because they are very small, hence unreliable, at the demographic group level. The summary statistics are in the lower panel of table 1.

Table 1: Summary statistics of app usage

Variables	Mean	Min	Max	StdDev	# Obs	Unit
App Usage Data						
Active user	28.05	5	1074.2	46.38	312724	ten thousands
Market share	0.1064	0.004	0.958	0.132	312724	-
Average time spent	58.65	0	800.17	74.86	312724	minutes
Smartphone Usage Data						
Active user	225.05	10.31	1238.75	193.92	3770	ten thousands
Average time spent	1006	561.5	1435.5	202.08	3770	minutes
Common User data						
Common user	599.21	6.41	29979.13	1387.733	79809	ten thousands

Note:
1, The smartphone and app usage data are weekly observations at the demographic group level from the first 13 weeks of 2017 in China. The common user data are weekly aggregate data for each pair of apps.
2. Active user of an app is the number of devices that used the app at least once during the week. Active user of smartphone is the number of Android smartphones that are used at least once during the week. Average time spent is the average number of minutes spent on the app per device during the week. Market share of an app is the active user of this app divided by the active user of Android smartphones in that market. Common user is the number of Android smartphones that used both apps at least once during the week.
3, The zeros in app usage data result from the technical difficulty of estimating usage of some apps, for example, input methods.
Data Source: iResearch.

3 Model

A consumer $i = 1, 2, , ..., I$ allocates her time T to J apps and an outside option denoted by $j = 0$. The utility from an allocation described by $\mathbf{t} = [t_{i0}, t_{i1}, t_{i2}, ..., t_{iJ}]'$ where t_{ij} is the amount of time allocated to option $j = 0, 1, 2, ..., J$ is given by

$$U(\mathbf{t}) = \boldsymbol{\mu}'\mathbf{t} + 0.5\mathbf{t}'\boldsymbol{\Gamma}\mathbf{t} \tag{1}$$

where

$$\boldsymbol{\mu} = [\mu_{i0}, \mu_{i1}, ..., \mu_{iJ}]'$$

and

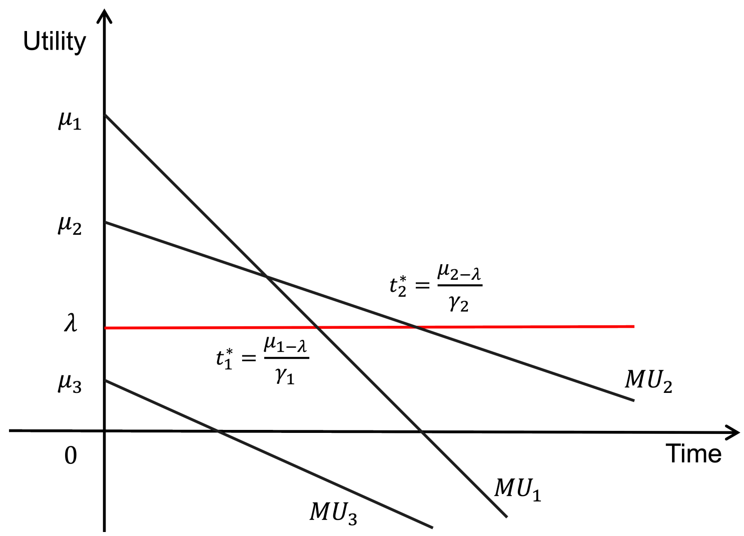
$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{i0} & \gamma_{i01} & \cdots & \gamma_{i0J} \\ & \gamma_{i1} & \cdots & \gamma_{i1J} \\ & & \ddots & \vdots \\ & & & \gamma_{iJ} \end{bmatrix}.$$

$\boldsymbol{\mu}$ is a $(J+1)\times 1$ vector of first order parameters and $\boldsymbol{\Gamma}$ is a $(J+1)\times(J+1)$ symmetric matrix of second order parameters. The marginal utility of app j is

$$MU_{ij} = \mu_{ij} + \gamma_{ij}t_{ij} + \sum_{j' \neq j} \gamma_{ijj'}t_{ij'}.$$

It is clear that the marginal utility of app j consists of three components. The first term μ_{ij} is the marginal utility of app j at zero usage and will be referred to as the taste parameter of app j. γ_{ij} in the second term determines how MU_{ij} changes as user spend more time on app j. Therefore, γ_{ij} should be negative and will be referred to as the satiation parameter of app j. The last term captures the impact of app j' on app j: if parameter $\gamma_{ijj'} > 0$ then MU_{ij} is increasing in $t_{j'}$ and they are complements; otherwise they are substitutes. Therefore, the interaction parameter $\gamma_{ijj'}$ determines if j and j' are likely to be used together.

At the optimal level $\mathbf{t}^*$, the marginal utilities of apps that are used should



Note: This graph plots the marginal utility of three apps: 1, 2, and 3. λ is the marginal utility at the optimal allocation. For simplicity, I ignore $\gamma_{jj'}$. MU is the marginal utility of each app.

Figure 1: Marginal Utilities and Optimal Allocation of Time

be equalized. Denote this as λ. Zero usage arises naturally when the marginal utility at zero is too small, i.e., $\mu_{ij} < \lambda$. In figure 1, I plot three apps with different combinations of μ_j and γ_j with no consideration of $\gamma_{jj'}$ for now. Intuitively, μ_j determines if an app is used and conditional on being used, γ_j determines the time spent on app j.[11]

I choose the quadratic utility function because it naturally models the discrete-continuous nature of app usage and the complementarity/substitutability between apps. Despite the advantages of the quadratic utility function, the size of $\mathbf{\Gamma}$ also increases quadratically in J. Therefore, instead of analyzing 100 apps at the same time, which involves a gigantic matrix $\mathbf{\Gamma}$, I analyze a smaller model with more assumptions and only two apps of interest. Two is certainly not an ideal number. However, many mergers in the mobile Internet industry apps is about two apps, (for example, Instagram and Facebook).

[11] This is not accurate because λ is a function of all μ_j and γ_j.

3.1 A Simplified Model

In the model to be estimated, there are four options $j = 0, 1, 2, 3$, where $j = 1, 2$ are the two apps of interest and $j = 0$ is the option of not using a smartphone and $j = 3$ is a generic app which is to use any other apps. The utility maximization problem of consumer i in market $m = 1, 2, ..., M$ is

$$\max_{t_{i0m},t_{i1m},t_{i2m},t_{i3m}\geq 0} t_{i0m} - 0.0005t_{i0m}^2 + \sum_{j=1}^{2} \mu_{ijm}t_{ijm} + 2t_{i3m} + \frac{1}{2}\sum_{j=1}^{3}\gamma_{ijm}t_{ijm}^2 + \gamma_{12}t_{i1m}t_{i2m} \tag{2}$$

$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = T$$

Note that I add more assumptions compared to equation (1). To normalize the level of the utility function, I assume $\mu_{i0m} = 1$. I assume $\mu_{i3m} = 2$ because the market shares of $j = 3$ are always 1. Because the time spent on $j = 0$ can be seen as a residual term ($t_0 = T - t_1 - t_2 - t_3$) in the model, I assume γ_{i0m} be a non-positive constant -0.001 ($\frac{1}{2} \times 0.001 = 0.0005$). I also assume $\gamma_{10} = \gamma_{20} = \gamma_{13} = \gamma_{23} = 0$ because those who use either app 1 or app 2 will always use the generic app and spend some time on offline activities.

3.2 Consumer Heterogeneity

Consumers have different preferences regarding apps. μ_{ijm} and γ_{ijm} are parameterized as

$$\mu_{i1m} = \mathbf{x}_m\boldsymbol{\beta}_1^{\mu} + \xi_{1m}^{\mu} + \varepsilon_{i1m} = \delta_{1m}^{\mu} + \varepsilon_{i1m} \tag{3}$$

$$\mu_{i2m} = \mathbf{x}_m\boldsymbol{\beta}_2^{\mu} + \xi_{2m}^{\mu} + \varepsilon_{i2m} = \delta_{2m}^{\mu} + \varepsilon_{i2m} \tag{4}$$

$$\gamma_{i1m} = \mathbf{x}_m\boldsymbol{\beta}_1^{\gamma} + \xi_{1m}^{\gamma} = \delta_{1m}^{\gamma} \tag{5}$$

$$\gamma_{i2m} = \mathbf{x}_m\boldsymbol{\beta}_2^{\gamma} + \xi_{2m}^{\gamma} = \delta_{2m}^{\gamma} \tag{6}$$

$$\gamma_{i3m} = \mathbf{x}_m\boldsymbol{\beta}_3^{\gamma} + \xi_{3m}^{\gamma} = \delta_{3m}^{\gamma} \tag{7}$$

where $\mathbf{x}_m$ is a set of exogenous variables. ξ^μ and ξ^γ capture app-market specific idiosyncratic error terms. For example, a weather shock to market m may increase the marginal utility of Uber but not that of Google Docs. ε_{i1m} and ε_{i2m} are individual error terms that are iid across individuals but can be correlated across apps. ε_{i1m} and ε_{i2m} capture unobserved individual characteristics that affect utilities derived from apps. For example, users with cars, compared to those without cars, derive higher utilities from Google Maps navigation apps and lower utilities from Uber. Therefore, the preference of Uber and the preference of Google Maps can be negatively correlated. As discussed in Train (2009), the variance of μ_{ijm} cannot be separately identified from the mean of μ_{ijm}. I assume $(\varepsilon_{i1m}, \varepsilon_{i2m})$ follows a normal distribution $N(\mathbf{0}, \mathbf{\Sigma})$, where

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

ρ captures the correlated preferences. As we add more controls in $\mathbf{x}_m$, ρ may be closer to zero. Given that we can never control for all relevant factors at the individual level, it is unwarranted to assume $\rho = 0$. γ_{12} and ρ together explains the common user between app 1 and app 2. An econometric challenge is to disentangle γ_{12} from ρ, which will be discussed in the next Section.

4 Estimation

I use GMM to match moments predicted by the model with the moments calculated from the data. The full set of parameters to be estimated are $\boldsymbol{\theta} = (\boldsymbol{\beta}_1^\mu, \boldsymbol{\beta}_2^\mu, \boldsymbol{\beta}_1^\gamma, \boldsymbol{\beta}_2^\gamma, \boldsymbol{\beta}_3^\gamma, \gamma_{12}, \rho)$. As in Nevo (1998), denote the linear parameters with $\boldsymbol{\theta}_1 = (\boldsymbol{\beta}_1^\mu, \boldsymbol{\beta}_2^\mu, \boldsymbol{\beta}_1^\gamma, \boldsymbol{\beta}_2^\gamma, \boldsymbol{\beta}_3^\gamma)$ as they enter the GMM function linearly and the nonlinear parameters with $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$. I observe a set of markets, which are defined to be demographic groups, for 13 weeks. Denote weeks with w.

For each market-week unit, I observe s^*_{1mw} and s^*_{2mw}, the share of users who uses app 1 and app 2, t^*_{1mw}, t^*_{2mw} and t^*_{3mw}, the average time spent on app 1, app 2, and all other apps in hours. For each week, I also observe the total number of common user between app 1 and app2, c^*_{12w}. The asterisks indict that they are observed variables. Hence the endogenous variables to be explained are $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ and c^*_{12w}. The exogenous variables $\mathbf{x}_{mw}$ are a set of week and market fixed effects. I follow Berry *et al.* (1995) and Nevo (1998) in denoting market-level parameters with $\boldsymbol{\delta} = (\delta^{\mu}_{1mw}, \delta^{\mu}_{2mw}, \delta^{\gamma}_{1mw}, \delta^{\gamma}_{2mw}, \delta^{\gamma}_{3mw})$. We have $\boldsymbol{\delta} = \mathbf{x}_{mw}\boldsymbol{\theta}_1 + \boldsymbol{\xi}$.

With those notations, the model can be succinctly summarized as

$$(\mathbf{y}^*_{mw}, c^*_{12w}) = f(\boldsymbol{\delta}, \gamma_{12}, \rho) = f(\mathbf{x}_{mw}\boldsymbol{\theta}_1 + \boldsymbol{\xi}, \gamma_{12}, \rho)$$

where $f(\cdot)$ is the nonlinear model described in the previous section and $\boldsymbol{\xi}$ is the stack of all market level error terms. Note that there are five components in $\boldsymbol{y}^*_{mw}$ and five components in $\boldsymbol{\delta}$. At the market level, we have six outcome variables but seven parameters. The model is not identified with the variables we have.

The econometric challenge is to identify γ_{12} from ρ. Intuitively, γ_{12} and ρ can both explain c^*_{12w}. If we observe many users use both NYTimes and WSJ, it could be that NYTimes and WSJ are complements as they offer different perspectives on the same events or that users have a strong demand of news in general. Complements and substitutes are defined with cross price elasticities of demand: if an exogenous increase in the price of product A leads to a decrease in the demand of product B, then they are complements; otherwise, they are substitutes. When there is no price, we can extend the definition: if an exogenous increase in the utility of product A leads to a decrease in the demand of product B, then they are complements; otherwise, they are substitutes. This definition is the basis of my identification strategy with updates as instruments. Updates of app 1 should change the *utility* of app 1 but not that of app 2. However, updates of app 1 can change the *usage* of app 2 through γ_{12}. Therefore, I use the following moments to identify nonlinear parameters γ_{12} and ρ,

$$E(c_{12}^* - c_{12}) = 0 \tag{8}$$

$$E(update_{2w} \cdot \xi_{1mw}^{\mu}) = 0 \tag{9}$$

$$E(update_{1w} \cdot \xi_{2mw}^{\mu}) = 0 \tag{10}$$

The moment in (8) matches the observed common user and the predicted common user given γ_{12} and ρ. The moments in (9) and (10) are based on the assumption that the update history of app 1 (app 2) should not enter the utility of app 2 (app 1) directly. More specifically, the update history is described by three variables: the cumulative numbers of small updates, medium updates, and major updates.[12] As shown in the subscript of $update_{1w}$, this update history is common to all users in China and co-linear with time fixed effects. To circumvent this, I create market-specific update history variables, which allows each market to respond to the updates differently. Therefore, there are at most $3 \times M$ moments implied by (9).

The identification of linear parameters $\boldsymbol{\theta}_1$ is straightforward and relies on the following moment conditions

$$E(\mathbf{x}_{mw}' \xi_{1mw}^{\mu}) = 0 \tag{11}$$

$$E(\mathbf{x}_{mw}' \xi_{2mw}^{\mu}) = 0 \tag{12}$$

$$E(\mathbf{x}_{mw}' \xi_{1mw}^{\gamma}) = 0 \tag{13}$$

$$E(\mathbf{x}_{mw}' \xi_{2mw}^{\gamma}) = 0 \tag{14}$$

$$E(\mathbf{x}_{mw}' \xi_{3mw}^{\gamma}) = 0 \tag{15}$$

Based on the above moments from (8) to (15), the GMM estimation is

[12] "Small", "medium", and "major"are defined by the digits of version numbers.

to minimize

$$\min_{\boldsymbol{\theta}} \boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi} + (c^*_{12} - c_{12})^2 \tag{16}$$

where $\boldsymbol{\xi}$ is the stack of all market level error terms and $\mathbf{z}_{mw} = (\mathbf{x}_{mw}, updata_{1w}, updata_{2w})$. I separate $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ from $(c^*_{12w} - c_{12w})^2$ to highlight the fact that $\boldsymbol{\theta}_1$ enters $\boldsymbol{\xi}'\mathbf{z}\mathbf{z}'\boldsymbol{\xi}$ linearly and does not enter $(c^*_{12w} - c_{12w})^2$ given $\boldsymbol{\delta}$. Therefore, we can limit the globe search to $\boldsymbol{\theta}_2 = (\gamma_{12}, \rho)$ as $\boldsymbol{\theta}_1$ is a linear function of $\boldsymbol{\delta}$.

The estimation follows Berry *et al.* (1995) with an inversion step and a global search step. I need to find the values of $\boldsymbol{\delta}$ that match the five observed market outcomes $\mathbf{y}^*_{mw} = (s^*_{1mw}, s^*_{2mw}, t^*_{1mw}, t^*_{2mw}, t^*_{3mw})$ given (γ_{12}, ρ). This is to solve the following system of nonlinear equations,

$$\mathbf{y}^*_{mw} = \mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho). \tag{17}$$

Note that each component in $\mathbf{y}_{mw}$ is monotonically increasing in the corresponding component in $\boldsymbol{\delta}$. For example, given $(\delta^{\mu}_{2mw}, \delta^{\gamma}_{1mw}, \delta^{\gamma}_{2mw}, \delta^{\gamma}_{3mw})$ and (γ_{12}, ρ), s_{1mw} is increasing in δ^{μ}_{1mw}. I solve (17) by iterating on $\boldsymbol{\delta}$ analogously to the contraction mapping used by Berry *et al.* (1995) and Gowrisankaran & Rysman (2012):

$$\boldsymbol{\delta}^{new} = \boldsymbol{\delta}^{old} + \boldsymbol{\phi} \cdot \{ln(\mathbf{y}^*_{mw}) - ln(\mathbf{y}_{mw}(\boldsymbol{\delta}^{old}, \gamma_{12}, \rho))\} \tag{18}$$

where $\boldsymbol{\phi}$ are five positive tuning parameter used in the iterations.

Despite the appealing features of quadratic utility functions, there is no analytical solution to quadratic optimization problems. Therefore, I use numerical integration to form expectations of $\mathbf{y}_{mw}$. Let N_s be the number of simulations used for integration. We have

$$\mathbf{y}_{mw}(\boldsymbol{\delta}, \gamma_{12}, \rho) = \frac{1}{N_s}\sum_{n=1}^{N_s} \mathbf{y}_{nmw}(\boldsymbol{\delta}, \gamma_{12}, \varepsilon_{n1mw}, \varepsilon_{n2mw}) \tag{19}$$

where $\mathbf{y}_{nmw}$ are the individual outcome for the nth draw of $(\varepsilon_1, \varepsilon_2)$. In practice, I use 1000 Halton draws in the integration.

To summarize, the estimation consists of the following steps:

1. For a pair of (γ_{12}, ρ), invert out $\boldsymbol{\delta}(\gamma_{12}, \rho)$ with the mapping described in (18).

2. Calculate $c_{12}(\boldsymbol{\delta}(\gamma_{12}, \rho), \gamma_{12}, \rho)$ and $\boldsymbol{\xi}(\boldsymbol{\delta}(\gamma_{12}, \rho), \mathbf{z})$. Based on them, calculate the value of GMM function in (16).

3. Find (γ_{12}, ρ) that minimizes the GMM value calculated in step 2.

5 Estimation Results

I estimate the model on three pairs of apps to see how the model performs in different situations. To reduce the computation burden, I aggregate market outcomes over provinces. Therefore, for each pair of apps, I have a panel of 11 markets[13] for 13 weeks.

5.1 Substitutes

The first pair of apps are Baidu Map (app 1) and Amap (app 2), two dominant players in the mobile map market in China. During the 13 weeks, the number of active users of Baidu Map increases from 90 million to 110 million and that of Amap increases from 75 million to 100 million. The number of common users between Baidu Map and Amap increases from 11 million to 18 million. The summary statistics of market level variables are in table 2.

[13]Gender and 5 age groups define 10 groups; and an “other”market to account for the difference between national usage and market level usage.

Table 2: Summary Statistics of Baidu Map and Amap

Variables	Mean	StdDev	Min	Max	Unit
$s^*_{BaiduMap}$	0.1463	0.032	0.0877	0.2414	-
s^*_{Amap}	0.1277	0.0286	0.0832	0.2164	-
$t^*_{BaiduMap}$	0.0367	0.0085	0.0195	0.0652	hour
t^*_{Amap}	0.0746	0.0192	0.0421	0.1564	hour
t^*_{3mw}	16.6915	3.1601	10.5834	21.8704	hour

Note: $s^*_{BaiduMap}$ (s^*_{Amap}) are the number of active users of Baidu Map (Amap) divided by the number of active users of android cellphones. $t^*_{BaiduMap}$ (t^*_{Amap}, t^*_{3mw}) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphones.
Data Source: iResearch.

The first three columns of table 3 presents the estimates of γ_{12} and ρ with different IVs. They have the same signs and similar magnitudes. I use column (3) as my main results. It is clear that Baidu Map and Amap are estimated to be substitutes ($\hat{\gamma}_{12} = -1.15$), which confirms our intuition. $\hat{\rho} = 0.7711$ suggests that Baidu map and Amap target the same group of users. Because they offer similar functions, users that need Baidu Map will also find Amap useful. For comparison, I also estimate γ_{12} with the assumption $\rho = 0$ in column (4) of table 3. In this specification, Baidu Map and Amap are estimated to be independent apps. It is also clear that γ_{12} and ρ "substitute" each other in explaining the common user data: from column (3) to column (4), as ρ decrease from 0.7711 to 0, γ_{12} increases from -1.15 to -0.02.

Table 3: Estimates for Baidu Map and Amap

	(1)	(2)	(3)	(4)
γ_{12}	-0.95	-1.15	-1.15	-0.02
	(0.0009)	(0.0003)	(0.0003)	(0.0015)
ρ	0.592	0.7711	0.7711	0
	(0.0001)	(0.0004)	(0.0004)	-
$update_1$ as IV	No	Yes	Yes	No
$update_2$ as IV	Yes	No	Yes	No

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
3, The weighting matrix is the identity matrix. However, I scale the sum of IV moments to match the scale of the common user moment.
Data Source: The author's calculations.

As with coefficients in logit or probit models, the economic meaning of the magnitude of γ_{12} is unclear. To understand the implication of γ_{12}, I shut down one of the two apps to see how the usage of the other map would change. In table 4, I simulate the market outcomes for females under 24 in China under two sets of (γ_{12}, ρ).[14] Columns (2) and (3) present counter-factuals for the baseline estimates and columns (4) and (5) present counter-factuals for the estimates in the last column of table 3 where we assume $\rho = 0$.

The two sets of simulated outcomes are very different. When $\gamma_{12} = -0.02$ and $\rho = 0$, shutting down one app has almost no effect on the other. In contrast, when $\gamma_{12} = -1.15$ and $\rho = 0.7711$, the market share of Baidu Map would increase by 3.34 percentage points if we shut down Amap and that of Amap would increase by 2 percentage points if we shut down Baidu Map. Let us focus on the case of shutting down Amap when $(\gamma_{12}, \rho) = (-1.15, 0.7711)$. The inversion process reveals that there are 1.4% users uses both Baidu Map and Amap. Therefore, there are 6.92% users using Amap but not Baidu Map. When Amap is shut down, 3.34% out of the 6.92% users turn to Baidu

[14]Specifically, I invert out $\boldsymbol{\delta}$ for the two pairs of (γ_{12}, ρ) and then set δ^{μ}_{2mw}, the mean marginal utilities of Amap, to a very small number, -20, and simulate the market outcomes.

Map. As Baidu Map and Amap are close competitors, we might expect that more of them should use Baidu Map. Note that there are other map apps consumers can use such as Tencent Map. Also note that the effect of shutting down Amap on Baidu Map is larger than the reverse. When Baidu Map is not available, the market share of Amap would only increase by 1.98 percentage points.

Table 4: Counter-factuals of Baidu Map and Amap

	Observed Outcomes	Baseline		Assume $\rho = 0$	
		No Baidu Map	No Amap	No Baidu Map	No Amap
	(1)	(2)	(3)	(4)	(5)
$s_{BaiduMap}$	0.1056	0	0.139	0	0.107
s_{Amap}	0.0832	0.103	0	0.084	0
$t_{BaiduMap}$	0.0224	0	0.0314	0	0.022
t_{Amap}	0.0421	0.0497	0	0.0426	0
t_3	15.559	15.559	15.56	15.559	15.559

Note:
1, This market is the female-under-24 group in China in the first week of 2017.
2, $s_{BaiduMap}$ (s_{Amap}) are the number of active users of Baidu Map (Amap) divided by the number of active users of android cellphones. $t_{BaiduMap}$ (t_{Amap}, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of Android cellphones.
Data Source: The author's calculations.

5.2 Complements

The second pair of apps are Baidu (app 1) and Baidu Map (app 2). As the names suggest, they are developed by the same company Baidu. The core function of the Baidu app is searching and news stream. We would expect search engines and maps, and hence Baidu and Baidu Map, are complements. For example, when users search for locations, the first results often direct users to map apps. During the 13 weeks, the number of active users of Baidu fluctuates around 177 million. The number of common users between Baidu and Baidu Map increases from 30 million to 37 million. The summary statistics of market level variables are in table 5. Note that there are slight differences between the summary statistics of s^*_{2mw} in table 5 and the summary statistics of s^*_{1mw} in table 2. This arises because the balanced panels of the two pairs are slightly different.

Table 5: Summary Statistics of Baidu and Baidu Map

Variables	Mean	StdDev	Min	Max	Unit
s^*_{Baidu}	0.2494	0.0464	0.1555	0.3321	-
$s^*_{BaiduMap}$	0.146	0.0323	0.0876	0.2414	-
t^*_{Baidu}	0.3086	0.0524	0.1475	0.4001	hour
$t^*_{BaiduMap}$	0.0366	0.0085	0.0195	0.0652	hour
t^*_{3mw}	16.4493	3.1086	10.4568	21.5881	hour

Note: s^*_{Baidu} ($s^*_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t^*_{Baidu} ($t^*_{BaiduMap}$, t^*_{3mw}) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphones.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in table 6. As before, I treat results in column (3) as the main results. The main results confirm our prior belief that Baidu and Baidu Map are complements. As in the previous subsection, I simulate what would happen if one of the app is shut down. The results are in table 7. It is clear that shutting down Baidu Map has negligible effect on Baidu. However, shutting down Baidu would reduce the market share of Baidu Map by 0.86 percentage point and the time spent on Baidu Map by more than 10% of the current level. It is therefore no surprise that the company Baidu treat the Baidu app as its core business. A caveat is that the discovery process of apps is not modeled in this paper. Cross-promotion between apps developed by the same company is a widely used marketing strategy.[15] Promoting Baidu Map with Baidu will lead to a persistent larger

[15]See The Ultimate Mobile Marketing Playbook by App Annie at https://www.appannie.com/en/insights/aso-app-store-optimization/ultimate-mobile-marketing-playbook/

number of common users if there are large switch costs.

Table 6: Estimates for Baidu and Baidu Map

	(1)	(2)	(3)
γ_{12}	-0.4613	0.296	0.1467
	(0.0006)	(0.0035)	(0.0005)
ρ	0.5522	-0.1642	-0.0448
	(0.0006)	(0.0001)	(0.0005)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
3, The weighting matrix is the identity matrix. However, I scale the sum of IV moments to match the scale of the common user moment.
Data Source: The author's calculations.

Table 7: Counter-factuals of Baidu and Baidu Map

	Observed Outcomes (1)	No Baidu (2)	No Baidu Map (3)
s_{Baidu}	0.2594	0	0.259
$s_{BaiduMap}$	0.1056	0.097	0
t_{Baidu}	0.3222	0	0.0320
$t_{BaiduMap}$	0.0225	0.0201	0
t_3	15.279	15.284	15.28

Note:
1, This market is the female-under-24 group in China in the first week of 2017.
2, s_{Baidu} ($s_{BaiduMap}$) is the number of active users of Baidu (Baidu Map) divided by the number of active users of android cellphones. t_{Baidu} ($t_{BaiduMap}$, t_3) are the total number of hours spent on Baidu Map (Amap, the generic app) divided by the number of active users of android cellphones.
Data Source: The author's calculations.

5.3 Independent Apps

The last pair of apps are WeChat (app 1) and Kwai (app 2). WeChat is the flagship app of Tencent first published in 2011. By the first quarter of 2017, the main functions includes instant messaging, social media (“Moments”), mobile payment (“WeChat Pay”), content distribution (“Subscriptions”), and app store (“mini program”). Kwai is a video-sharing app featuring short videos and live-streaming. Thanks to the recommendation algorithms, short video apps like Kwai and Tik Tok are often described as “a black hole of time”. In terms of functions, WeChat and Kwai seems to be independent or weak substitutes in the broad sense of social networking. However, WeChat and Kwai share a lot of common users. During the 13 weeks, the number of active users of WeChat fluctuates around 555 million, and that of Kwai increase from 78 million to 81 million. The number of common users between WeChat and Kwai is about 70 million. It is tempting to conjecture that they are complements based on the number of common users.

Table 8: Summary Statistics of WeChat and Kwai

Variables	Mean	StdDev	Min	Max	Unit
s^*_{WeChat}	0.8352	0.0532	0.731	0.9346	-
s^*_{Kwai}	0.1154	0.0148	0.0864	0.1451	-
t^*_{WeChat}	4.3685	0.5473	3.2078	5.5869	hour
t^*_{Kwai}	0.1896	0.017	0.1519	0.2246	hour
t^*_{3mw}	12.2633	2.6741	6.7176	16.3154	hour

Note: s^*_{WeChat} (s^*_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t^*_{WeChat} (t^*_{Kwai}, t^*_{3mw}) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphones.
Data Source: iResearch.

The estimates of (γ_{12}, ρ) are in table 9. As before, I treat results in column (3) as the main results. $\hat{\gamma}_{12} = -0.08$ refutes the conjecture that WeChat and Kwai are complements. The large number of common users is explained by the positive correlation between the preference for WeChat and that for Kwai ($\hat{\rho} = 0.42$). This implies that the wealth effect of one on the other may be large. Given that WeChat is the dominant player and Kwai is the entrant, it would be more interesting to focus on the effect of Kwai on WeChat. The counter-factuals with the main estimates are in table 10. Note that a representative consumer spend 0.2025 hour on Kwai, out of which about 0.06 comes would have been spent on WeChat which means about 30% of the time spent on Kwai comes from WeChat. The remaining 70% mostly comes from the offline activities.[16] 0.06 hour seems to be small. However, note that the denominator of this metric is the size of the market. For the 121 users who uses Kwai among the 1000 simulations, their time spent on WeChat increases on average by 0.486 hour if Kwai is shut down.

Table 9: Estimates for WeChat and Kwai

	(1)	(2)	(3)
γ_{12}	-0.14	-0.02	-0.08
	(0.0001)	(0.0001)	(0.0015)
ρ	0.76	0.18	0.42
	(0.0001)	(0.0001)	(0.0024)
$update_1$ as IV	No	Yes	Yes
$update_2$ as IV	Yes	No	Yes

Note:
1, Standard errors are in parentheses.
2, There are 143 market-week observations.
3, The weighting matrix is the identity matrix. However, I scale the sum of IV moments to match the scale of the common user moment.
Data Source: The author's calculations.

[16]Note that I assume away complementarity and correlated preference between app 1 (app 2) and the generic app.

Table 10: Counter-factuals of WeChat and Kwai

	Observed Outcomes (1)	No WeChat (2)	No Kwai (3)
s_{WeChat}	0.7725	0	0.779
s_{Kwai}	0.1221	0.222	0
t_{WeChat}	3.7391	0	3.7984
t_{Kwai}	0.2025	0.4257	0
t_3	11.6821	11.7173	11.6835

Note:
1, This market is the female-under-24 group in China in the first week of 2017.
2, s_{WeChat} (s_{Kwai}) is the number of active users of WeChat (Kwai) divided by the number of active users of android cellphones. t_{WeChat} (t_{Kwai}, t_3) is the total number of hours spent on WeChat (Kwai, the generic app) divided by the number of active users of android cellphones.
Data Source: The author's calculations.

With the model and the estimates, I can further quantify the welfare gain of this entrant using compensating variation. More specifically, I increase the total amount of time a Kwai user has to compensate for the loss of Kwai such that the maximized utilities are the same before and after shutting down Kwai. My calculation indicates that if Kwai is shut down, Kwai users should be compensated on average 47 minutes for the female-under-24 group in China in the first week of 2017. In other words, the welfare gain from Kwai is on average 47 minutes per Kwai user for a week.

6 Mergers

6.1 Price and Profit Functions

The motivation of this paper is to evaluate the effects of mergers of apps on firms and consumers. To do so, I need to specify the profit function and how money enters the utility function. For simplicity, I assume the utility function is linear in money:

$$\max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} U(\mathbf{t}) - \alpha(p_1 \mathbb{I}\{t_{i1m} > 0\} + p_2 \mathbb{I}\{t_{i2m} > 0\}) \tag{20}$$
$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168$$

where $U(\mathbf{t})$ is from equation (2). p_1 and p_2 are the weekly subscription prices of app 1 and app 2. Alternatively, if firms charge users based on usage, the utility function is

$$\max_{t_{i0m}, t_{i1m}, t_{i2m}, t_{i3m} \geq 0} U(\mathbf{t}) - \alpha(p_1 t_{i1m} + p_2 t_{i2m}) \tag{21}$$
$$s.t. \quad t_{i0m} + t_{i1m} + t_{i2m} + t_{i3m} = 168$$

Note that in both equation (20) and equation (21), the marginal value of time in terms of money is $\frac{1-0.001t_0^*}{\alpha}$. This ratio is estimated to be \$1.65 per hour for Taiwan in 2001(Shiaw, 2004). Therefore, I assume $\frac{1}{\alpha} = 8.5$ yuan.[17]

Corresponding to the two pricing schemes, the profit functions of a monopolist with an app $j = 1, 2$ in market m are

$$\Pi_{jm} = p_j \sum_i^{I_m} \mathbb{I}\{t_{ij} > 0\} + r_{jm} \sum_i^{I_m} t_{ij} - \Psi \tag{22}$$

and

$$\Pi_{jm} = p_j \sum_i^{I_m} t_{ij} + r_{jm} \sum_i^{I_m} t_{ij} - \Psi \tag{23}$$

where Ψ is the fixed cost and r_{jm} is the advertising revenue per hour. r_{jm} is not observed. However, we observe that $p_j = 0$ for the three pairs of apps. Therefore, we can back out the thresholds of r_j above which it is optimal to set $p_j = 0$. All three components in the profit function can change after a

[17]Considering the GDP per capita of Taiwan in 2001 and that of Mainland China in 2017, a comparable estimate for $\frac{1-0.001t_0^*}{\alpha}$ in our context is 7.2322 yuan. As the average of t_0 is 16.76 hours, $\frac{1-0.001t_0^*}{\alpha} \approx \frac{0.85}{\alpha} = 7.2322$. Therefore, $\frac{1}{\alpha} = 8.5$ yuan.

merger. If there are cost synergies, then Ψ would change. Prat & Valletti (2018) is concerned about the increased market power in the advertising market after a merger and r_{jm} would change in that case. In the following analysis, I assume $r_{jm} = \Psi = 0$ and focus on how p_j would change after a merger.

How can a merger have welfare effects if the prices in reality are zero before and after the merger? If we expand $U(\mathbf{t})$ in equation (21), it is clear that the "net marginal utility"is now $\mu_j - \alpha P_j$. Therefore a price change is equivalent to a change in μ_j for consumers. If consumers dislike advertisements, an increase in price is equivalent to an increase in advertisement intensity, which reduces μ_j. In equation (23), the "total revenue per hour"is $p_j + r_{jm}$. Hence, a price change is equivalent to a change in r_{jn} for firms. Therefore, my merger analysis has implications for the real world.

6.2 A Merger of Substitutes

The market outcomes before and after a merger of Baidu Map and Amap are in table 11. Column (1) displays the results when firms charge users weekly subscription prices. Column (2) displays the results when firms charge users usage-based prices. The results are intuitive for a pair of substitutes: in both cases, the prices and profits increase and consumer surplus decreases.

6.3 A Merger of Complements

The market outcomes before and after a merger of Baidu and Baidu Map are in table 12. Note that Baidu and Baidu Map are always owned by the same company. Therefore, this merger analysis should be seen as a divestment analysis. Column (1) displays the results when firms charge users weekly subscription prices. Column (2) displays the results when firms charge users usage-based prices.

The results in column (1) are strange: the market outcomes are the same

Table 11: Mergers of Baidu Map and Amap

	Subscription Pricing		Usage-Based Pricing	
	Baidu Map	Amap	Baidu Map	Amap
	(1)		(2)	
Pre-Merger				
Prices	1.405	1.70	3.188	3.059
Active User	20	30	58	49
Total Usage	10.316	27.913	10.984	19.535
Consumer Surplus	68.187		98.423	
Profits	79.099		94.774	
Total Surplus	147.286		193.197	
Post-Merger				
Prices	2.346	1.738	3.961	3.66
Active User	11	32	49	41
Total Usage	7.048	28.79	9.016	16.656
Consumer Surplus	57.237		79.869	
Profits	81.443		96.669	
Total Surplus	138.68		176.539	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

before and after the merger. A key reason is that there are strong asymmetry between the two apps. Baidu is much larger in terms of active users and usage than Baidu Map. More importantly, Baidu has a significant effect on Baidu Map in terms of active users and the reverse is not true.[18] The post-merger monopolist has no incentive to change the optimal price of Baidu Map as it has a negligible effect on Baidu. Because Baidu and WeChat have a larger user base, deviations from the optimal price of Baidu would lead to a large loss from Baidu and a small gain from Baidu Map. Another reason could be simulation error. Because I only simulate for 1000 users, the demand of an app in terms of active user is a step function of price and not responsive to a small change of price.

The results in column (2) are intuitive. After a merger of a pair of complements, we expect the monopolist to internalize the complementarity between Baidu and Baidu Map by lowering prices. The prices of Baidu and Baidu Map is lower after the merger. Consumers benefit more from the merger: consumer surplus increases by about 1.2% whereas profits increase by 0.15%.[19] Therefore, the results in column (2) suggest that developing complementary apps around a flagship app can be a profitable strategy for tech firms.

6.4 A Merger of Independent Apps

The market outcomes before and after a merger of WeChat and Kwai are in table 13. Column (1) displays the results when firms charge users weekly subscription prices. Column (2) displays the results when firms charge users usage-based prices. As in table 12, the results in column (1) do not change after the merger for the same reason in the previous subsection. Let us now focus on column (2).

Though WeChat and Kwai are not competing in functions, they are com-

[18]The same is true for WeChat and Kwai. See table 7 and table 10.

[19]The increase in profits should be a lower bound of the benefit of having the two apps in the same firm as I assume $r_{jm} = \Psi = 0$.

Table 12: Mergers of Baidu and Baidu Map

	Subscription Pricing		Usage-Based Pricing	
	Baidu	Baidu Map	Baidu	Baidu Map
	(1)		(2)	
Pre-Merger				
Prices	8.331	0.954	4.254	3.548
Active User	53	28	126	45
Total Usage	152.422	13.037	129.0	8.216
Consumer Surplus	570.417		506.959	
Profits	468.303		577.093	
Total Surplus	1038.72		1084.89	
Post-Merger				
Prices	8.331	0.954	4.215	3.45
Active User	53	28	128	45
Total Usage	152.422	13.037	130.212	8.447
Consumer Surplus	570.417		512.867	
Profits	468.303		577.961	
Total Surplus	1038.72		1090.831	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

peting for user time. After the merger, the prices of WeChat and Kwai increase by 1.5% and 39.7%. As a result, profits increase by 0.3% whereas consumers surplus decreases by 3.3%. Therefore, a seemingly innocuous merger of two apps with independent functions can hurt consumers due to the binding time constraint of users.[20] The size of wealth effect in this case is large enough to be considered by anti-trust authorities. After failed attempts to promote its own short-video app WeSee, Tencent invested $2 billion in Kwai in December 2019.[21]

6.5 Discussions

Some patterns in tables 11, 12, and 13 warrant further discussions.

The prices of Baidu Map in table 11 and table 12 are different. The two different prices are both correct because they are calculated with different assumptions. The price of Baidu Map in table 11 is the equilibrium price when the price of Baidu is fixed at 0. And the price of Baidu Map in table 12 is the equilibrium price when the price of Amap is fixed at 0. The price of Baidu and the price of Amap can both shift the demand of Baidu Map. Ideally, we should include all relevant factors in a demand estimation. However, this is not realistic due to data availability and computational feasibility.[22]

From tables 11, 12, and 13, We can also find that profits and total surplus are always higher with usage-based pricing. However, consumer surplus can be lower with usage-based pricing. This is because usage-based pricing is a price discrimination tool that enables firms to discriminate users based on usage. If a user has a higher μ_{ij}, then she uses app j more and pays more with usage-based pricing. In contrast, users pay the same price with subscription

[20]This is different from the anti-trust concern raised in Prat & Valletti (2018). In Prat & Valletti (2018), smaller advertiser find it more difficult to reach consumers after the merger due to the increased concentration of consumer attention.

[21]See https://www.scmp.com/tech/apps-social/article/3041747/tencent-said-invest-us2-billion-short-video-app-kuaishou

[22]See I.C in Gentzkow (2007) for a related discussion.

Table 13: Mergers of WeChat and Kwai

	Subscription Pricing		Usage-Based Pricing	
	WeChat	Kwai	WeChat	Kwai
	(1)		(2)	
Pre-Merger				
Prices	44.639	19.219	6.651	3.565
Active User	207	17	489	81
Total Usage	1877.542	77.605	1636.603	121.592
Consumer Surplus	8318.069		9258.009	
Profits	9630.798		11318.536	
Total Surplus	17948.866		20576.545	
Post-Merger				
Prices	44.639	19.219	6.751	4.981
Active User	207	17	486	56
Total Usage	1877.542	77.605	1620.697	82.357
Consumer Surplus	8318.069		8951.244	
Profits	9630.798		11352.952	
Total Surplus	17948.866		20304.196	

Note:
1, I simulate 1000 consumers with parameters estimated for the female-under-24 group in China in the first week of 2017.
2, All monetary values are in yuan.
3, The Total Usage variable is in hour.
Data Source: The author's calculations.

pricing. We can also find that subscription pricing suppresses the number of active users whereas usage-based pricing suppresses usage. Users with higher μ_{ij} will benefit from subscription pricing and users with lower μ_{ij} will benefit from usage-based pricing.

7 Conclusion

The rapid development of the mobile Internet industry and its profound effects on our society warrant better understanding of this industry. This paper develops a discrete-continuous model of consumer demand for apps that allows for complements as well as substitutes and incorporates a binding time constraint. I estimate the model with a weekly panel of app usage in the first quarter of 2017 in China. I separate complements from substitutes with the help of IVs. Updates of an app can change the utility of this app bu not those of other apps. However, updates of an app can change the *usage* of other apps through complementarity/substitutability.

I apply the model to three pairs of apps, each featuring an important aspect of the competition landscape in this industry (substitutes, complements, and independent apps). The estimation results recover a reasonable competition pattern. My estimation results show that allowing for correlated preferences and using both common user data and IV are crucial to getting reasonable competition patterns. I also find that within a pair of apps, the effect of one on another is often asymmetric: the larger app has a larger effect on the smaller app and the smaller app has a smaller or negligible effect on the larger app.

I then simulate mergers of the three pairs of apps. I find that both firms and consumers can benefit from a merger of complements because the post-merger monopolist can internalize complementarity between apps. Therefore, developing complementary apps around a flagship app can be a profitable strategy for tech firms. I also find that a seemingly innocuous merger of independent apps can hurt consumers due to the binding time constraint. The price of Kwai increase 39.7% after its merger with WeChat and consumers surplus decreases by 3.3%. Wealth effect due to a binding budget constraint is not unique to the mobile Internet industry. However, the size of the effect is large enough to be taken into consideration by anti-trust authorities. The simulation results also suggest that usage-based pricing can lead to higher profits and total surplus compared with subscription pricing as it enables firms to discriminate users based on usage.

The demand model in this paper incorporates four desirable features: discrete-continuous decisions, interactions between products, wealth effect, and estimation with instruments. The model can further incorporate other important features in the mobile Internet industry (for example, advertisement and two-sidedness) or be adapted to study consumer demand for other goods and services.

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