# An endogenous regime switching model of ordered choice with an application to federal funds rate target

Andrei Sirchenko*
Higher School of Economics, Moscow

September 11, 2017

**Abstract**

This paper introduces a class of ordered probit models with endogenous switching among $N$ latent regimes and possibly endogenous explanatory variables. The paper contributes to and bridges two strands of microeconometric literature. First, it extends endogenous switching regressions to models of ordered choice with $N$ unknown regimes. Second, it generalizes the existing zero-inflated ordered probit models to make them suitable for ordinal data that take on negative, zero and positive values and characterized by abundant and heterogeneous zero observations. From a macroeconomic perspective, it is the first attempt to implement regime switching and accommodate endogenous regressors in discrete-choice monetary policy rules. Recurring oscillating regime switches in the three regimes evolving endogenously in response to the state of economy are detected during a relatively stable policy period such as the Greenspan era. The Monte Carlo experiments and an application to the federal funds rate target demonstrate that ignoring endogeneity and regime-switching environment can lead to seriously distorted statistical inference. In the simulations, the new models perform well in small samples. In the application, they not only have better in-sample fit for the Greenspan era than the existing models but also forecast better out of sample for the entire Bernanke era, correctly predicting 91 percent of policy decisions.

Keywords. Ordered discrete responses, switching regression, endogenous switching, endogenous regressors, federal funds rate target, real-time data.

JEL classification. C34, C35, C36, E52.

---

*Andrei Sirchenko: asirchenko@hse.ru. I am thankful to Michael Beenstock, James D. Hamilton, Mark Harris, Peter R. Hansen, Helmut Lütkepohl, Massimiliano Marcellino, Francis Vella, the co-editor, three anonymous referees, seminars participants at EUI, U. of Bologna, New Economic School, U. of Melbourne, Augsburg U., Ca' Foscari U. and U. of Cyprus, and conference participants of the MOOD in Rome, ESEM in Málaga, ESAM in Hobart, IAAE in London, World Congress of Econometric Society in Montreal, and NBER-NSF in Vienna for useful discussions and comments. I gratefully acknowledge financial support from the Global Development Network under Grant R10-0221, the National Bank of Poland, the Basic Research Program of the Higher School of Economics, and the Economics Education and Research Consortium via the Zvi Griliches Excellence Award.

# 1 Introduction

This paper develops a class of endogenous regime switching models with $N$ latent regimes, a discrete ordered dependent variable and possibly endogenous explanatory variables. I illustrate the new models with an empirical application that predicts the next decision of the U.S. Federal Open Market Committee (FOMC) on the federal funds rate target (*target* henceforth), using the vintages of real-time data that do not include subsequent revisions and were truly available right before each FOMC target-setting decision (both scheduled and unscheduled). The paper makes the methodological and empirical contributions to the following aspects of monetary policy modeling.

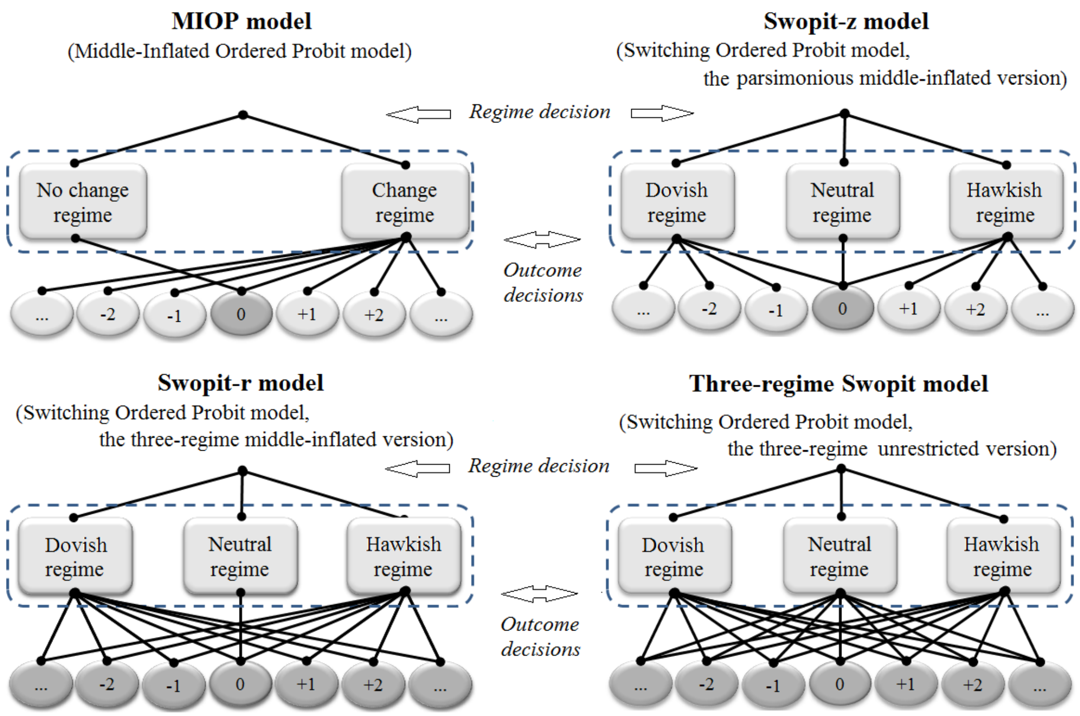
The U.S. Federal Reserve System (Fed) as many other central banks adjusts policy interest rates by discrete increments. This feature of modern monetary policy motivated the usage of nonlinear econometric techniques for a discrete ordered dependent variable (Vanderhart (2000), Hu and Philips (2004), Dolado, Maria-Dolores and Naveira (2005), Piazzesi (2005), Gerlach (2007, 2010), Kauppi (2012), Van den Hauwe, Paap and van Dijk (2013)). This literature employs single-equation models (with just a few exceptions of Hamilton and Jorda (2002) and Brooks, Harris, and Spencer (2012)) that cannot allow central bank actions to be generated by different policy regimes. But despite the increasing popularity of a regime-switching approach in macroeconomics, the existing regime-switching applications focus exclusively on models for continuous outcomes with mostly exogenous switching. Extensions to endogenous switching are typically limited to two regimes.[1] Finally, the literature on monetary policy rules largely ignores the problem of possible correlation between observed explanatory variables and unobserved monetary shocks.[2]

To accommodate the discreteness of dependent variable, the possibly endogenous explanatory variables and the unobserved endogenous switching of data-generating process (DGP), I develop a class of the Swopit (**sw**itching **o**rdered **p**rob**it**) models in which an ordinal dependent variable can be generated by $N$ latent regimes. The models consist of a latent regime-switching equation and $N$ latent regime-specific outcome equations. The bottom right panel of Figure 1 shows the decision tree of the Swopit model with three regimes (interpreted in the interest rate setting context as hawkish, neutral and dovish policy stances). The regime decision is endogenously driven by central bank response to observed and unobserved economic data. The outcome decisions, which are conditional on the regime, are driven by the observed data and unobservables that can

[1] See Section 2 for a brief review of the literature on regime switching in monetary policy modeling.
[2] See de Vries and Li (2014) for a discussion of the problem of endogeneity in modeling monetary policy.

be contemporaneously correlated with the unobservables in the regime equation (in this sense the regime switching is endogenous). All implicit decisions are represented by the ordered probit (OP) equations (McKelvey and Zavoina (1975)) with possibly endogenous (correlated with the unobservables) regressors.

Figure 1. The Swopit model is a generalization of two-part zero-inflated OP models and an ordered-choice $N$-regime extention of endogenous switching models




From a macroeconomic perspective, to the best of my knowledge, it is the first attempt to implement regime switching and accommodate endogenous regressors in the discrete-choice modeling of monetary policy rules. Such approach is shown to provide a high economic relevance for the analysis of monetary policy. The data favor the idea of slowly fluctuating latent policy regimes during the relatively stable period under the Greenspan chairmanship. The conventional single-equation and two-equation models lead to inferences that are markedly different from those in the new model, which correctly predicts 83 percent of FOMC decisions in the 11/1992–1/2006 period and also clearly outperforms the existing models in the out-of-sample forecasting (68 steps ahead). The new model estimated for the Greenspan era correctly predicts 91 percent of FOMC decisions in the Bernanke era. The empirical results and simulations make it clear that the endogeneity of explanatory variables does matter and can cause a severe bias in the estimates of discrete-choice monetary policy rules.

From a microeconomic perspective, the proposed Swopit models bridge two strands of literature for discrete ordinal responses: switching regression models and zero-inflated models. On the one hand, the Swopit model is an extension of endogenous switching models with unknown sample separation to multiple ordinal responses and multiple ordinal selection into three or more latent segments (regimes). The switching models with exogenous switching (when after controlling for observables the regime-switching (selection) decision is independent from the outcome decisions) lead to considerable computational simplicity but inconsistent estimates if the endogeneity assumption is invalid.[3] The idea of endogenous switching (when the unobservables in the regime-switching equation are correlated with the unobservables in the outcome equations) can be traced to Roy discussion of earnings distribution and self-selection between two professions (Roy (1951)). The first econometric implementations belong to Gronau (1974), Heckman (1974), Lewis (1974), Maddala and Nelson (1975) (see Maddala (1983) for an early survey). Numerous variants of these models are often concerned with sample selection[4] or treatment effects[5], and deal mainly with two regimes and continuous outcomes. Models with endogenous switching and ordinal outcomes have received considerably less attention and are mostly limited either to binary switching with two regimes (Hill (1990), Li and Tobias (2008), and Gregory (2015) deal with observed regimes, while Greene, Harris, Hollingsworth and Maitra (2008) consider unobserved regimes), or to endogenous treatment models, which include a single outcome equation with endogenous dummies representing an observed choice of treatment (Geweke, Gowrisankaran and Town (2003) consider unordered multiple treatment and binary outcomes, Miranda and Rabe-Hesketh (2006) consider binary treatment and multiple ordered outcomes, and Munkin and Trivedi (2008) consider unordered multiple treatment and multiple ordered outcomes). Models involving endogenous ordinal selection, ordinal outcomes and more than two regimes with a distinct outcome equation in each regime are scarce and

[3]The exogenous switching models for ordinal outcomes with two or more unobserved regimes (latent segments) can be found in Desantis, Houseman, Coull, Stemmer-Rachamimov, and Betensky (2008) with adjacent categories logit outcome equations, and in Eluru, Bagheri, Miranda-Moreno and Fu (2012) with ordered logit outcome equations.

[4]In the sample selection models, which are similar in structure to the regime switching models, the outcomes from one regime are never observed (not selected to the sample) and the regimes are known, whereas in the regime switching modes the outcomes can be generated by any regime but the regimes may not be known. The switching model can be viewed as two or more selection models merged together.

[5]The typical treatment-effects (or program-evaluation) models contain a selection-into-treatment equation and a single outcome equation with a set of endogenous dummy variables (treatment indicators), so the rest of parameters in the outcome equation is the same for both treated and untreated individuals, which are observed, in contrast to the switching regression models, where the regimes may not be obseved, and the outcomes in each regime are handled separately with the regime-specific values of all parameters in outcome equations.

deal only with the observed regimes (the choices of multiple ordinal treatment) (Chib and Hamilton (2000) consider binary outcomes, Carneiro, Hansen and Heckman (2003) consider multiple ordinal and categorical outcomes).

On the other hand, under some restrictions on the parameters, the Swopit model can be described as a three-part zero-inflated OP model. Figure 1 shows the decision trees of two restricted zero-inflated versions of the Swopit model with three regimes: the Swopit-r version, in which the neutral regime can generate only zeros (see the bottom left panel), and the more parsimonious Swopit-z version, in which the zeros can be generated by any regime while the cuts/hikes are only generated by the dovish/hawkish regime, respectively (see the upper right panel). The two-part zero-inflated OP models, developed to address the unobserved heterogeneity of zeros, include the zero-inflated OP (ZIOP) model of Harris and Zhao (2007) for a non-negative ordinal outcome, and the middle-inflated OP (MIOP) model of Brooks, Harris, and Spencer (2012) and Bagozzi and Mukherjee (2012) for an ordinal outcome, which ranges from negative to positive responses with an abundant zero response situated in the middle of the choice spectrum (see the upper left panel). In these models, the regime decision and the outcome decision are represented by binary probit and OP equations, respectively, and the two latent regimes overlap at the zero outcome, generating "inflated" zeros.

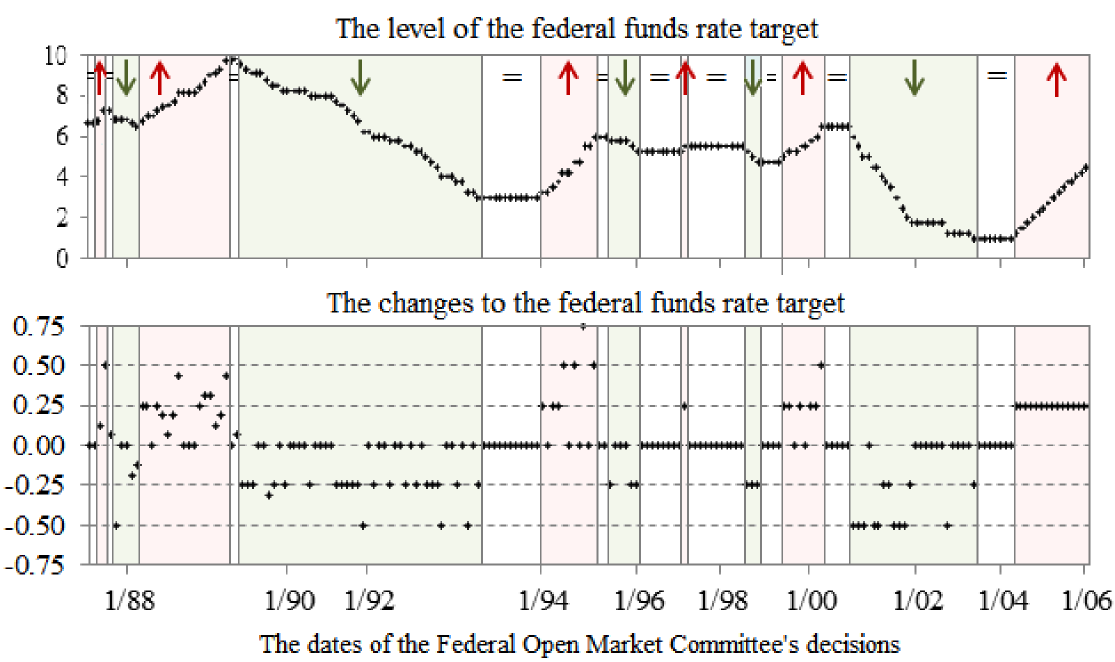
The three-regime Swopit-z model is a natural extension of the two-regime MIOP model.[6] A trichotomous regime decision (no increase, no change, or no decrease) seems to be more realistic than a binary decision (change or no change) if applied to ordinal data that assume negative, zero and positive values. The policymakers, who consider adjusting the rate, have already decided in which direction they are going to change it. Furthermore, the decision to raise or cut the rate may be driven by asymmetric reaction to economic data. Combining these two distinct decisions into one category in the MIOP model may seriously distort the inference. The OP, ZIOP, MIOP and Swopit-z models are actually special cases of the Swopit-r model, which in turn is a special case of the three-regime Swopit model. The empirical rejections of the OP and MIOP models in favor of the Swopit-z model provide a compelling empirical evidence of the regime switching and asymmetric effects of explanatory variables on the decisions to cut or hike the target.

The rationale behind the three-regime zero-inflated approach can be motivated by the stylized fact that for many central banks the majority of policy decisions are status quo decisions (zeros), which can be made in three different circumstances, namely: in contractionary periods; in policy maintaining periods between rate reversals; and in eas-

[6] If inflated response is located at the *end* of the ordered scale, the Swopit-z model reduces to the ZIOP model of Harris and Zhao (2007).

ing periods (see Figure 2). Many of the zeros, situated between rate hikes during policy contraction, are likely to be driven by different economic conditions compared with many of those that are situated between cuts during policy easing. Many of the zeros, clustered between rate reversals during maintaining periods, are also likely to differ from the zeros in easing or contractionary periods. In addition, the rate cuts and hikes can also be generated by distinct processes. According to the Swopit-z model, the average probability of neutral policy stance in the Greenspan era is 0.35, whereas the observed frequency of status quo decisions is 0.54. Only two thirds of zeros are the "neutral" zeros generated by the neutral policy reaction to economic conditions; the remaining zeros originate under the dovish or hawkish policy regime. The contractionary/maintaining/easing periods are dominated by, respectively, the hawkish/neutral/dovish regimes; however, in the maintaining periods one third of status quo outcomes are either the "dovish" or "hawkish" zeros. One third of all outcomes in the dovish and hawkish regimes are zeros — the outcome decisions tend to smooth the target rate by weakening the up- and downward policy inclinations.

Figure 2. Federal funds rate target can remains unchanged in different circumstances: during the easing, maintaining or contractionary periods




Notes. ↓/=/↑ denote the easing/maintaining/contractionary periods. The easing/contractionary periods are periods when the rate only moves down/up, from the first to the last sequential unidirectional rate change (decrease/increase, respectively). The maintaining period is a status quo period between the rate reversals.

The next section makes a brief review of the literature on regime switching in monetary policy modeling. Section 3 describes the econometric framework of the new models, which are estimated via maximum likelihood (ML). A consistent estimator in the presence of endogenous regressors is implemented using a control function approach, and allows for a simple test of the null hypothesis of endogeneity. The problem of the comparisons of the OP, ZIOP, MIOP and Swopit models is also discussed. The Monte Carlo experiments reported in Section 4 demonstrate a good performance of the Swopit models in small samples (both without and with endogenous regressors) and its superiority with respect to the OP and MIOP models, which deliver asymptotically biased estimates if the underlying DGP is characterized by three switching regimes. The simulations demonstrate that ignoring the endogeneity of regime switching and explanatory variables leads to asymptotically biased estimates. The new and existing models are applied in Section 5 to predict the next FOMC decision on the target. Section 6 concludes. Four appendices with supporting materials and replication files are available online.

## 2 Regime switching in monetary policy modeling

The regime-switching applications in monetary economics focus exclusively on models for a continuous dependent variable. Policy regime change is often modelled as a permanent, non-stochastic switch; empirical studies divide data samples into regime-specific subperiods (Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Schorfheide (2005), Boivin and Giannoni (2006), Coibion and Gorodnichenko (2011)) or employ a gradual regime switching approach with smoothed transitions between regimes (Mankiw, Miron and Weil (1987), Gerlach (2010), Gerlach and Lewis (2014)). However, treating regime changes as breaks rather than as a slowly fluctuating stochastic process is largely a matter of convenience and even logically inconsistent, as Cooley, Leroy and Raymon (1984) argue.

The significant attempts in modeling short-lived oscillating switches in monetary policy are made, among others, by Sims and Zha (2006) and Bikbov and Chernov (2013) in the structural vector autoregression context, as well as by Davig and Leeper (2007), Bianchi (2013) and Baele, Bekaert, Cho, Inghelbrecht and Moreno (2015), Foerster (2016) in the dynamic stochastic general equilibrium framework. These and other studies demonstrate that dividing the U.S. post-World War II sample into distinct regimes can distort qualitative and quantitative inferences; the U.S. monetary policy changes are better modeled as stochastic and repeated fluctuations between two or more regimes; and from a theoretical point of view, regime switching has the expectation formation effects and crucial consequencies for the uniqueness and existence of the economy's equlibrium and the distributions of macroeconomic outcomes. Short-term market interest rates are also often modelled and forecasted using the regime switching approaches (Hamilton (1988), Garcia and Perron (1996) and Ang and Bekaert (2002)).

Building on the seminal work of Hamilton (1989), recurring policy regimes are typically modelled as the unobserved states generated by a stochastic exogenous Markov-chain process, which is independent of the endogenous economic variables and has constant probabilities of transition from one regime to another. This assumption is realistic in a wide range of contexts but highly unlikely in monetary policy applications. It is natural to assume that central banks react systematically to changes in the economic conditions; thus, regime switches are endogenous to the state of the economy and have time-varying transition probabilities. Models, in which the time-varying probabilities of regime changes are driven by the observed exogenous variables and past values of dependent variable, are considered by Diebold, Lee, and Weinbach (1994), Filardo (1994), Chib and Dueker (2004), and Bazzi, Blasques, Koopman and Lucas (2017) in the framework of univariate linear regression, and by Davig and Leeper (2008) and Barthélemy and Marx (2017) in the dynamic stochastic general equilibrium framework framework. Kim, Piger, and Startz (2008) and Chang, Choi and Park (2017) develop the endogenous Markov regime-switching models, in which the regression disturbance is correlated with the disturbance to the latent state variable controlling the regime. The existing extensions to endogenous switching with time-varying probabilities are typically limited to two regimes.[7] All the above-mentioned models assume continuous outcomes.

# 3 Model

This section describes the econometric framework of the Swopit models in the most general form with $N$ regimes and endogenous switching as well as in the restricted zero-inflated forms with three regimes. I also present the Swopit estimator that allows for endogenous explanatory variables. The relationship and discrimination among the OP, MIOP and Swopit models are also discussed.

## 3.1 $N$-regime Swopit model with endogenous switching

Let $t$ ($t = 1, 2, ..., T$) be one of the available $T$ observations. Let $y_t$ be an observed dependent variable that can take on a finite number $J$ of ordinal values coded by index $j$ ($j = 1, 2, ..., J$). The observed outcome $y_t$ can be generated in any of $N$ ($N \geq 2$) states,

[7] In a recent paper, Hwu, Kim and Piger (2017) propose an $N$-state Markov-switching regression model in which the state indicator variable is correlated with the regression disturbance term.

coded by index $s$ ($s = 1, 2, ..., N$) and interpreted as latent regimes in the time-series context or as latent segments of population in the cross-section context. The realized regimes $r_t$ are unobserved and of an ordinal nature. The regime switching decision is determined by the continuous latent variable $r_t^*$, endogenously driven in response to the observed data and unobservables according to the OP regime equation. The correspondence between $r_t^*$ and $r_t$ is determined by unobserved thresholds in the usual ordered-response fashion according to a matching rule. For each $t$, only one out of $N$ potential realizations of $y_t$ is observed. The observed outcome $y_t$ is determined (also in the usual ordered-response fashion) by one out of $N$ unobserved continuous latent variables $y_{s,t}^*$, representing the potential outcomes in each regime, and driven in response to the observed data and unobservables according to $N$ outcome equations.

To summarize, the $N$-regime Swopit model can be described by the following system

$$
\begin{array}{ll}
r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t,\ t = 1, 2, ..., T & \text{(regime equation)}, \\
r_t|\mathbf{x}_t = s \text{ if } \mu_{s-1} < r_t^*|\mathbf{x}_t \leq \mu_s,\ s = 1, 2, ..., N & \text{(regime matching rule)}, \\
y_{s,t}^* = \mathbf{x}_{s,t}'\beta_s + \varepsilon_{s,t} & \text{(outcome equations)}, \\
y_t|(\mathbf{x}_{s,t}, r_t = s) = j \text{ if } \alpha_{s,j-1} < y_{s,t}^*|\mathbf{x}_{s,t} \leq \alpha_{s,j} & \text{(outcome matching rules)}, \\
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} Normal\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho_s \\ \rho_s & 1 \end{bmatrix} \right) & \begin{array}{l}\text{(interdependence between the} \\ \text{regime and outcome decisions)},\end{array}
\end{array}
\tag{1}
$$

where $\mathbf{x}_t$ and $\mathbf{x}_{s,t}$ are the $t^{\text{th}}$ rows of the observed data matrices $\mathbf{X}$ and $\mathbf{X}_s$, respectively; $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_s$ are the vectors of unknown slope parameters; $\varepsilon_t$ and $\varepsilon_{s,t}$ are the $t^{\text{th}}$ rows of the independently and identically distributed (iid) across $t$ disturbance terms $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$; $\varepsilon_t$ is independent of $\varepsilon_{s,t}$ at leads and lags: $E(\varepsilon_t \varepsilon_{s,t+\tau}) = 0$ for $\forall \tau \neq 0$; $-\infty \equiv \mu_0 \leq \mu_1 \leq ... \leq \mu_N \equiv \infty$ and $-\infty \equiv \alpha_{s,0} \leq \alpha_{s,1} \leq ... \leq \alpha_{s,J} \equiv \infty$ are the unknown threshold parameters; the joint probability density between $\varepsilon_t$ and each $\varepsilon_{s,t}$ is standardized bivariate normal with cumulative distribution function (CDF) $\Phi_2(\varepsilon_t; \varepsilon_{s,t}; \rho_s)$.

The probabilities of the outcome $j$ are given by

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}) \\
&= \sum_{s=1}^{N} \Pr(\mu_{s-1} < r_t^*|\mathbf{x}_t \leq \mu_s \text{ and } \alpha_{s,j-1} < y_{s,t}^*|\mathbf{x}_{s,t} \leq \alpha_{s,j}) \\
&= \sum_{s=1}^{N} \Pr(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} < \varepsilon_t \leq \mu_s - \mathbf{x}_t'\boldsymbol{\beta} \text{ and } \alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s < \varepsilon_{s,t} \leq \alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) \\
&= \sum_{s=1}^{N} [\Phi_2(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s) - \Phi_2(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s)
\end{aligned}
\tag{2}
$$

$$-\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)+\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j-1}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)].$$

More specifically, these probabilities can be computed as

$$\begin{aligned}
&\Pr(y_t=1|\mathbf{x}_t,\mathbf{x}_{1,t},\mathbf{x}_{2,t},...,\mathbf{x}_{N,t})\\
&=\Phi_2(\mu_1-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{1,1}-\mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1)+\sum_{s=2}^{N-1}[\Phi_2(\mu_s-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,1}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)\\
&-\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,1}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)]+\Phi_2(-\mu_{N-1}+\mathbf{x}_t'\boldsymbol{\beta};\alpha_{N,1}-\mathbf{x}_{N,t}'\boldsymbol{\beta}_N;-\rho_N);
\end{aligned}$$

$$\begin{aligned}
&\Pr(y_t=j|\mathbf{x}_t,\mathbf{x}_{1,t},\mathbf{x}_{2,t},...,\mathbf{x}_{N,t})\\
&=\Phi_2(\mu_1-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{1,j}-\mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1)-\Phi_2(\mu_1-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{1,j-1}-\mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1)\\
&+\sum_{s=2}^{N-1}[\Phi_2(\mu_s-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)-\Phi_2(\mu_s-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j-1}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)\\
&-\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)+\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};\alpha_{s,j-1}-\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;\rho_s)]\\
&+\Phi_2(-\mu_{N-1}+\mathbf{x}_t'\boldsymbol{\beta};\alpha_{N,j}-\mathbf{x}_{N,t}'\boldsymbol{\beta}_N;-\rho_N)\\
&-\Phi_2(-\mu_{N-1}+\mathbf{x}_t'\boldsymbol{\beta};\alpha_{N,j-1}-\mathbf{x}_{N,t}'\boldsymbol{\beta}_N;-\rho_N)\text{ for }2\leq j\leq J-1;
\end{aligned}$$

$$\begin{aligned}
&\Pr(y_t=J|\mathbf{x}_t,\mathbf{x}_{1,t},\mathbf{x}_{2,t},...,\mathbf{x}_{N,t})\\
&=\Phi_2(\mu_1-\mathbf{x}_t'\boldsymbol{\beta};-\alpha_{1,J-1}+\mathbf{x}_{1,t}'\boldsymbol{\beta}_1;-\rho_1)+\sum_{s=2}^{N-1}[\Phi_2(\mu_s-\mathbf{x}_t'\boldsymbol{\beta};-\alpha_{s,J-1}+\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;-\rho_s)\\
&-\Phi_2(\mu_{s-1}-\mathbf{x}_t'\boldsymbol{\beta};-\alpha_{s,J-1}+\mathbf{x}_{s,t}'\boldsymbol{\beta}_s;-\rho_s)]+\Phi_2(-\mu_{N-1}+\mathbf{x}_t'\boldsymbol{\beta};-\alpha_{N,J-1}+\mathbf{x}_{N,t}'\boldsymbol{\beta}_N;\rho_N).
\end{aligned}$$

If $\rho_s=0,\forall s$ then $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$ are mutually independent, and regime switching is exogenous. The above probabilities in the case of exogenous switching are shown in Appendix A.

The intercept components of $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_s$ are identified only jointly with the corresponding threshold parameters $\boldsymbol{\mu}$ and $\boldsymbol{\alpha}_s$, and the variances of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$. To identify the model, the intercept components of $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_s$ are fixed to zero, and the variances of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$ are fixed to one. The probabilities in (2), however, are absolutely estimable and invariant to the identifying assumptions. They can be estimated using an ML estimator of the vector of the parameters $\boldsymbol{\theta}=(\boldsymbol{\mu}',\boldsymbol{\beta}',\boldsymbol{\alpha}_1',\boldsymbol{\beta}_1',...,\boldsymbol{\alpha}_N',\boldsymbol{\beta}_N')'$ that solves

$$\max_{\boldsymbol{\theta}\epsilon\boldsymbol{\Theta}}\sum_{t=1}^{T}\sum_{j=1}^{J}h_{t,j}\ln[\Pr(y_t=j|\mathbf{x}_t,\mathbf{x}_{1,t},\mathbf{x}_{2,t},...,\mathbf{x}_{N,t},\boldsymbol{\theta})],\tag{3}$$

where $h_{tj}$ is an indicator function such that $h_{t,j}=1$ if $y_t=j$ and $h_{tj}=0$ otherwise; and $\boldsymbol{\Theta}$ is a parameter space. In general, the parameters in $\boldsymbol{\theta}$ are separately identified (up to scale and location) by construction via the functional form due to the nonlinearity of the OP equations (Wilde 2000). There is no need for exclusion restrictions on the specification of covariates in the latent equations to avoid collinearity problems. In practice, however, the collinearity problems might still exist if many observations lie within the middle quasi-linear range of normal CDF. Thus, if $\mathbf{X}$ and $\mathbf{X}_s$ are identical or have many variables in common, the simultaneous estimation of $N+1$ OP equations may be subject to the collinearity and weak identification (the common symptoms of this problem are large standard errors and close-to-singular Hessian matrix). In this case, the exclusion restrictions (ensuring that $\mathbf{X}$ and $\mathbf{X}_s$ are not identical) may be necessary. The starting values for $\boldsymbol{\theta}$ can be obtained by, for example, using the independent OP estimations of each latent equation. The asymptotic standard errors of $\widehat{\boldsymbol{\theta}}$ can be estimated from the Hessian matrix.

## 3.2 Restricted zero-inflated versions of the Swopit model

The estimation of the Swopit model even with only three regimes can be cumbersome and infeasible if the sample size is not large enough as is often the case in monetary policy modeling. The Swopit estimator can suffer from problems with the invertibility of the Hessian matrix, because in small samples the likelihood function at the maximum can be flat for an infinitely wide range of parameters' values. I present two less parameterized restricted versions of the three-regime Swopit model: the Swopit-r and Swopit-z models (see Figure 1). Although these models may seem restrictive, they allow to obtain parsimony in modeling and have clear economic interpretation in the context of policy interest rates. The three regimes ($s = 1, 2, 3$) can be interpreted as monetary policy stances (dovish, neutral, or hawkish, respectively). The observed outcome $y_t$ — the change to the central bank policy rate made at a monetary policy meeting $t$ (usually, 6-12 times per year) — typically takes on a finite number of positive, zero, and negative ordinal values with 0.25-percent increment. The most popular outcome — a zero (no change to the rate) — is typically observed for more than a half of central bank decisions. The zero-inflated Swopit-r and Swopit-z models can accommodate the possible unobserved heterogeneity of abundant zeros by allowing them to be generated by three distinct processes. The observed outcomes $y_t$ are coded as $j = 1, 2, ..., J$, among which the potentially heterogeneous ("inflated") and often abundant response is coded as $q$.

### 3.2.1 The Swopit-r model

The three-regime Swopit model reduces to the Swopit-r model if the only possible outcome in the neutral regime ($s = 2$) is $q$. There is no need for an outcome equation in the neutral regime. To describe the Swopit-r model, modify the system (1) as follows

$$
\begin{array}{ll}
r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t,\ t = 1, 2, ..., T & \text{(regime equation),} \\
r_t|\mathbf{x}_t = s \text{ if } \mu_{s-1} < r_t^*|\mathbf{x}_t \le \mu_s,\ s = 1, 2, 3 & \text{(regime matching rule),} \\
y_{s,t}^* = \mathbf{x}_{s,t}'\boldsymbol{\beta}_s + \varepsilon_{s,t},\ s = 1, 3 & \text{(outcome equations),} \\
y_t|(\mathbf{x}_{s,t}, r_t = s) = j \text{ if } \alpha_{s,j-1} < y_{s,t}^*|\mathbf{x}_{s,t} \le \alpha_{s,j},\ s = 1, 3 & \text{(outcome} \\
y_t|(r_t = 2) = q & \text{matching rules),} \\
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} Normal\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho_s \\ \rho_s & 1 \end{bmatrix} \right),\ s = 1, 3 & \begin{array}{l}\text{(interdependence between the} \\ \text{regime and outcome decisions).}\end{array}
\end{array}
$$

The probabilities of the outcome $j$ in the Swopit-r model are given by

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = I_{j=q} \Pr(\mu_1 < r_t^*|\mathbf{x}_t \le \mu_2) \\
&+ \sum_{s=1,3} \Pr(\mu_{s-1} < r_t^*|\mathbf{x}_t \le \mu_s \text{ and } \alpha_{s,j-1} < y_{s,t}^*|\mathbf{x}_{s,t} \le \alpha_{s,j}) \\
&= \sum_{s=1,3} \Pr(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} < \varepsilon_t \le \mu_s - \mathbf{x}_t'\boldsymbol{\beta} \text{ and } \alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s < \varepsilon_{s,t} \le \alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) \\
&+ I_{j=q} \Pr(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta} < \varepsilon_t \le \mu_2 - \mathbf{z}_t'\boldsymbol{\gamma}) = \sum_{s=1,3} [\Phi_2(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s) \\
&- \Phi_2(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s) - \Phi_2(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s) \\
&+ \Phi_2(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s; \rho_s)] + I_{j=q} \left( \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}) \right),
\end{aligned} \tag{4}
$$

where $I_{j=q}$ is an indicator function such that $I_{j=q} = 1$ if $j = q$, and $I_{j=q} = 0$ if $j \neq q$.

These probabilities can be computed as

$$
\begin{aligned}
&\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; \rho_1) \\
&+ \Phi_2(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; \rho_3) + \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3);
\end{aligned}
$$

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) \\
&= \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; \rho_1) - \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; \rho_1) \\
&+ I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] + \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3) \\
&- \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3) \text{ for } 2 \le j \le J - 1;
\end{aligned}
$$

$$
\begin{aligned}
&\Pr(y_t = J|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; -\alpha_{1,J-1} + \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; -\rho_1) \\
&+ \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; -\alpha_{3,J-1} + \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; \rho_3).
\end{aligned}
$$

The above probabilities in the case of exogenous switching (when $\rho_s = 0, \forall s$) are shown in Appendix A.

### 3.2.2 The Swopit-z model

The three-regime Swopit model reduces to the Swopit-z model if (i) the possible outcomes in the dovish regime ($s = 1$) are $j \leq q$; (ii) the only possible outcome in the neutral regime ($s = 2$) is $j = q$; (iii) the possible outcomes in the hawkish regime ($s = 3$) are $j \geq q$. There is no need for an outcome equation in the neutral regime. To describe the Swopit-z model, modify the system (1) as follows

$$
\begin{array}{ll}
r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \varepsilon_t & \text{(regime equation),} \\
r_t|\mathbf{x}_t = s \text{ if } \mu_{s-1} < r_t^*|\mathbf{x}_t \leq \mu_s,\ s = 1, 2, 3 & \text{(regime matching rule),} \\
y_{s,t}^* = \mathbf{x}_{s,t}'\boldsymbol{\beta}_s + \varepsilon_{s,t},\ s = 1, 3 & \text{(outcome equations),} \\
y_t|(\mathbf{x}_{1t}, r_t = 1) = j \text{ if } \alpha_{1,j-1} < y_{1,t}^*|\mathbf{x}_{1,t} \leq \alpha_{1,j},\ j \leq q & \text{(outcome} \\
y_t|(r_t = 2) = q & \text{matching} \\
y_t|(\mathbf{x}_{3,t}, r_t = 3) = j \text{ if } \alpha_{3,j-1} < y_{3,t}^*|\mathbf{x}_{3,t} \leq \alpha_{3,j},\ j \geq q & \text{rules),} \\
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} Normal\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho_s \\ \rho_s & 1 \end{bmatrix} \right),\ s = 1, 3 & \begin{array}{l}\text{(interdependence} \\ \text{between the regime} \\ \text{and outcome decisions).}\end{array}
\end{array}
$$

The probabilities of the outcome $j$ in the Swopit-z model are given by

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = I_{j\leq q} \Pr(r_t^*|\mathbf{x}_t \leq \mu_1 \text{ and } \alpha_{1,j-1} < y_{1,t}^*|\mathbf{x}_{1,t} \leq \alpha_{1,j}) \\
&+I_{j=q} \Pr(\mu_1 < r_t^*|\mathbf{x}_t \leq \mu_2) + I_{j\geq q} \Pr(\mu_2 < r_t^*|\mathbf{x}_t \text{ and } \alpha_{3,j-1} < y_{3,t}^*|\mathbf{x}_{3,t} \leq \alpha_{3,j}) \\
&= I_{j\leq q} \Pr(\varepsilon_t \leq \mu_1 - \mathbf{x}_t'\boldsymbol{\beta} \text{ and } \alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1 < \varepsilon_{1,t} \leq \alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) \\
&+I_{j=q} \Pr(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta} < \varepsilon_t \leq \mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) \\
&+I_{j\geq q} \Pr(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta} < \varepsilon_t \text{ and } \alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3 < \varepsilon_{3,t} \leq \alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3) \\
&= I_{j\leq q}[\Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1) - \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1)] \\
&+I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] \\
&+I_{j\geq q}[\Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3) - \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3)],
\end{aligned}
\tag{5}
$$

where $I_{j\leq q}$ is an indicator function such that $I_{j\leq q} = 1$ if $j \leq q$, and $I_{j\leq q} = 0$ if $j > q$ (analogously for $I_{j=q}$ and $I_{j\leq q}$).

These probabilities can be computed as

$$
\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; \rho_1);
$$

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) \\
&= I_{j\leq q}[\Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1) - \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1;\rho_1)] \\
&+I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] + I_{j\geq q}[\Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3) \\
&-\Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3)] \text{ for } 1 < j < J;
\end{aligned}
$$

$$\Pr(y_t = J|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; -\alpha_{3,J-1} + \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; \rho_3).$$

The above probabilities in the case of exogenous switching (when $\rho_s = 0, \forall s$) are shown in Appendix A.

### 3.3 Allowing for endogenous explanatory variables

The Swopit model can be extended by relaxing the assumption that explanatory variables in $\mathbf{X}$ and $\mathbf{X}_s$ are not correlated with corresponding error terms $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$. Accounting for endogeneity in nonlinear models, even relatively simple single-equation ones, is more difficult to implement than in linear models. The simple mimicking of two-stage least squares estimation of linear models (i.e. inserting fitted values from the reduced form in place of endogenous regressors in the structural equation) does not generally work for nonlinear models and often makes the endogeneity bias worse (Bhattacharya, Goldman, and McCaffrey (2006), Terza, Basu and Rathouz (2008)).

To accommodate endogenous explanatory variables (EEVs) in the Swopit framework I implement the two-step control function (CF) approach with bias correction terms as additional regressors. This approach, which introduces residuals from the reduced form for EEVs into the structural equation as controls for endogeneity, was first suggested by Hausman (1978) in the linear context as a tool of testing for endogeneity, further developed for nonlinear models by Smith and Blundell (1986), Newey (1987) and Rivers and Vuong (1988), among others, and extensively applied to the sample-selection and disequilibrium models in the microeconometric literature. In the time-series literature, the CF approach is employed by Kim (2004) in the context of Markov-switching linear regressions. Kim (2009) considers both a joint and a two-step estimation procedures and concludes that although the former is asymptotically most efficient, the latter provides a reasonable alternative: the Monte Carlo experiments show that the two-step procedure can be more efficient than the joint one in finite samples.

To obtain the CF Swopit estimator, let $\mathbf{x}_t$ and $\mathbf{x}_{s,t}$ contain only exogenous regressors, let $\mathbf{w}_t$ and $\mathbf{w}_{s,t}$ be the $t^{\text{th}}$ rows of the observed data matrices $\mathbf{W}$ and $\mathbf{W}_s$ that contain continuous EEVs, let $\widetilde{\mathbf{w}}_t$ be the $t^{\text{th}}$ row of the $T \times \widetilde{K}$ matrix of EEVs $\widetilde{\mathbf{W}}$ that combines all variables from $\mathbf{W}$, $\mathbf{W}_1$, ..., $\mathbf{W}_N$, let $\mathbf{z}_t$ be the $t^{\text{th}}$ row of the observed $T \times K_\delta$ data matrix $\mathbf{Z}$ that contains exogenous instrumental variables, modify the system (1) and augment it with the reduced forms for EEVs as follows:

$$
\begin{array}{ll}
r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{w}_t'\boldsymbol{\gamma} + \varepsilon_t,\ t = 1,2,...,T & \text{(regime equation)},\\
r_t|(\mathbf{x}_t,\mathbf{w}_t) = s \text{ if } \mu_{s-1} < r_t^*|(\mathbf{x}_t,\mathbf{w}_t) \le \mu_s,\ s = 1,2,...,N & \text{(regime matching rule)},\\
y_{s,t}^* = \mathbf{x}_{s,t}'\beta_s + \mathbf{w}_{s,t}'\boldsymbol{\gamma}_s + \varepsilon_{s,t},\ s = 1,2,...,N & \text{(outcome equations)},\\
y_t|(\mathbf{x}_{s,t},\mathbf{w}_{s,t},r_t = s) = j \text{ if } \alpha_{s,j-1} < y_{s,t}^*|(\mathbf{x}_{s,t},\mathbf{w}_{s,t}) \le \alpha_{s,j} & \text{(outcome matching rules)},\\
\widetilde{\mathbf{w}}_t = \mathbf{z}_t\boldsymbol{\delta} + \widetilde{\boldsymbol{\nu}}_t & \text{(reduced form for EEVs)},\\
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} Normal\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho_s \\ \rho_s & 1 \end{bmatrix} \right),\ s = 1,2,...,N & \begin{array}{l}\text{(interdependence between}\\ \text{the regime and outcome}\\ \text{decisions)},\end{array}
\end{array}
$$

where $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}_s$ are the $K_\gamma \times 1$ and $K_{s\gamma} \times 1$ vectors of unknown parameters; $\boldsymbol{\delta}$ is the $K_\delta \times \widetilde{K}$ matrix of unknown parameters; $\widetilde{\boldsymbol{\nu}}_t$ is the $T \times \widetilde{K}$ matrix of normally distributed error terms with zero means; and the rest of variables and parameters are defined as in (1).

Denote $\boldsymbol{\nu}_t$ and $\boldsymbol{\nu}_{st}$ the corresponding subsets of $\widetilde{\boldsymbol{\nu}}_t$ that match the subsets $\mathbf{w}_t$ and $\mathbf{w}_{st}$ of $\widetilde{\mathbf{w}}_t$, respectively. Assume that each of the $N+1$ pairs $(\varepsilon_t, \boldsymbol{\nu}_t)$, $(\varepsilon_{1,t}, \boldsymbol{\nu}_{1,t})$, ..., $(\varepsilon_{N,t}, \boldsymbol{\nu}_{N,t})$ is independent of $\mathbf{z}_t$ and jointly normally distributed with zero mean. The regressors $\mathbf{w}_t$, $\mathbf{w}_{1,t}$, ..., $\mathbf{w}_{N,t}$ are endogenous if the structural error terms $\varepsilon_t$, $\varepsilon_{1,t}$, ..., $\varepsilon_{N,t}$ in the above $N+1$ pairs are correspondingly correlated with the reduced form error terms $\boldsymbol{\nu}_t$, $\boldsymbol{\nu}_{1,t}$, ..., $\boldsymbol{\nu}_{N,t}$. If there is no correlation in any of these pairs, there is no endogeneity problem.

Under the joint normality of $(\varepsilon_t, \boldsymbol{\nu}_t)$, $(\varepsilon_{1,t}, \boldsymbol{\nu}_{1,t})$, ..., $(\varepsilon_{N,t}, \boldsymbol{\nu}_{N,t})$, assume that $\varepsilon_t = \boldsymbol{\nu}_t\boldsymbol{\lambda} + \zeta_t$ and $\varepsilon_{s,t} = \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s + \zeta_{s,t}$ to obtain

$$
\begin{aligned}
&r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{w}_t'\boldsymbol{\gamma} + \boldsymbol{\nu}_t\boldsymbol{\lambda} + \zeta_t,\\
&y_{s,t}^* = \mathbf{x}_{s,t}'\boldsymbol{\beta}_s + \mathbf{w}_{s,t}\boldsymbol{\gamma}_s + \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s + \zeta_{s,t},\ s = 1,2,...,N,
\end{aligned}
$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}_s$ are the $K_\gamma \times 1$ and $K_{s\gamma} \times 1$ vectors of unknown parameters; $\zeta_t$ and $\zeta_{s,t}$ are the error terms that are assumed to be normally distributed with zero means and independent of $\mathbf{z}_t$, and also of $\mathbf{x}_t$ and $\mathbf{x}_{s,t}$, and $\boldsymbol{\nu}_t$ and $\boldsymbol{\nu}_{s,t}$, respectively (and therefore independent of $\mathbf{w}_t$ and $\mathbf{w}_{s,t}$). Since $\varepsilon_t$ and $\varepsilon_{s,t}$ have unit variance due to normalization, $var(\zeta_t) = var(\varepsilon_t - \boldsymbol{\nu}_t\boldsymbol{\lambda}) = 1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})$, and analogously $var(\zeta_{s,t}) = 1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)$.

Rewrite the probabilities to observe the outcome $j$ in (2) as

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}, \widetilde{\mathbf{w}}_t)\\
&= \sum_{s=1}^{N} \left[ \Phi_2\left( \tfrac{\mu_s - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}}; \tfrac{\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}}; \rho_s \right) \right.\\
&-\Phi_2\left( \tfrac{\mu_s - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}}; \tfrac{\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}}; \rho_s \right)\\
&-\Phi_2\left( \tfrac{\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}}; \tfrac{\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}}; \rho_s \right)\\
&\left. +\Phi_2\left( \tfrac{\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}}; \tfrac{\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}}; \rho_s \right) \right].
\end{aligned}
$$

The probabilities in (4) for the Swopit-r model and in (5) for the Swopit-z model can be rewritten analogously. The probabilities in the case of exogenous switching (when $\rho_s = 0, \forall s$) are shown in Appendix A.

Since $\widetilde{\boldsymbol{\nu}}_t$ is not observed, in order to consistently estimate these probabilities perform the following two-step procedure:

1. Run the OLS regressions of each EEV in $\mathbf{w}_t$ on the set of instruments $\mathbf{z}_t$ to consistently estimate $\boldsymbol{\delta}$ and variance of each component of $\widetilde{\boldsymbol{\nu}}_t$, and save the residuals $\widehat{\widetilde{\boldsymbol{\nu}}}_t$.

2. Run the Swopit regression with $\mathbf{x}_t$, $\mathbf{w}_t$, $\widehat{\boldsymbol{\nu}}_t$ in the regime equation and with $\mathbf{x}_{s,t}$, $\mathbf{w}_{s,t}$, $\widehat{\boldsymbol{\nu}}_{s,t}$ in the outcome equations to consistently estimate the scaled parameters $(\underline{\boldsymbol{\mu}}', \underline{\boldsymbol{\beta}}', \underline{\boldsymbol{\gamma}}', \underline{\boldsymbol{\lambda}})$ and $(\underline{\boldsymbol{\alpha}}'_s, \underline{\boldsymbol{\beta}}'_s, \underline{\boldsymbol{\gamma}}'_s, \underline{\boldsymbol{\lambda}}_s)$.

The scaled parameters from the second step relate to the original (unscaled) parameters as

$$(\underline{\boldsymbol{\mu}}', \underline{\boldsymbol{\beta}}', \underline{\boldsymbol{\gamma}}', \underline{\boldsymbol{\lambda}}) \equiv (\boldsymbol{\mu}', \boldsymbol{\beta}', \boldsymbol{\gamma}', \boldsymbol{\lambda})/\sqrt{1 - var(\boldsymbol{\nu}_t \boldsymbol{\lambda})},$$
$$(\underline{\boldsymbol{\alpha}}'_s, \underline{\boldsymbol{\beta}}'_s, \underline{\boldsymbol{\gamma}}'_s, \underline{\boldsymbol{\lambda}}_s) \equiv (\boldsymbol{\alpha}'_s, \boldsymbol{\beta}'_s, \boldsymbol{\gamma}'_s, \boldsymbol{\lambda}_s)/\sqrt{1 - var(\boldsymbol{\nu}_{s,t} \boldsymbol{\lambda}_s)}.$$

To obtain the estimates of the original parameters, notice that $1 - var(\boldsymbol{\nu}_t \boldsymbol{\lambda}) = 1 - [1 - var(\boldsymbol{\nu}_t \boldsymbol{\lambda})]var(\boldsymbol{\nu}_t \underline{\boldsymbol{\lambda}})$; hence, $1/[1 - var(\boldsymbol{\nu}_t \boldsymbol{\lambda})] = 1 + var(\boldsymbol{\nu}_t \underline{\boldsymbol{\lambda}})$. Analogously, $1/[1 - var(\boldsymbol{\nu}_{s,t} \boldsymbol{\lambda}_s)] = 1 + var(\boldsymbol{\nu}_{s,t} \underline{\boldsymbol{\lambda}}_s)$. It follows that, for example, $\boldsymbol{\beta} \equiv \underline{\boldsymbol{\beta}}/\sqrt{1 + var(\boldsymbol{\nu}_t \underline{\boldsymbol{\lambda}})}$. Thus, we can obtain the consistent estimators of the original parameters as

$$(\widehat{\boldsymbol{\mu}}', \widehat{\boldsymbol{\beta}}', \widehat{\boldsymbol{\gamma}}', \widehat{\boldsymbol{\lambda}}) \equiv (\widehat{\underline{\boldsymbol{\mu}}}', \widehat{\underline{\boldsymbol{\beta}}}', \widehat{\underline{\boldsymbol{\gamma}}}', \widehat{\underline{\boldsymbol{\lambda}}})/\sqrt{1 + \widehat{var(\widehat{\boldsymbol{\nu}}_t \widehat{\underline{\boldsymbol{\lambda}}})}},$$
$$(\widehat{\boldsymbol{\alpha}}'_s, \widehat{\boldsymbol{\beta}}'_s, \widehat{\boldsymbol{\gamma}}'_s, \widehat{\boldsymbol{\lambda}}_s) \equiv (\widehat{\underline{\boldsymbol{\alpha}}}'_s, \widehat{\underline{\boldsymbol{\beta}}}'_s, \widehat{\underline{\boldsymbol{\gamma}}}'_s, \widehat{\underline{\boldsymbol{\lambda}}}_s)/\sqrt{1 + \widehat{var(\widehat{\boldsymbol{\nu}}_{s,t} \widehat{\underline{\boldsymbol{\lambda}}}_s)}},$$

where all terms on the right-hand side are available from the two-step CF estimation procedure.

The usual $t$ statistics on $\widehat{\boldsymbol{\nu}}_t$ and $\widehat{\boldsymbol{\nu}}_{s,t}$ from the second step can be used to test the null hypotheses that $\mathbf{w}_t$ and $\mathbf{w}_s$ are exogenous, that is $\boldsymbol{\lambda} = 0$ and $\boldsymbol{\lambda}_s = 0$. Under the null of exogeneity, $\varepsilon_t = \zeta_t$ and $\varepsilon_{s,t} = \zeta_{s,t}$, so the distributions of $\widehat{\boldsymbol{\nu}}_t$ and $\widehat{\boldsymbol{\nu}}_{s,t}$ do not matter. However, if $\mathbf{w}_t$ or $\mathbf{w}_s$ are endogenous, the normality of $\widehat{\boldsymbol{\nu}}_t$ and $\widehat{\boldsymbol{\nu}}_{s,t}$ is critical. The CF method imposes strong assumptions on the process generating EEVs and does not generally work if they are discrete. Allowing for discrete EEVs is notoriously difficult even in a single-equation instrumental-variable model of discrete choice (Bhattacharya, Goldman, and McCaffrey (2006)). Relaxing the distributional restrictions on the genesis of EEVs in the CF method, as Chesher and Smolinski (2012) demonstrate, results in uncertainty — the model becomes only *set,* not *point,* identifying. However, in the case of discrete EEVs, as suggested by Terza, Basu and Rathouz (2008), we can run a discrete-choice first-stage estimation and obtain the generalized residuals. Wooldridge (2014) argues that the CF approach with generalized residuals might yield good estimates of average MEs and provide a reasonable approximate solution to the endogeneity problem.

## 3.4 Discriminating among the OP, MIOP and Swopit models

This section discusses the relations among the OP, MIOP and Swopit models and the choice of a formal statistical test for model selection.

In general, neither the OP nor MIOP model is nested in the Swopit-z model, and vice versa. However, these models are not strictly non-nested. They overlap under certain parameter restrictions, namely, if their slope coefficients are all fixed to zero and only the thresholds are estimated. Therefore, the comparison of the OP or MIOP model with the Swopit-z model can be performed using a test for non-nested overlapping models, such as the Vuong test (Vuong 1989).

An interesting special case when the Swopit-z model nests the MIOP model occurs under certain parameter restrictions provided (i) the dependent variable has three outcome categories, (ii) both $\mathbf{X}_1$ and $\mathbf{X}_3$ contain all covariates in the MIOP regime equation, and (iii) $\mathbf{X}$ includes all covariates in the MIOP outcome equation. See Appendix B for a proof. In this case, the comparison of the Swopit-z and MIOP models can be performed using a test for nested models, such as the likelihood ratio (LR) test.

The MIOP reduces to the OP model if (i) the outcome equation of the former contains all covariates in the latter and (ii) the threshold parameter in the regime equation of the former is infinitely close to the lower bound of its parameter space (that is close enough to $-\infty$) to ensure that all observations always occur in the "change" regime. In this special case, not only one parameter (the threshold in the regime equation) is not the interior point of the parameter space in the null hypothesis, but also all the slope coefficients in the regime equation become the nuisance parameters that are not identified under the null; therefore, since some standard regularity conditions of the classical LR test fail to hold, the comparison of the MIOP and OP models can be performed using the LR test with the simulated critical values suggested in Andrews (2001).

The OP, MIOP and Swopit-z models are the special cases of the Swopit-r model, which is in turn a special case of the three-regime Swopit model. For example, the Swopit-r model reduces to the MIOP if either the smallest threshold parameter in the Swopit-r regime equation is infinitely close to $-\infty$ or the largest threshold parameter is infinitely close to $\infty$. Their comparison can be performed using the LR test with the critical values suggested in Andrews (2001).

# 4 Finite sample performance

This section summarizes the results of conducted Monte Carlo experiments. In brief, the simulations suggest that (i) the proposed ML estimators of the Swopit-z and Swopit-r models are consistent and demonstrate good performance in small samples; (ii) it requires roughly twice as many observations per parameter for the Swopit-z model with no overlap among the covariates in latent equations (and four times as many observations per parameter with full overlap) to achieve the same accuracy of the estimates as in the standard OP model; (iii) if the underlying DGP is characterized by switching among three regimes, both the OP and MIOP estimators are inconsistent, whereas the Swopit-r estimator is consistent under either the OP, or MIOP, or Swopit-z DGP; (iv) the Swopit model is identified even with no exclusion restrictions; though, the more exclusion restrictions, the more accurate the estimates; (v) ignoring the endogeneity of regime switching leads to asymptotically biased estimates; and (vi) the implemented CF Swopit estimator is consistent and provides a reliable small-sample inference in the presence of EEVs.

The dependent variable is generated in all experiments with five observed choices (large decrease, small decrease, no change, small increase, and large increase) with the following population frequencies: 7%, 14%, 58%, 14% and 7%. The number of explanatory variables is three in all models. The number of replications is 5,000 in each experiment.

Unlike the parameters, which are identified up to scale and location only, and are of little interest in their own right, the choice probabilities and MEs of covariates on them are absolutely estimable functions (invariant to the identifying assumptions), and are of main interest in empirical research.[8] The reported Monte Carlo results assess the accuracy and uncertainty of the estimates of the MEs of covariates on choice probabilities, estimated at the population medians of the covariates.[9] The following measures of finite sample performance (averaged for all covariates and choices, and across all replications) are reported: bias — the absolute difference between the estimated and true ME values;

[8]The ME of a continuous covariate on the probability of each discrete choice is computed as the partial derivative with respect to this covariate, holding all other covariates fixed. For a discrete-valued covariate, the ME is computed as the change in the probabilities when this covariate changes by one increment and all other covariates are fixed.

[9]The Monte Carlo assessments of the accuracy and uncertaity of the Swopit parameter estimates are reported in Appendix C.

RMSE — the root mean square error of ME estimates relative to their true values; CP — the percentage of times the estimated asymptotic 95% confidence intervals cover the true values of the MEs; and s.e. bias — the absolute difference between the average of estimated standard errors and standard deviation of ME estimates divided by the standard deviation of ME estimates, in percent.

## 4.1 Performance under alternative DGPs

This set of experiments assesses the capability of the competing models (OP, MIOP and Swopit-z with exogenous and endogenous switching) to estimate the MEs from the data generated by each of four DGPs. Three vectors of covariates $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are drawn once (and held fixed in all simulations) as $\mathbf{v}_1 \overset{iid}{\sim} Normal(2,1)$, $\mathbf{v}_2 \overset{iid}{\sim} Normal(0,1)$ and $\mathbf{v}_3 \overset{iid}{\sim} Normal(0,1)$. The repeated samples with 250, 500 and 1000 observations are generated as follows: (i) by the OP DGP — with the covariates $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the vector of slope coefficients (0.6, 0.4, 0.8)$'$ and the vector of cutpoints (-1.45, -0.55, 0.75, 1.65)$'$; (ii) by the MIOP DGP with exogenous switching — with $\mathbf{X}^0 = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{Z}^0 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta}^0 =$ (0.6, 0.8)$'$, $\boldsymbol{\gamma}^0 =$ (0.5, 0.6)$'$, $\mu^0 = 0.45$, $\boldsymbol{\alpha}^0 =$ (-1.45, -0.55, 0.75, 1.65)$'$ and $\rho^0 = 0$; (iii) by the Swopit-z DGP with exogenous switching — with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} =$ (0.6, 0.4)$'$, $\boldsymbol{\beta}_1 =$ (0.3, -0.8)$'$, $\boldsymbol{\beta}_3 =$ (0.9, -0.3)$'$, $\boldsymbol{\mu} =$ (0.91, 1.5)$'$, $\boldsymbol{\alpha}_1 =$ (-0.95, 0.25)$'$, $\boldsymbol{\alpha}_3 =$ (-0.13, 1.2), $\rho_1 = \rho_3 = 0$; and (iv) by the Swopit-z DGP with endogenous switching — with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} =$ (0.6, 0.4)$'$, $\boldsymbol{\beta}_1 =$ (0.3, -0.8)$'$, $\boldsymbol{\beta}_3 =$ (0.9, -0.3)$'$, $\boldsymbol{\mu} =$ (0.9, 1.5)$'$, $\boldsymbol{\alpha}_1 =$ (-1.22, 0.08)$'$, $\boldsymbol{\alpha}_3 =$ (0.3, 1.63), $\rho_1 = 0.3$, and $\rho_3 = 0.6$.

Under each DGP, several competing models are estimated using the same set of covariates: (i) under the OP DGP — the OP model with the covariates $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the exogenous MIOP model with $\mathbf{X}^0 = \mathbf{Z}^0 = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, and the exogenous Swopit-z model with $\mathbf{X} = \mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$; (ii) under the exogenous MIOP DGP — the OP model with $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the exogenous MIOP model with $\mathbf{X}^0 = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{Z}^0 = (\mathbf{v}_2, \mathbf{v}_3)$, and the exogenous Swopit-z and Swopit-r models with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$ and $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$; (iii) under the exogenous Swopit-z DGP — the OP model with $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the exogenous MIOP model with $\mathbf{X}^0 = \mathbf{Z}^0 = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, and the exogenous Swopit-z and Swopit-r models with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$ and $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$; and (iv) under the endogenous Swopit-z DGP — the OP model with $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, the exogenous MIOP model with $\mathbf{X}^0 = \mathbf{Z}^0 = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, and the exogenous and endogenous Swopit-z models with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$ and $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$.

Table 1 summarizes the performance of competing models under each DGP. The simulations suggest that, under the Swopit-z DGP, the proposed ML estimator of the Swopit-z model is consistent: as the sample size grows, the biases and RMSE reduce more than twice, and the CP moves closer to the 95% nominal level and reaches 94.7%. In contrast, under the Swopit-z DGP, both the OP and MIOP estimates are severely biased (under endogenous switching the biases are three times as large as under exogenous one): as the sample size grows, the biases almost do not change, the s.e. biases increase, and the CPs move away from the nominal level (down to 45% with exogenous switching and to 12% with endogenous one). The exogenous Swopit-z model is also biased under the endogenous Swopit-z DGP: as the sample size grows, the bias almost does not change, and the CP moves away from the nominal level down to 82%.

Table 1. Performance of competing models under alternative DGPs

| | True DGP: | OP | | | MIOP | | | |
|---|---|---|---|---|---|---|---|---|
| Sample size | Estimated model: | OP | MIOP | Swopit-z ($\rho_s = 0$) | OP | MIOP | Swopit-z ($\rho_s = 0$) | Swopit-r ($\rho_s = 0$) |
| 250 | Bias of ME estimates, $\times 10^3$ | 0.48 | 3.7 | 5.6 | 54.1 | 3.0 | 19.0 | 2.4 |
| 500 | | 0.19 | 2.2 | 3.4 | 55.0 | 1.5 | 18.3 | 1.4 |
| 1000 | | 0.17 | 1.6 | 2.4 | 53.3 | 0.7 | 20.5 | 0.8 |
| 250 | RMSE of ME estimates, $\times 10^2$ | 1.5 | 1.8 | 2.5 | 5.6 | 2.1 | 3.2 | 9.3 |
| 500 | | 1.1 | 1.2 | 1.6 | 5.6 | 1.4 | 2.7 | 4.7 |
| 1000 | | 0.8 | 0.8 | 1.1 | 5.4 | 1.0 | 2.6 | 1.1 |
| 250 | Coverage probability, % (at 95% level) | 93.2 | 93.3 | 91.4 | 18.0 | 91.4 | 67.6 | 85.0 |
| 500 | | 94.0 | 94.2 | 92.9 | 10.5 | 94.0 | 63.7 | 90.9 |
| 1000 | | 94.5 | 94.5 | 93.5 | 6.0 | 94.6 | 52.4 | 93.6 |
| 250 | Bias of ME s.e. estimates, % | 1.2 | 1.8 | 8.3 | 3.1 | 4.2 | 16.1 | 26.8 |
| 500 | | 1.1 | 1.3 | 5.3 | 4.8 | 1.6 | 23.2 | 31.2 |
| 1000 | | 0.7 | 0.7 | 3.8 | 3.8 | 0.9 | 30.2 | 7.2 |

| | True DGP: | Swopit-z ($\rho_s = 0$) | | | | Swopit-z | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sample size | Estimated model: | OP | MIOP | Swopit-z ($\rho_s = 0$) | Swopit-r ($\rho_s = 0$) | OP | MIOP | Swopit-z ($\rho_s = 0$) | Swopit-z |
| 250 | Bias of ME estimates, $\times 10^3$ | 14.5 | 15.2 | 2.0 | 4.0 | 50.0 | 54.1 | 5.6 | 2.7 |
| 500 | | 14.4 | 14.9 | 1.1 | 2.4 | 49.7 | 54.5 | 6.2 | 1.5 |
| 1000 | | 14.6 | 14.8 | 0.6 | 1.1 | 49.5 | 53.3 | 6.0 | 0.9 |
| 250 | RMSE of ME estimates, $\times 10^2$ | 2.2 | 2.4 | 2.3 | 3.8 | 5.2 | 6.2 | 2.8 | 3.2 |
| 500 | | 1.9 | 2.1 | 1.6 | 2.4 | 5.1 | 6.0 | 2.0 | 2.2 |
| 1000 | | 1.7 | 1.8 | 1.1 | 1.2 | 5.0 | 5.7 | 1.4 | 1.5 |
| 250 | Coverage probability, % (at 95% level) | 69.1 | 74.5 | 93.7 | 89.4 | 25.7 | 34.3 | 92.9 | 93.3 |
| 500 | | 55.6 | 59.8 | 94.2 | 92.1 | 18.1 | 25.9 | 88.9 | 93.5 |
| 1000 | | 45.0 | 46.0 | 94.7 | 94.0 | 11.9 | 18.0 | 85.2 | 94.4 |
| 250 | Bias of ME s.e. estimates, % | 2.0 | 5.6 | 3.4 | 26.2 | 4.2 | 12.2 | 2.0 | 0.0 |
| 500 | | 2.0 | 7.9 | 2.4 | 24.0 | 4.7 | 15.8 | 1.4 | 2.0 |
| 1000 | | 2.4 | 7.5 | 1.3 | 6.7 | 5.4 | 21.0 | 0.4 | 1.2 |

Under the OP DGP, though the Swopit-z model does not nest the OP one, the Swopit-z estimator performs much better than the OP estimator under the Swopit-z DGP; as the sample size increases, the Swopit-z performance under the OP DGP improves substantially, whereas the OP performance under the Swopit-z DGP becomes even worse. Since the MIOP nests the OP model, the MIOP estimator under the OP DGP performs better than the Swopit-z one.

Under the MIOP DGP, the Swopit-z estimator seems not to be consistent — as the sample size grows, its bias remains approximately the same (and with 1000 observations is 26 times as large as the bias of the MIOP estimator), its s.e. bias increases, and its CP moves away from the nominal level down to 52.4%. In contrast, the estimator of the Swopit-r model that nests all three competitors performs well under both the MIOP and Swopit-z DGPs — its biases are only slightly larger than those of the MIOP and Swopit-z estimators (however, its s.e. biases are significantly larger). As the samples size grows, both biases reduce sharply, and with 1000 observations the CP approaches 94%. Under the OP DGP, the Swopit-r estimator is also consistent, but in small samples experiences problems with the invertibility of the Hessian matrix, because its likelihood function at the maximum can be flat for an infinitely wide range of parameters' values. For that reason the Swopit-r model is not included under the OP DGP. In practice, the problems with the invertibility of the Hessian matrix in the Swopit estimator might signal that either the sample is not large enough or the underlying DGP is not consistent with the complicated Swopit process, and the simpler models should be estimated instead.

## 4.2 Effect of exclusion restrictions

To assess the effect of exclusion restrictions, the repeated samples in this set of experiments are generated by the Swopit-z DGP under three different scenarios of the overlap among the covariates in three latent equations: "no overlap" (each covariate belongs only to one equation), "partial overlap" (each covariate belongs to two equations) and "complete overlap" (all three equations have the same set of covariates). Under each scenario, the samples are generated using the same number of observations per parameter: 25 (with 225, 300 and 375 observations, respectively), 50 (with 450, 600 and 750 observations, respectively) and 75 observations per parameter (with 675, 900 and 1125 observations, respectively).

Three vectors of covariates $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are drawn once and held fixed in all replications as $\mathbf{v}_1 \overset{iid}{\sim} \mathit{Normal}(2,1)$, $\mathbf{v}_2 \overset{iid}{\sim} \mathit{Normal}(0,1)$ and $\mathbf{v}_3$ = -1 if $\mathbf{u} \leq 0.3$, $\mathbf{v}_3 = 0$ if $0.3 < \mathbf{u} \leq 0.7$, or $\mathbf{v}_3 = 1$ if $0.7 < \mathbf{u}$, where $\mathbf{u} \overset{iid}{\sim} \mathit{Uniform}[0,1]$. The repeated samples are generated as follows: under no overlap scenario — with $\mathbf{X} = \mathbf{v}_1$, $\mathbf{X}_1 = \mathbf{v}_2$, $\mathbf{X}_3 = \mathbf{v}_3$, $\boldsymbol{\beta}$

$= 0.6$, $\boldsymbol{\beta}_1 = 0.8$, $\boldsymbol{\beta}_3 = 0.9$, $\boldsymbol{\mu} = (0.95,\ 1.45)'$, $\boldsymbol{\alpha}_1 = (\text{-}1.22,\ 0.03)'$, $\boldsymbol{\alpha}_3 = (\text{-}0.03,\ 1.18)'$, and $\rho_1 = \rho_3 = 0$; under partial overlap scenario — with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_2)$, $\mathbf{X}_1 = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} = (0.6,\ 0.4)'$, $\boldsymbol{\beta}_1 = (0.2,\ 0.3)'$, $\boldsymbol{\beta}_3 = (0.3,\ 0.9)'$, $\boldsymbol{\mu} = (0.9,\ 1.5)'$, $\boldsymbol{\alpha}_1 = (\text{-}0.67,\ 0.36)'$, $\boldsymbol{\alpha}_3 = (0.02,\ 1.28)'$, and $\rho_1 = \rho_3 = 0$; and under complete overlap scenario — with $\mathbf{X} = \mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} = (0.6,\ 0.4,\ 0.8)'$, $\boldsymbol{\beta}_1 = (0.2,\ 0.8,\ 0.3)'$, $\boldsymbol{\beta}_3 = (0.4,\ 0.3,\ 0.9)'$, $\boldsymbol{\mu} = (0.85,\ 1.55)'$, $\boldsymbol{\alpha}_1 = (\text{-}1.2,\ 0.07)'$, $\boldsymbol{\alpha}_3 = (1.28,\ 2.5)'$, and $\rho_1 = \rho_3 = 0$. The samples are estimated using the exogenous Swopit-z model with the same specifications of the latent equations as in the above DGPs.

The Monte Carlo results summarized in Table 2 suggest that the exclusion restrictions are not necessary for the identification and consistent estimation of the Swopit-z model. As the sample size grows, the biases and RMSE reduce, and the CP approaches the nominal value both with and without exclusion restrictions. Not surprisingly, the more exclusion restrictions, the more accurate the estimates — for example, without the exclusion restrictions (under the complete overlap) the bias is 100% larger, and the s.e. bias is 250% larger than in the case of no overlap among the covariates.

Table 2. Effect of exclusion restrictions on the performance of Swopit-z estimator

| Number of observations per parameter: | 25 | 50 | 75 |
|---|---|---|---|
| The overlap among the covariates in the regime and amount equations: | no overlap \| partial overlap \| complete overlap | | |
| Bias of ME estimates (=1 for "no overlap" with 25 obs/parameter) | 1.0 \| 1.5 \| 1.6 | 0.4 \| 0.8 \| 0.7 | 0.3 \| 0.6 \| 0.6 |
| RMSE of ME estimates (=1 for "no overlap" with 25 obs/parameter) | 1.0 \| 1.1 \| 1.2 | 0.7 \| 0.8 \| 0.8 | 0.6 \| 0.6 \| 0.6 |
| Coverage probability of ME estimates, % (at 95% level) | 93.0 \| 92.0 \| 92.1 | 93.5 \| 93.7 \| 93.4 | 94.2 \| 94.1 \| 93.5 |
| Bias of ME standard error estimates (=1 for "no overlap" with 25 obs/parameter) | 1.0 \| 1.4 \| 2.5 | 0.9 \| 0.8 \| 2.2 | 0.4 \| 0.6 \| 1.4 |

## 4.3 Performance with endogenous explanatory variables

To analyze the finite sample performance of the CF Swopit-z estimator in the presence of EEVs, the repeated samples in this set of experiments are generated according to

$$r_t^* = \mathbf{x}_t'\boldsymbol{\beta} + \mathbf{w}_t'\gamma + \varepsilon_t,\ t = 1, 2, ..., T$$
$$r_t|(\mathbf{x}_t, \mathbf{w}_t) = s \text{ if } \mu_{s-1} < r_t^*|(\mathbf{x}_t, \mathbf{w}_t) \leq \mu_s,\ s = 1, 2, 3$$
$$y_{s,t}^* = \mathbf{x}_{s,t}'\beta_s + \mathbf{w}_{s,t}'\gamma_s + \varepsilon_{s,t},\ s = 1, 3$$

$$
\begin{aligned}
&y_t|(\mathbf{x}_{1t}, \mathbf{w}_{1,t}, r_t = 1) = j \text{ if } \alpha_{1,j-1} < y^*_{1,t}|(\mathbf{x}_{1,t}, \mathbf{w}_{1,t}) \leq \alpha_{1,j},\ j = 1, 2, ..., q \\
&y_t|(r_t = 2) = q \\
&y_t|(\mathbf{x}_{3,t}, \mathbf{w}_{3,t}, r_t = 3) = j \text{ if } \alpha_{3,j-1} < y^*_{3,t}|(\mathbf{x}_{3,t}, \mathbf{w}_{3,t}) \leq \alpha_{3,j},\ j = q, q+1, ..., J \\
&\widetilde{\mathbf{w}}_t = \mathbf{z}_t \boldsymbol{\delta} + \widetilde{\boldsymbol{\nu}}_t \\
&\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_t \end{bmatrix} \sim N\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right),\ s = 1, 3
\end{aligned}
$$

with $\mathbf{X} = \mathbf{0}$, $\mathbf{X}_1 = \mathbf{X}_3 = \mathbf{v}_2$, $\mathbf{Z} = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{v}_1 \overset{iid}{\sim} Normal(2, 1)$, $\mathbf{v}_2 \overset{iid}{\sim} Normal(0, 1)$, $\mathbf{v}_3 \overset{iid}{\sim} Normal(0, 1)$, $\boldsymbol{\gamma} = (0.6,\ 0.4)'$, $\boldsymbol{\gamma}_1 = 0.5$, $\boldsymbol{\gamma}_3 = 0.5$, $\boldsymbol{\beta}_1 = 0.6$, $\boldsymbol{\beta}_3 = 0.6$, $\boldsymbol{\mu} = (0.91,\ 1.49)'$, $\boldsymbol{\alpha}^1 = (\text{-}1.25,\ \text{-}0.04)'$, $\boldsymbol{\alpha}^2 = (0.4,\ 1.25)'$, $\boldsymbol{\lambda} = (0.75,\ 0.75)'$, $\lambda_1 = \lambda_3 = 0.75$, $var(\widetilde{\boldsymbol{\nu}}_t) = (0.36,\ 0.36)$, and $\boldsymbol{\delta} = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix}$.

There are two endogenous explanatory variables: the first one (instrumented by $\mathbf{v}_1$) enters $\mathbf{w}_t$ in the regime equation, the second one (instrumented by $\mathbf{v}_3$) enters $\mathbf{w}_t$, $\mathbf{w}_{1t}$ and $\mathbf{w}_{2t}$ and belongs to all three equations. The only exogenous regressor $\mathbf{v}_2$ is included in $\mathbf{x}_{1t}$ and $\mathbf{x}_{3t}$ in the outcome equations. The values of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are drawn repeatedly for each replication. The estimations are performed using the standard and CF Swopit-z estimators.

Table 3. Performance of standard and CF Swopit-z estimators in the presense of endogenous explanatory variables

| | Estimator | Sample size | | | |
|---|---|---|---|---|---|
| | | 250 | 500 | 1000 | 2000 |
| Bias of ME estimates | Standard | 10.9 | 10.6 | 10.5 | 10.4 |
| (= 1 for CF estimator with 250 observations) | CF | 1.0 | 0.5 | 0.3 | 0.1 |
| RMSE of ME estimates | Standard | 1.8 | 1.6 | 1.4 | 1.3 |
| (= 1 for CF estimator with 250 observations) | CF | 1.0 | 0.7 | 0.4 | 0.3 |
| Coverage probability of ME estimates, | Standard | 61.7 | 48.9 | 41.8 | 39.0 |
| % (at 95% level) | CF | 91.8 | 93.4 | 94.0 | 94.4 |
| Bias of ME standard error estimates | Standard | 1.0 | 0.7 | 0.5 | 0.3 |
| (= 1 for standard estimator with 250 observations) | CF | 2.6 | 1.4 | 1.0 | 0.7 |

In the first round of experiments, the repeated samples are generated with 250, 500, 1000 and 2000 observations. The Monte Carlo results demonstrate (see Table 3) that the CF estimator is consistent and provides the reliable estimates in small samples: as the sample size grows, the bias reduces by 90%; the RMSE and s.e. bias reduce by 70%; and the CP moves from 91.8% to 94.4%, closer to the 95% nominal value. The standard estimator that ignores the endogeneity performs much worse: the bias reduces by 5% only; the RMSE reduces by 30% only; the s.e. bias reduces by 70%; and the CP moves from 61.7% to 39.0%, away from the nominal level. The relative performance of the standard estimator with respect to the CF one worsens as the sample size grows: with 250 observations, its bias is eleven times as large as the bias of the CF estimator, whereas with 2000 observations, it is hundred times as large. The s.e. bias of the standard estimator is only twice as small as that of the CF one.

In the second round of experiments, the repeated samples are generated with 1000 observations but with the various degree of endogeneity strength (measured by the values of $\boldsymbol{\lambda}$, $\lambda_1$ and $\lambda_3$). The simulations show (see Table 4) that as endogeneity becomes stronger, the performance of the CF estimator decreases but not sharply: the bias grows by 10%; the RMSE grows by 20%; the s.e. bias is affected more severe — it increases by 170%; consequently, the CP slowly moves from 94.5% to 93.7%, away from the nominal value. The performance of the standard Swopit estimator deteriorates much severe: the bias almost doubles; the RMSE increases by 80%; the CP stays around 40%; the s.e. bias, however, remains low. The relative performance of the standard estimator with respect to the CF one worsens as the endogeneity strengthens: when it is weak (strong) the bias is 29 (51) times as large as the bias of the CF estimator.

Table 4. Effect of endogeneity strength on the performance of standard and CF Swopit-z estimators

| | Estimator | The strength of endogeneity (the values of $\lambda = \lambda^+ = \lambda^-$) | | | | |
|---|---|---|---|---|---|---|
| | | 0.10 | 0.25 | 0.50 | 0.75 | 0.90 |
| Bias of ME estimates (=1 for CF estimator with $\lambda$ = 0.1) | Standard | 29.4 | 31.9 | 39.3 | 49.1 | 57.4 |
| | CF | 1.0 | 1.0 | 1.0 | 1.2 | 1.1 |
| RMSE of ME estimates (=1 for CF estimator with $\lambda$ = 0.1) | Standard | 2.3 | 2.5 | 2.9 | 3.6 | 4.2 |
| | CF | 1.0 | 1.0 | 1.0 | 1.1 | 1.2 |
| Coverage probability of ME estimates, % (at 95% level) | Standard | 43.6 | 39.9 | 37.4 | 41.8 | 43.7 |
| | CF | 94.5 | 94.5 | 94.4 | 94.0 | 93.7 |
| Bias of ME standard error estimates (=1 for standard estimator with $\lambda$ = 0.1) | Standard | 1.0 | 1.0 | 0.8 | 1.1 | 0.9 |
| | CF | 1.3 | 1.2 | 1.7 | 2.5 | 3.5 |

In the third round of experiments, the repeated samples are generated with 1000 observations but with the various degree of instruments' strength (measured by the values of $\boldsymbol{\delta}$). The simulations suggest (see Table 5) that as the instruments become stronger, the performance of the CF estimator improves sharply: the bias reduces by 94%; the RMSE reduces by 87%; the s.e. bias decreases by 84%; and the CP moves from

79% to 94%, closer to the nominal value. The performance of the standard estimator also improves but not so sharply — the CP changes from 29% to 42% only, and its relative performance with respect to the CF estimator worsens significantly: with weak (strong) instruments the bias is 8 (40) times as large as the bias of the CF estimator, the RMSE is larger by 37% (220%), and the s.e. bias is smaller by 328% (122%).

Table 5. Effect of the strength of instruments on the performance of standard and CF Swopit-z estimators

| | Estimator | The strength of instruments (the values of $\delta_{11} = \delta_{22}$) | | | | |
|---|---|---|---|---|---|---|
| | | 0.10 | 0.25 | 0.50 | 0.75 | 0.90 |
| Bias of ME estimates | Standard | 8.2 | 7.1 | 4.6 | 3.0 | 2.4 |
| (= 1 for CF estimator with $\delta$ = 0.1) | CF | 1.0 | 0.2 | 0.07 | 0.05 | 0.06 |
| RMSE of ME estimates | Standard | 1.4 | 1.2 | 0.8 | 0.5 | 0.4 |
| (= 1 for CF estimator with $\delta$ = 0.1) | CF | 1.0 | 0.4 | 0.2 | 0.14 | 0.13 |
| Coverage probability of ME | Standard | 29.0 | 35.5 | 37.0 | 38.5 | 41.8 |
| estimates, % (at 95% level) | CF | 79.3 | 90.9 | 93.1 | 93.9 | 94.0 |
| Bias of ME standard error estimates | Standard | 1.0 | 0.5 | 0.4 | 0.4 | 0.3 |
| (= 1 for standard estimator with $\delta$ = 0.1) | CF | 4.3 | 1.0 | 1.0 | 0.8 | 0.7 |

# 5 Empirical application

I applied the OP, MIOP and Swopit models to predict the next FOMC decision on the target during the Greenspan era, provide the out-of-sample forecast for the Bernanke era, compare the in- and out-of-sample performance of the competing models, compare the fit of the Swopit model and the discrete-choice models for the target from the literature, and address the problem of EEVs using the CF estimator.

## 5.1 FOMC decisions as sample observations

The FOMC makes interest rate decisions either at prescheduled meetings eight times per year or occasionally at unscheduled meetings and by the discretion of the chairman during intermeeting periods. Neither quarterly nor monthly interval matches a natural FOMC decision-making cycle. Modeling relationship between monthly or quarterly averages of the federal funds rate and economic variables can be subject to a problem of reverse causation, especially if financial market data are included. The Fed closely monitors daily market interest rates; on the other hand, market rates respond immediately to any Fed action. The identification of policy rules using aggregated (monthly or quarterly) financial market data is not plausible. I instead use all FOMC decisions (scheduled and unscheduled) on the target as sample observations, and estimate Fed response to recent economic and daily financial data truly available right before each FOMC decision, using the vintages of real-time data that do not include subsequent revisions.

The dates of FOMC decisions, the sizes of target changes, and the values of dependent variable $y_t$ are reported in Table 14 of Appendix D. I use the chronology of FOMC decisions from the Fed's Board of Governors, available from ALFRED[10] and derived from Thornton (2005) prior to 1994 and from FOMC meeting transcripts and statements after 1994. I made the following consolidations of target changes: (i) two hikes by 37.5 basis points (bp) and 12.5 bp made on September 3$^{rd}$ and 4$^{th}$, 1987, respectively, are merged into a single 50 bp hike; (ii) no change on September 22$^{nd}$, 1987 and 6.25 bp hike on September 24$^{th}$, 1987 are merged into a single 6.25 bp hike; (iii) two hikes by 6.25 bp and 37.5 bp made on August 8$^{th}$ and 9$^{th}$, 1988, respectively, are merged into a single 43.75 bp hike; and (iv) two hikes by 6.25 bp and 18.75 bp made on November 17$^{th}$ and 22$^{nd}$, 1988, respectively, are merged into a single 25 bp hike. The sample consists of 190 observations during the 7/1987–1/2006 period under the Greenspan chairmanship.[11]

Prior to October 1989 the Fed often changed the target in multiples of 6.25 bp, but later on the changes have been always made in multiples of 25 bp. The sample frequencies of original target changes are as follows:

| Change, bp | -50 | -31.25 | -25 | -18.75 | -12.5 | 0 | 6.25 |
|---|---|---|---|---|---|---|---|
| Frequency | 13 | 1 | 30 | 1 | 1 | 99 | 3 |

| Change, bp | 12.5 | 18.75 | 25 | 31.25 | 43.75 | 50 | 75 |
|---|---|---|---|---|---|---|---|
| Frequency | 2 | 3 | 27 | 2 | 2 | 5 | 1 |

I classified these 190 observations into the following five categories of the dependent variable $y_t$:

| $y_t$ | large cut | small cut | no change | small hike | large hike |
|---|---|---|---|---|---|
| Frequency | 14 | 32 | 102 | 32 | 10 |

,

where "large hike/cut" is an increase/decrease more than 25 bp, "small hike/cut" is an increase/decrease 25 bp or less but more than 6.25 bp, and "no change" is either no change or a change no more than 6.25 bp.[12]

[10] ALFRED, ArchivaL Federal Reserve Economic Data, is a collection of vintage versions of U.S. economic data, compiled by the Federal Reserve Bank of St. Louis (https://alfred.stlouisfed.org/).

[11] With an exception for the first observation in the sample – the FOMC meeting on July 7$^{th}$, 1987 under the Volcker chairmanship.

[12] The estimations results are robust to the alternative classification: "large hike/cut" is an in-

## 5.2 Data and model specification

A detailed list of candidate explanatory variables I tried is reported in Table 15 of Appendix D. The list includes various measures of inflation, output, employment, monetary aggregates, interest rates, exchange rates, and other indicators of economic activity that could influence FOMC decisions and actively monitored by the Fed according to the minutes and transcripts of FOMC meetings. For quarterly and monthly variables, I used the real-time data from the latest release available right before the FOMC makes its decision. For daily variables, I used information available by the end of the previous day. The preferred specification includes the following four variables:

(1) $spread_t$ — the difference between the one-year treasury constant maturity rate and effective federal funds rate, five-business-day moving average. The studies that model target changes using discrete choice approach report that the Taylor-rule variables (such as inflation and output gap) do not provide the best forecasting performance. Piazzesi (2005) finds that the two-year yield describes Fed policy better than the Taylor rules because: (i) yield data summarize the market anticipations of future target moves; (ii) these market anticipations are based on a wide spectrum of information, not just on a couple of variables; and (iii) yield data are available at higher frequencies and are less affected by measurement errors than macroeconomic variables. Hamilton and Jorda (2002), Kauppi (2012) and Van den Hauwe, Paap and van Dijk (2013) report that despite an extensive literature relating Fed policy to such macroeconomic variables as inflation, output gap, capacity utilization, the spread between the six-month Treasury bill and federal funds rate appears to be by far the most important predictor of the target changes. The term spread can be seen as a low-dimension market-based precursor of future inflation and economic activity (Mishkin (1990), Bernanke (1990), Estrella and Hardouvelis (1991), Frankel and Lown (1994), Estrella and Mishkin (1998)). Bikbov and Chernov (2013) argue that monetary policy regimes may not be estimated precisely if one uses information from the short interest rate only. The real-time data on $spread_t$ are retrieved from ALFRED.

(2) $tight_{t-1}$ and (3) $ease_{t-1}$ — the two binary indicators derived from the "policy bias" statement or "balance-of-risks" assessment at the previous FOMC meeting. During the 1983–1999 period at each meeting the FOMC issued a statement in its domestic policy directive about its expectations for changes in the stance of monetary policy in the nearest future. The directive was symmetric if it is stated that a tightening or an easing of policy were equally likely; otherwise, the directive was asymmetric toward either a tightening or an easing (see Thornton and Wheelock (2000) for the history of the

crease/decrease more than 25 bp, "small hike/cut" is an increase/decrease 25 bp or less, and "no change" is no change.

policy directive). Since 2000, the policy directive was replaced by FOMC assessment of the balance of risks between heightened inflation pressure and economic weakness over the foreseeable future; and since 2003, the FOMC issued separate risk assessments for both inflation and economic growth. The balance-of-risks assessment indicates FOMC evaluation of whether the risks for the economy are biased towards an economic slowdown (easing bias), towards higher inflationary pressure (tightening bias) or whether the both risks are balanced (symmetrical assessment). As the earlier policy bias directive, the balance-of-risks statement has persistently been interpreted as an indicator of the likely future policy actions. Lapp and Pearce (2000) and Pakko (2005) report that these FOMC statements have predictive power for the next decisions on the target. I classified the policy bias and balance-of-risks statements as "easing", "symmetrical" or "tightening" and constructed two indicator variables: $tight_{t-1}$ that takes value one if the statement at the previous FOMC meeting was tightening, and zero otherwise; and $ease_{t-1}$ that takes value one if the statement was easing, and zero otherwise. The indicators are derived from FOMC statements and minutes.[13]

(4) $house_t$ — the total number of new privately owned housing units started — a critical leading indicator of economic strength, actively monitored by the Fed and frequently mentioned in FOMC statements. The real-time data on housing starts are retrieved from ALFRED and the Philadelphia Fed's real-time data set.[14]

The values of employed variables are shown in Table 14 of Appendix D. Sample descriptive statistics are reported in Table 16 of Appendix D. All variables are stationary at the 0.01 significance level according to the augmented Dickey-Fuller unit root test as documented in Table 17 of Appendix D.

## 5.3 Estimation results

Table 6 reports the estimates of the exogenous Swopit-z parameters. The regime equation contains all four above variables. The outcome equations contain $tight_{t-1}$ and $spread_t$ in the hawkish regime, and $ease_{t-1}$ and $spread_t$ in the dovish regime. I did not impose any exclusion restrictions a priori. The exclusions are completely data-driven: those variables, coefficients on which are not significantly different from zero at the 0.05 level, are excluded from the final specification (with an exception of $ease_{t-1}$ in the outcome equation under the dovish regime). The estimation results without exclusion restrictions are reported in Table 18 of Appendix D. Among 190 FOMC actions during almost twenty years, the Swopit-z model correctly predicts 134 decisions (in terms of

[13] https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm
[14] https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/

five choices) with the 8.2 bp mean absolute error (MAE) and 71% hit rate. In terms of three policy choices (up, no change, down) the hit rate is 78%. The model never wrongly predicts the direction of change.

Table 6. Modeling changes to federal funds rate target: the estimates of the exogenous Swopit-z model parameters

| Variables | Regime equation | Outcome equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| $tight_{t-1}$ | 1.77 (0.37)*** | | 2.44 (0.91)*** |
| $ease_{t-1}$ | -2.74 (0.73)*** | -0.34 (0.34) | |
| $spread_t$ | 2.01 (0.37)*** | 0.99 (0.29)*** | 1.67 (0.72)** |
| $house_t$ | 5.21 (0.97)*** | | |
| $threshold_1$ | 7.09 (1.56)*** | -1.52 (0.34)*** | -2.33 (5.49) |
| $threshold_2$ | 9.95 (1.64)*** | -0.22 (0.34) | 3.18 (1.00)*** |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses. The explanatory variables are defined in Section 5.2.

## 5.4 Comparison of alternative models

Table 7 compares the in-sample fit of the Swopit-z and Swopit-r models (estimated with exogenous switching and the same specifications of covariates), the standard OP model (estimated with all explanatory variables in the Swopit models), and the MIOP model (in which its both equations include all covariates from the Swopit models).[15] The OP model, as a natural starting point for discrete-choice modeling of monetary policy, is used in many studies (Vanderhart (2000), Dolado, Maria-Dolores and Naveira (2005), Gerlach (2007), among others). The MIOP model is applied in Brooks, Harris and Spencer (2012) to the policy rate of Bank of England.

For the 7/1987–1/2006 period, according to the Akaike (AIC) and Bayesian (BIC) information criteria and the Vuong test (at the 0.01 significance level), the Swopit-z model is superior to the OP and MIOP models, and also preferred to the Swopit-r model by the AIC and BIC. The empirical rejection of the MIOP model in favor of the Swopit-z model implies that the impacts of explanatory variables on the outcome decisions are asymmetric; hence, combining these two distinct decisions into one branch of the decision

[15] The OP and MIOP estimation results are shown in Table 19 of Appendix D. The Swopit-r results are shown in Table 20 of Appendix D.

tree, as implemented in the MIOP model, may seriously distort an inference. The Swopit-z model has the best hit rate, MAE and adjusted noise-to-signal ratios, especially for status quo outcomes. The OP and MOP models predict more zeros (130 and 121) than the Swopit model (119), but they correctly predict only 88 and 87 zeros, respectively, whereas the Swopit-z model correctly predicts 90 zeros. The Swopit-r model with 18 parameters has the identical measures of fit as the Swopit-z model with 14 parameters and delivers insignificant increase in the likelihood. Indeed, the estimates of common 14 parameters in both models are similar, and the four extra threshold parameters in the Swopit-r model have a little effect on the fit. The parsimonious Swopit-z model suffices to capture the regime-switching behavior of the Fed, and seems to be comprehensive enough to characterize the conduct of monetary policy during the Greenspan era. The three regimes can be naturally interpreted as latent monetary policy stances (dovish, neutral, or hawkish).

Table 7. Comparison of competing models: the Swopit-z model is favoured by the data

| Model: | OP | MIOP | Swopit-z | Swopit-r |
|---|---|---|---|---|
| Log likelihood | -159.5 | -149.8 | -139.1 | -137.5 |
| AIC | 335.0 | 325.6 | **306.2** | 311.0 |
| BIC | 361.0 | 367.8 | **351.6** | 369.5 |
| Vuong test versus OP model | | -2.26** | **-4.41*** ** | -4.38*** |
| Vuong test versus MIOP model | | | **-2.46*** ** | -2.40*** |
| Hit rate (in terms of 5 choices) | 0.64 | 0.66 | **0.71** | **0.71** |
| MAE, bp | 10.4 | 10.0 | **8.2** | **8.2** |
| Adj. noise-to-signal ratio for cuts | **0.10** | 0.11 | **0.10** | **0.10** |
| Adj. noise-to-signal ratio for no changes | 0.55 | 0.45 | **0.37** | **0.37** |
| Adj. noise-to-signal ratio for hikes | 0.08 | 0.05 | **0.03** | **0.03** |

Notes. Sample period: 7/1987–1/2006. ***/**/* denote the statistical significance at the 1/5/10 percent level. Hit rate is the percentage of correct predictions among five choices, where the predicted choice is that with the highest probability. MAE is the mean absolute difference between the observed change and expected change, computed as $\sum_j j \Pr(\Delta y_t = j | \mathbf{x}_t, \mathbf{x}_t^-, \mathbf{x}_t^+)$. Adjusted noise-to-signal ratio (Kaminsky and Reinhart (1999)), is defined as $[B/(B+D)]/[A/(A+C)]$, where $A$ denotes the event that the decision is predicted and occurred, $B$ denotes the event that the decision is predicted but not occurred, $C$ denotes the event that the decision is not predicted but occurred, and $D$ denotes the event that the decision is not predicted and not occurred.

In the past literature, two or three regimes are usually considered enough to characterize monetary policy switching even for longer time spans. It seems that in this empirical application with only 190 observations in the sample, the Swopit-z and Swopit-r models with exogenous switching is the most that available data can bear without serious computational problems and substantial small-sample biases. The estimation of the endogenous Swopit-z model that has 16 parameters and the unrestricted three-regime Swopit model that has 27 parameters experience problems with convergence and invertibility of the Hessian matrix, because in small samples their likelihood functions at the maximum can be flat for an infinitely wide range of parameters' values.[16]

How good is the fit of the Swopit model in comparison with the fit of other discrete-choice models for target changes in the literature? Hu and Philips (2004) and Piazzesi (2005) model the target changes (in terms of three choices: up, no change, down) made only at the scheduled FOMC meetings in the 2/1994–12/2001 and 2/1994–12/1998 periods with 64 and 40 observations in the samples, respectively. Though the Swopit-z model is estimated over the far longer 1987–2006 period with much larger sample size (190 observations), it has slightly better overall hit rate of 78% (in terms of three choices) and outperforms the competitors in terms of the adjusted noise-to-signal ratios.

Table 8. Comparison with Hu and Phillips (2004) and Piazzesi (2005): the Swopit-z model is favoured by the data

| | Model (the number of observations in the sample) | | | | | |
|---|---|---|---|---|---|---|
| | Hu and Phillips (64 obs) | Piazzesi (40 obs) | Swopit-z model (114 obs) | Hu and Phillips (64 obs) | Piazzesi (40 obs) | Swopit-z model (114 obs) |
| Actual outcome | Hit rate | | | Adjusted noise-to-signal ratio | | |
| Decrease | 0.69 | 0.00 | **0.84** | 0.08 | n/a | **0.03** |
| No change | 0.90 | **0.93** | 0.91 | 0.45 | 0.72 | **0.14** |
| Increase | 0.50 | 0.57 | **0.89** | **0.04** | 0.11 | 0.05 |
| All | 0.78 | 0.75 | **0.89** | | | |

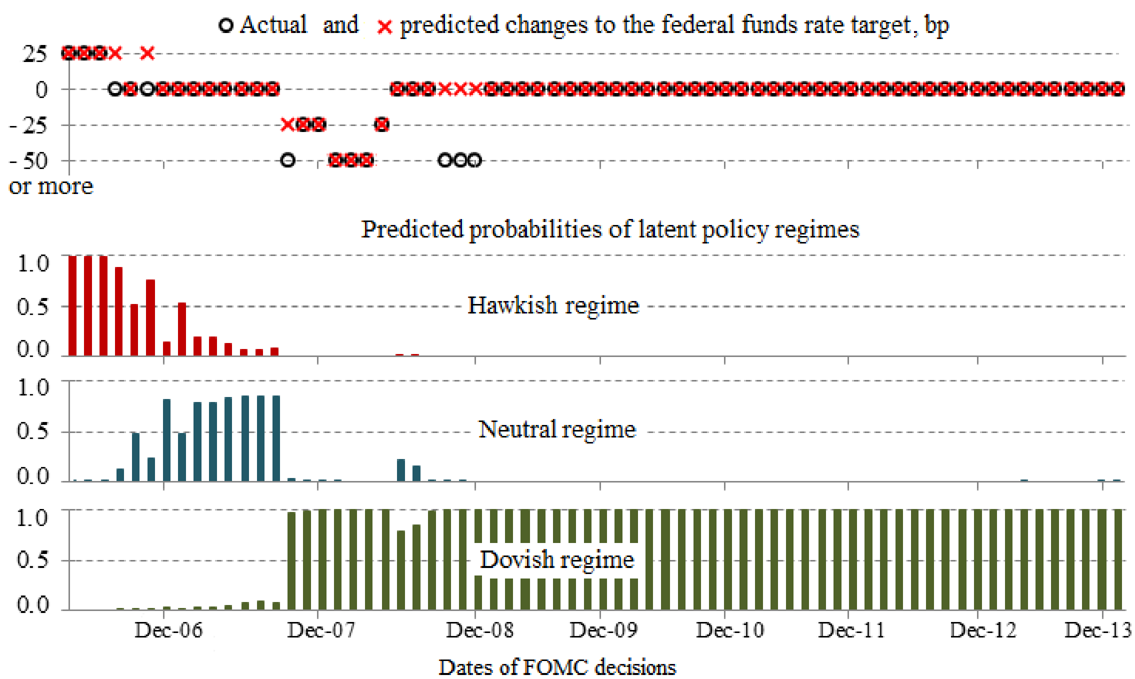
I re-estimated the Swopit model for a shorter period from 11/1992 to 1/2006 with 114 observations, still much more than in Hu and Philips (2004) and Piazzesi (2005). I omitted the first 76 observations from 7/1987 to 10/1992, among which there are 33 decisions made at the unscheduled FOMC meetings and teleconferences; such intermeeting FOMC decisions are more difficult to predict (among 114 decisions from 11/1992 to

[16]The estimation of the endogenous Swopit-z model for a larger sample with 281 observations for the 7/1987–12/2016 period converges but the covariance of the parameters still fails to invert. The estimation results for both exogenous and endogenous Swopit-z models for this period are reported in Tables 21 and 22 of Appendix D. The rise in the likelihood of the endogenous Swopit-z model is significant according to the LR test at the 0.01 level. The endogenous switching is preferred by the AIC, while the BIC prefers the exogenous switching.

1/2006 there are seven intermeeting decisions only). As Table 8 reports, the Swopit-z model demonstrates much higher overall hit rate (89% versus 78% and 75% in Hu and Philips (2004) and Piazzesi (2005), respectively), much higher hit rate for cuts (84% versus 69% and 0%, respectively) with better noise-to-signal ratio, much higher hit rate for hikes (89% versus 50% and 57%, respectively) with similar or better noise-to-signal ratio, and similar hit rate for no changes (but with much better noise-to-signal ratio of 0.14 versus 0.45 and 0.72, respectively). The models in Hu and Philips (2004) and Piazzesi (2005) predict too many zeros, and many of these predictions are wrong. The Swopit model overcomes this typical shortcoming of discrete choice models, which tend to overfit the most popular choice.

### 5.4.1 Out-of-sample forecasting performance

Figure 3. Out-of-sample forecast for the Bernanke era (68 steps ahead): the Swopit-z model correctly predicts 91 percent of FOMC decisions




Notes. The 68-step ahead forecast for the 3/2006–12/2014 period is obtained from the Swopit-z model estimated for the 7/1897–1/2006 period (see Table 6).

How well does the Swopit model, estimated for the Greenspan era, forecast out of sample? Figure 3 shows the actual and predicted target changes as well as the predicted probabilities of latent regimes for the next 68 decisions (without rolling re-estimations), made during the 3/2006–12/2014 period under the Bernanke chairmanship. The Swopit-z model correctly predicts all three hikes, seven out of ten cuts, and 49 out of 51 status quo decisions.

Table 9 compares the out-of-sample forecasting performance of the competing models. The Swopit-z model correctly predicts 91% of the decisions (in terms of five choices) and clearly outperforms the OP (29%) and MIOP (78%) models. The MAE of the forecast in the OP and MIOP models are, respectively, six times and two times as large as in the Swopit-z model. If we compare the forecasts only for the first 27 decisions before the onset of the zero-lower-bound (ZLB) period in December 2008, the Swopit-z model also has the best hit rate of 78%, whereas the OP and MIOP models correctly predict 74% and 48% of the policy decisions, respectively. Among the next 41 decisions after December 2008 during the ZLB period (all of them were status quo outcomes), the Swopit-z and MIOP models correctly predict 100% and 98% of them, respectively, while the OP model makes no correct predictions (it predicts 41 cuts). According to the Swopit-z forecast, the status quo decisions in the 8/2006–8/2007 period are either the "hawkish" or "neutral" zeros, while since 1/2009 the zeros are generated exclusively by the dovish regime. The Swopit-z model predicts that the probability of a dovish regime is close to one during the entire 10/2007–12/2014 period; nevertheless, it predicts no cuts during the ZLB period when there was actually no scope for reducing target further.

Table 9. Comparison with the OP and MIOP models: the Swopit-z model provides the best out-of-sample forecast for the Bernanke era (68 steps ahead)

| Model: | OP | MIOP | Swopit-z |
|---|---|---|---|
| Hit rate (in terms of 5 choices) | 0.29 | 0.78 | **0.91** |
| MAE, basis points | 19.5 | 7.0 | **3.3** |

Notes. The 68-step ahead forecasts for the 3/2006–12/2014 period are obtained from the models estimated for the 7/1897–1/2006 period.

## 5.5 Surmounting the endogeneity problem

The model is cast in a predictive format. If our interest is not in forecasting per se but rather in estimating Fed reaction to economic and financial developments, we must pay attention to possible correlation between the regressors and monetary policy shocks in order to avoid a bias in the estimates.[17] Although the model predicts the next Fed

[17] See de Vries and Li (2014) for a discussion of the problem of endogeneity and an assessment of the magnitude of the bias in the conventional estimates of linear monetary policy rules; see also Kim (2004) for endogeneity problem in the estimation of a forward-looking linear reaction function of the Fed with an unknown break date.

decision using the predetermined values of explanatory variables as they were known prior to each FOMC meeting (so there is no reverse causal effect from the shocks to the regressors), we however should worry about the possible endogeneity problem for two reasons. First, the FOMC policy bias directives or balance-of-risks assessments may contain internal Fed information about monetary shock at the next policy meeting; hence, the variables $tight_{t-1}$ and $ease_{t-1}$ may be endogenous to the shocks. Second, according to the efficient market hypothesis, the market participants use all available information to predict the upcoming policy actions, and the market interest rates may move in anticipation of FOMC decisions; thus, the variable $spread_t$ may be endogenous.

To overcome the possible endogeneity problem, I apply the CF estimation approach that introduces residuals from the reduced form for the endogenous regressors into the structural equation as controls for endogeneity. These residuals are presumably those components of the endogenous variables that are attributable to the unobserved monetary policy shocks. The estimation of these residuals raises a host of issues (such as the lack, validity and strength of the instruments)[18] that we can avoid in this application by computing instead the market-based proxies for monetary shocks in order to use them as controls for endogeneity.

Cochrane and Piazzesi (2002) point out that monetary shocks derived from daily interest rate data are nearly the ideal measures of unanticipated changes to the target. Financial markets can flexibly employ the boundless spectrum of information in order to predict Fed decisions, surmounting the omitted-variable and time-varying-parameter problems common in econometric estimations. Krueger and Kuttner (1996) document that the federal funds futures rates provide the efficient forecasts of funds rate changes. Gürkaynak, Sack and Swanson (2007) report that the federal funds futures dominate the other securities in forecasting monetary policy at horizons up to six months.

*Measuring monetary policy shocks.*

I estimate monetary policy shocks as surprises, that is as FOMC decisions on the target unanticipated by the market. Following Kuttner (2001), the one-day surprises are measured by market reaction to FOMC actions using the change in the implied rate of current-month federal funds futures on the day of policy action:

[18] As Sims and Zha (2006) point out, identification in instrumental variable model is based on claiming that a list of instrumental variables is available to control for the endogeneity of explanatory variables, but these instruments are available only because of a claim that we know a priori that they do not belong directly to the central bank reaction function and can affect monetary policy only through their effects on the future values of regressors. However, it seems inherently implausable that, for instance, the central bank response to an expected future 3-percent inflation rate does not depend on whether the past inflation rate was 1.5 percent or 6 percent.

$$surprise_\tau = \frac{n}{n-\tau}(f_\tau - f_{\tau-1}), \tag{6}$$

where $\tau$ is the day of month when the policy action became known to market participants, $n$ is the number of days in the current month, $f_\tau$ and $f_{\tau-1}$ are the current-month futures rate on the day of policy action and on the day prior to policy action, respectively. The futures rate is computed as 100 minus the contract's settlement price. The contracts, known as "30 day federal funds futures", are traded on the Chicago Mercantile Exchange. Each contract is for interest on federal funds for one month calculated on a 30-day basis at a rate equal to the average overnight effective federal funds rate for the contract month. The data are retrieved from Quandl.[19] The first multiplier is a scaling factor that accounts for the number of remaining days in the month affected by a policy action. If a policy action occurs on the first day of the month, the one-month futures rate on the last day of the previous month is subtracted from the current-month futures rate on the first day of the current month. If an FOMC decision comes on one of the last three days of the month, the unscaled difference in the one-month futures rates is used instead, following Bernanke and Kuttner (2005).

The timing of Fed actions (the dates when the market participants became aware of them) is crucial for measuring monetary policy surprises. The effective dates of Fed actions and the estimated monetary policy shocks are reported in Table 14 of Appendix D. The employed chronology is taken from the Board of Governors of the Fed (via ALFRED) with several adjustments suggested by Kuttner (2001, 2003) on the basis of the analysis of market reaction.[20]

*The CF estimates of the Swopit-z model*

The exogeneity of estimated monetary policy shocks is first tested by regressing them on all explanatory variables. The null hypothesis of exogeneity is rejected (the $p$-value of $F$-test is 0.0000). Table 10 reports the second-step CF estimates of the Swopit-z parameters with monetary policy surprises included as controls for endogeneity in all three equations. The usual $t$ statistics for the coefficients on monetary policy surprises from the second step of CF estimation procedure can also be used to test exogeneity. The coefficients on $surprise_t$ are significant at the 0.001 level in each equation. The null of exogeneity is rejected again.[21] The inclusion of $surprise_t$ substantially improves the model fit: the log likelihood increases from -139.1 to -104.3, the hit rate rises from

[19] https://www.quandl.com

[20] See notes to Table 14 of Appendix D for the details.

[21] Evidence in favor of endo/exogeneity should be interpreted conditional on the correct model specification.

0.71 to 0.79, and the MAE decreases from 8.2 bp to 5.5 bp. The CF Swopit-z model is clearly preferred to the standard Swopit-z model by the AIC and BIC.

The endogeneity of explanatory variables causes a severe bias in the standard estimates of the Swopit-z model. The null hypothesis that the parameters in the standard and CF Swopit-z models do not differ jointly is rejected by the LR test (the $p$-value is 0.0000); the slope parameters in two models do also differ jointly (the $p$-value is 0.0000). Individually, only three slope parameters differ in two models at the 0.001 significance level: the coefficients on $spread_t$ and $house_t$ in the regime equation and on $spread_t$ in the output equation under the hawkish regime. The other slope parameters are not individually different from their counterparts at the 0.10 significance level.

Table 10. Endogeneity does matter: the CF estimates of the Swopit-z model parameters

| Variables | Regime equation | Output equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| $tight_{t-1}$ | 2.03 (0.46)*** | | 2.56 (1.23)** |
| $ease_{t-1}$ | -4.04 (1.44)*** | -0.14 (0.38) | |
| $spread_t$ | 2.90 (0.59)*** | 1.36 (0.33)*** | 0.46 (0.75) |
| $house_t$ | 5.44 (1.22)*** | | |
| $surprise_t$ | 13.45 (3.98)*** | 8.68 (1.66)*** | 22.00 (7.02)*** |
| $threshold_1$ | 7.17 (2.04)*** | -2.48 (0.46)*** | -2.21 (1.35) |
| $threshold_2$ | 10.56 (2.11)*** | -0.61 (0.38) | 3.60 (1.37)*** |
| Log likelihood | AIC | BIC | Hit rate |
| -104.3 | 242.7 | 297.9 | 0.79 |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses.

The null hypothesis that the policy reactions are symmetrical (that is, the CF Swopit-z coefficients on $spread_t$ are equal and coefficients on $tight_{t-1}$ and $ease_{t-1}$ have the same absolute value but opposite signs in the dovish and hawkish regimes) is rejected by the LR-test (the $p$-value is 0.003). The CF Swopit-z model is also preferred to the CF MIOP and CF OP models by the AIC and BIC, and the Vuong test (the $p$-values are 0.007 and 0.000, respectively). The CF estimates of the MEs of the covariates on choice probabilities can differ significantly in the OP, MIOP and Swopit models. For example, as Table 11 shows, the ME of $ease_{t-1}$ on the probability of no change in the Swopit-z model is -0.23. It means that if the policy bias directive at the last FOMC meeting changes from "symmetrical" to "easing", the probability of no change reduces by 0.23

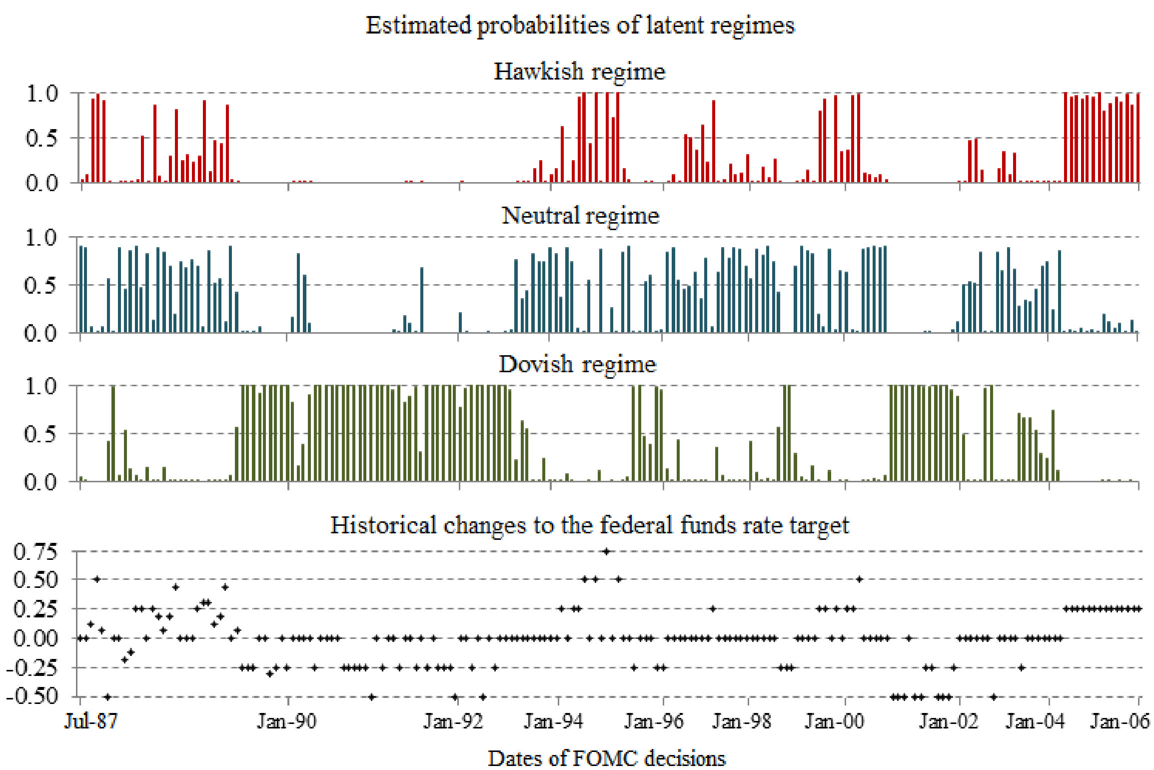
(holding all other covariates at their sample medians). In the OP and MIOP models, the ME estimates are five times as small as in the Swopit-z model, and statistically insignificant even at the 0.2 level.

Table 11. The marginal effects of the covariates on choice probabilities can differ significantly in the competing models

| Model | OP | MIOP | Swopit-z |
|---|---|---|---|
| ME of $ease_{t-1}$ on $\Pr(y_t = 0)$ | -0.05 (0.04) | -0.04 (0.04) | -0.23 (0.09)*** |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The MEs are computed using CF estimators and fixing the other variables at their sample median values.

Figure 4. Estimated probabilities of latent regimes and actual FOMC decisions during the Greenspan era

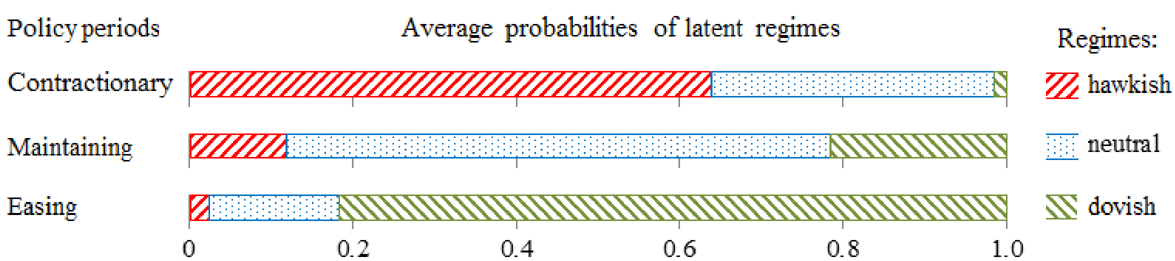



Notes. Sample period: 7/1987–1/2006 (190 observations). The estimates are obtained from the CF Swopit-z model (Table 10).

Figure 4 shows the estimated probabilities of latent policy regimes for each FOMC decision in the Greenspan era. The average probabilities of the hawkish, neutral and dovish regimes are 0.23, 0.35 and 0.42, whereas the observed frequencies of hikes, zeros and cuts are 0.22, 0.54 and 0.24, respectively. On average, the ratio of the probability of no change conditional on the neutral regime to the unconditional probability of no change $\Pr(y_t = 0|r_t = 0)/\Pr(y_t = 0)$ is 0.66. It means that only about two thirds of zeros are the neutral zeros; one third of zeros are generated in either dovish or hawkish policy regime. It turns out that one third of all outcomes in the dovish and hawkish regimes are zeros — the outcome decisions tend to leave the rate unchanged by weakening the tightening and easing policy inclinations.

Figure 5 illustrates the correspondence of latent regimes to the contractionary, maintaining and easing policy periods shown in Figure 2. The profiles of regime probabilities differ considerably. Monetary policy decisions in the contractionary/maintaining/easing periods are dominated by the hawkish/neutral/dovish regimes, respectively; however, in the maintaining periods one third of status quo outcomes are the dovish or hawkish zeros.

Figure 5. Average probabilities of latent regimes in different policy periods




Notes. Sample period: 7/1987–1/2006 (190 observations). The estimates are obtained from the CF Swopit-z model (Table 10). The policy periods are shown in Figure 2.

# Concluding remarks

Data measured on an ordinal scale are frequently encountered in many fields and can be generated by a heterogeneous process. In the cross-sectional context, the data can be generated by different unobserved groups (latent segments) of the population; in the time-series context — by different unobserved states (latent regimes). These issues are well recognized and adequately addressed by various switching regression techniques developed for continuous outcomes, with either exogenous or endogenous switching among two or more known or unknown regimes. For ordered categorical responses, the regime switching approach is less developed and mostly limited to binary outcomes, two regimes, exogenous switching, known sample separation and exogenous regressors.

This study fills a gap in the literature with methodological contribution of new tools for analyzing ordered-choice models with endogenous switching among $N$ latent regimes and with endogenous regressors. Different aspects of the methodology are illustrated through the Monte Carlo experiments and empirical application to the federal funds rate target. The simulations and empirical results make it clear that the regime switching and the endogeneity of explanatory variables do matter and can cause a severe bias in the estimates of monetary policy rules. The proposed model overwhelmingly outperforms the existing ordered-choice models both in- and out-of sample.

The flexible multi-part structure of the proposed model is also able to overcome some typical shortcomings of the conventional single-regime ordered-choice models. The new model allows a certain variable to have the same sign of the marginal effect on the probabilities of both the largest and smallest categories, and also allows the sign of the marginal effect to change more than once when moving from the smallest category to the largest one. In the conventional models, the marginal effects on the probabilities of the outcomes at the opposite ends of the ordered scale always have the opposite sign, and the sign of the marginal effect can only change once when moving from the smallest category to the largest one. The new model also overcomes another typical shortcoming of single-equation models, which tend to overfit the most popular choice.

The new method can be used to more adequately represent monetary policy rules of many central banks, can be embedded in multivariate macroeconomic models to better understand the monetary policy effects, and can be fruitfully applied to various ordinal data with an unobserved sample separation or a regime switching environment.

# References


Andrews, D. W. K. (2001), "Testing when a parameter is on the boundary of the maintained hypothesis." *Econometrica,* 69 (3), 683–734.

Ang, A. and G. Bekaert (2002), "Regime switches in interest rates." *Journal of Business and Economic Statistics*, 20 (2), 163–182.

Baele, L., G. Bekaert, S. Cho, K. Inghelbrecht, and A. Moreno (2015), "Macroeconomic regimes." *Journal of Monetary Economics*, 70, 51–71.

Bagozzi, B. E. and B. Mukherjee (2012), "A mixture model for middle category inflation in ordered survey responses." *Political Analysis,* 20, 369–386.

Barthélemy, J. and M. Marx (2017), "Solving endogenous regime switching models." *Journal of Economic Dynamics and Control*, 77, 1–25.

Bazzi, M., F. Blasques, S. J. Koopman, and A. Lucas (2017), "Time-Varying transition probabilities for Markov regime switching models." *Journal of Time Series Analysis*, 38 (3), 458–478.

Bernanke, B. S. (1990), "On the predictive power of interest rates and interest rate spreads." *New England Economic Review,* November/December, 51–68.

Bernanke, B. S. and K. N. Kuttner (2005), "What explains the stock market's reaction to Federal Reserve policy?" *Journal of Finance,* 60 (3), 1221–1257.

Bhattacharya, J., D. Goldman, and D. McCaffrey (2006), "Estimating probit models with self-selected treatments." *Statistics in Medicine,* 25, 389–413.

Bianchi, F. (2013), "Regime switches, agents' beliefs, and post-World War II U.S. macroeconomic dynamics." *Review of Economic Studies*, 80 (2), 463–490.

Bikbov, R. and M. Chernov (2013), "Monetary policy regimes and the term structure of interest rates." *Journal of Econometrics*, 174 (1), 27–43.

Boivin, J. and M. Giannoni (2006), "Has monetary policy become more effective?" *Review of Economics and Statistics,* 88 (3), 445–462.

Brooks, R., M. N. Harris, and C. Spencer (2012), "Inflated ordered outcomes." *Economics Letters,* 117 (3), 683–686.

Carneiro, P., K. Hansen, and J. Heckman (2003), "Estimating distributions of treatment effects with an application to the returns to schooling and measurement of the effects of uncertainty on college choice." *International Economic Review,* 44, 361–422.

Chang, Y., Y. Choi, and J. Y. Park (2017), "A new approach to model regime switching." *Journal of Econometrics*, 196 (1), 127–143.

Chesher, A. and K. Smolinski (2012), "IV models of ordered choice." *Journal of Econometrics,* 166, 33–48.

Chib, S. and M. J. Dueker (2004), "Non-Markovian regime switching with endogenous states and time-varying state strengths." Federal Rererve Bank of St. Louis Working Paper No. 2004-030A.

Chib, S. and B. H. Hamilton (2000), "Bayesian analysis of cross-section and clustered data treatment models." *Journal of Econometrics*, 97 (1), 25–50.

Clarida, R., J. Galí, and M. Gertler (2000), "Monetary policy rules and macroeconomic stability: Evidence and some theory." *Quarterly Journal of Economics*, 115 (1), 147–180.

Cochrane, J. H. and M. Piazzesi (2002), "The Fed and interest rates: A high-frequency identification." *American Economic Review,* 92 (2), 90–95.

Coibion, O. and Y. Gorodnichenko (2011), "Monetary policy, trend inflation, and the great moderation: An alternative interpretation." *American Economic Review*, 101 (1), 341–70.

Cooley, T. F., S. F. LeRoy, and N. Raymon (1984), "Econometric policy evaluation: Note." *American Economic Review*, 74 (3), 467–470.

Davig, T. and E. M. Leeper (2007), "Generalizing the Taylor principle." *American Economic Review*, 97 (3), 607–635.

Davig, T. and E. M. Leeper (2008), "Endogenous monetary policy regime change." in *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, 345–391, National Bureau of Economic Research, Inc.

Desantis, S. M., E. A. Houseman, B. A. Coull, A. Stemmer-Rachamimov, and R. A. Betensky (2008), "A penalized latent class model for ordinal data." *Biostatistics,* 9 (2), 249–262.

de Vries, C. G. and W. Li (2014), "Identifying monetary policy rules with serially correlated monetary policy shocks." Working paper, Erasmus University Rotterdam and Tinbergen Institute.

Diebold, F., J.-H. Lee, and G. Weinbach (1994), "Regime switching with time-varying transition probabilities." In: Hargreaves, C. (ed.), *Nonstationary Time Series Analysis and Cointegration* (Advanced Texts in Econometrics, C. W. J. Granger and G. Mizon, eds.), Oxford University Press, Oxford, 283–302.

Dolado, J., R. Maria-Dolores, and M. Naveira (2005), "Are monetary-policy reaction functions asymmetric?: The role of nonlinearity in the Phillips curve." *European Economic Review*, 49 (2), 485–503.

Eluru, N., M. Bagheri, L. F. Miranda-Moreno, and L. Fu (2012), "A latent class modeling approach for identifying vehicle driver injury severity factors at highway-railway crossings." *Accident Analysis and Prevention*, 47, 119–127.

Estrella, A. and G. A. Hardouvelis (1991), "The term structure as a predictor of real economic activity." *Journal of Finance,* 46 (2), 555–576.

Estrella, A. and F. S. Mishkin (1998), "Predicting U.S. recessions: Financial variables as leading indicators." *Review of Economics and Statistics,* 80 (1), 45–61.

Filardo, A. (1994), "Business-cycle phases and their transitional dynamics." *Journal of Business and Economic Statistics*, 12 (3), 299–308.

Foerster, A. T. (2016), "Monetary policy regime switches and macroeconomic dynamics." *International Economic Review*, 57, 211–230.

Frankel, J. A. and C. S. Lown (1994), "An indicator of future inflation expected from the steepness of the interest rate yield curve along its entire length." *Quarterly Journal of Economics,* 109 (2), 517–530.

Garcia, R. and P. Perron (1996), "An analysis of the real interest rate under regime shifts." *Review of Economics and Statistics*, 78 (1), 111–125.

Gerlach, S. (2007), "Interest rate setting by the ECB, 1999–2006: Words and deeds."

*International Journal of Central Banking,* 3 (3), 1–46.

Gerlach, S. (2011), "ECB repo rate setting during the financial crisis." *Economics Letters*, 112 (2), 186–188.

Gerlach, S. and J. Lewis (2014), "Zero lower bound, ECB interest rate policy and the financial crisis." *Empirical Economics*, 46 (3), 865–886.

Geweke, J., G. Gowrisankaran, and R. J. Town (2003), "Bayesian inference for hospital quality in a selection model." *Econometrica*, 71 (4), 1215–1238.

Greene, W. H., M. Harris, B. Hollingsworth, and P. Maitra (2008), "A bivariate latent class correlated generalized ordered probit model with an application to modeling observed obesity levels." Working paper No. 08-18, New York University.

Gregory, C. A. (2015), "Estimating treatment effects for ordered outcomes using maximum simulated likelihood." *Stata Journal,* 15 (3), 756–774.

Gronau, R. (1974), "Wage comparisons — a selectivity bias." *Journal of Political Economy*, 82 (6), 1119–1143.

Gürkaynak, R. S., B. P. Sack, and E. T. Swanson (2007), "Market-based measures of monetary policy expectations." *Journal of Business and Economic Statistics,* 25 (2), 201–212.

Hamilton, J. D. (1988), "Rational-expectations econometric analysis of changes in regime: An investigation of the term structure of interest rates." *Journal of Economic Dynamics and Control*, 12 (2-3), 385–423.

Hamilton, J. D. (1989), "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica,* 57 (2), 357–384.

Hamilton, J. D. and O. Jorda (2002), "A model for the federal funds rate target." *Journal of Political Economy,* 110 (5), 1135–1167.

Harris, M. N. and X. Zhao (2007), "A zero-inflated ordered probit model, with an application to modelling tobacco consumption." *Journal of Econometrics,* 141 (2), 1073–1099.

Hausman, J. A. (1978), "Specification tests in econometrics." *Econometrica,* 46, 1251–1271.

Heckman, J. (1974), "Shadow prices, market wages, and labor supply." *Econometrica*, 42, 679–694.

Hill, D. H. (1990), "An endogenously-switching ordered-response model of information, eligibility and participation in SSI." *Review of Economics and Statistics*, 72 (2), 368–371.

Hu, L. and P. C. B. Phillips (2004), "Dynamics of the federal funds target rate: a nonstationary discrete choice approach." *Journal of Applied Econometrics*, 19, 851–867.

Hwu, S.-T., C.-J. Kim, and J. Piger (2017), "An N-state endogenous Markov-switching model with applications in macroeconomics and finance." Manuscript, University of Washington.

Kaminsky, G. L. and C. M. Reinhart (1999), "The twin crises: the causes of banking and balance-of-payments problems." *American Economic Review,* 89 (3), 473–500.

Kauppi, H. (2012), "Predicting the direction of the Fed's target rate." *Journal of Forecasting,* 31, 47–67.

Kelley, M. E. and S. J. Anderson (2008), "Zero inflation in ordinal data: incorporating susceptibility to response through the use of a mixture model." *Statistics in Medicine,* 27, 3674–3688.

Kim, C.-J. (2004), "Markov-switching models with endogenous explanatory variables." *Journal of Econometrics*, 122 (1), 127–136.

Kim, C.-J. (2009), "Markov-switching models with endogenous explanatory variables II: A two-step MLE procedure." *Journal of Econometrics*, 148 (1), 46–55.

Kim, C.-J., J. Piger, and R. Startz (2008), "Estimation of Markov regime-switching regression models with endogenous switching." *Journal of Econometrics*, 143 (2), 263–273.

Kuttner, K. N. (2001), "Monetary policy surprises and interest rates: Evidence from the Fed funds futures market." *Journal of Monetary Economics,* 47 (3), 523–544.

Kuttner, K. N. (2003), "Dating changes in the federal funds rate, 1989–92." Manuscript, Federal Reserve Bank of New York.

Lapp, J. S. and D. K. Pearce (2000), "Does a bias in FOMC policy directives help predict intermeeting policy changes?" *Journal of Money, Credit and Banking,* 32 (3), 435–441.

Lewis, H. (1974), "Comments on selectivity biases in wage comparisons." *Journal of Political Economy*, 82 (6), 1145–1155.

Li, M. and J. L. Tobias (2008), "Bayesian analysis of treatment effects in an ordered potential outcomes model." In: Millimet, D., J. Smith, and E. Vytlacil (eds), *Modelling and Evaluating Treatment Effects in Econometrics (* Advances in Econometrics, 21, Fomby, T. B., R. C. Hill, I. Jeliazkov, J. C. Escanciano, and E. Hillebrand, eds), Elsevier Ltd., 57–91.

Lubik, T. A. and F. Schorfheide (2004), "Testing for indeterminacy: An application to U.S. monetary policy." *American Economic Review*, 94 (1), 190–217.

MacKinnon, J. G. (1996), "Numerical distribution functions for unit root and cointegration tests." *Journal of Applied Econometrics*, 11, 601–618.

Maddala, G. S. (1983), "Limited dependent and qualitative variables in econometrics." Cambridge University Press, New York.

Maddala, G. S. and F. Nelson (1975), "Switching regression models with exogenous and endogenous switching." *Proceedings of the American Statistical Association* (Business and Economics Section), 423–426.

Mankiw, N. G., J. A. Miron and D. N. Weil (1987), "The adjustment of expectations to a change in regime: A study of the founding of the Federal Reserve." *American Economic Review*, 77 (3), 358–374.

McKelvey, R. D. and W. Zavoina (1975), "A statistical model for the analysis of ordinal level dependent variables." *Journal of Mathematical Sociology*, 4, 103–120.

Miranda, A. and S. Rabe-Hesketh (2006), "Maximum likelihood estimation of endogenous switching and sample selection models for binary, ordinal, and count variables." *Stata Journal*, 6 (3), 285–308.

Mishkin, F. S. (1990), "The information in the longer-maturity term structure about future inflation." *Quarterly Journal of Economics,* 105 (3), 815–28.

Munkin, M. K. and P. K. Trivedi (2008), "Bayesian analysis of the ordered probit model with endogenous selection" *Journal of Econometrics,* 143, 334–348.

Newey, W. K. (1987), "Efficient estimation of limited dependent variable models with endogenous explanatory variables." *Journal of Econometrics*, 36 (3), 231–250.

Pakko, M. R. (2005), "On the information content of asymmetric FOMC policy statements: evidence from a Taylor-rule perspective." *Economic Inquiry,* 43 (3), 558–569.

Piazzesi, M. (2005), "Bond yields and the Federal Reserve." *Journal of Political Economy,* 113 (2), 311–344.

Rivers, D. and Q. Vuong (1988), "Limited information estimators and exogeneity tests for simultaneous probit models." *Journal of Econometrics,* 39, 347–366.

Roy, A. D. (1951), "Some thoughts on the distribution of earnings." *Oxford Economic Papers*, 3, 135–146.

Schorfheide, F. (2005), "Learning and monetary policy shifts." *Review of Economic Dynamics*, 8 (2), 392–419.

Sims, C. A. and T. Zha (2006), "Were there regime switches in U.S. monetary policy?" *American Economic Review,* 96 (1), 54–81.

Smith, R. J. and R. W. Blundell (1986), "An exogeneity test for a simultaneous equation tobit model with an application to labor supply." *Econometrica,* 54, 679–685.

Terza, J. V., A. Basu, and P. J. Rathouz (2008), "Two-stage residual inclusion estimation: Addressing endogeneity in health econometric modeling." *Journal of Health Economics*, 27 (3), 531–543.

Thornton, D. L. (2005), "A new federal funds rate target series: September 27, 1982 – December 31, 1993." Federal Reserve Bank of St. Louis Working Paper No. 2005-032A, May.

Thornton, D. L. and D. C. Wheelock (2000), "A history of the asymmetric policy directive." Federal Reserve Bank of St. Louis *Review,* 82 (5), 1–16.

Van den Hauwe, R. Paap, and D. van Dijk (2013), "Bayesian forecasting of federal funds target rate decisions." *Journal of Macroeconomics,* 37, 19–40.

Vanderhart, P. G. (2000), "The Federal Reserve's reaction function under Greenspan: An ordinal probit analysis." *Journal of Macroeconomics*, 22 (4), 631–644.

Vuong, Q. (1989), "Likelihood ratio tests for model selection and non-nested hypotheses." *Econometrica,* 57 (2), 307–333.

Wilde, J. (2000), "Identification of multiple equation probit models with endogenous dummy regressors." *Economics Letters,* 69 (3), 309–312.

Wooldridge, J. M. (2014), "Quasi-maximum likelihood estimation and testing for nonlinear models with endogenous explanatory variables." *Journal of Econometrics,* 182, 226–234.

——— *not for publication* ————

# Supplementary Online Material

**for**

**"An endogenous regime switching model of ordered choice with an application to federal funds rate target"**

Andrei Sirchenko

Higher School of Economics, Moscow

## Appendix A. The Swopit models with exogenous regime switching

If $\rho_s = 0, \forall s$ then $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$ are mutually independent and regime switching is exogenous. Assuming that $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}_s$ are iid with standard normal CDF $\Phi$, the probabilities of the outcome $j$ with exogenous switching are given as follows.

### The exogenous $N$-regime Swopit model

$$\begin{aligned}&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}) = \sum_{s=1}^{N} \Pr(r_t = s|\mathbf{x}_t) \Pr(y_t = j|\mathbf{x}_{s,t}, r_t = s) \\ &= \sum_{s=1}^{N} \left\{ [\Phi(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta})][\Phi(\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) - \Phi(\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s)] \right\}.\end{aligned} \tag{7}$$

These probabilities can be computed as

$$\begin{aligned}&\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})\Phi(\alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) \\ &+ \sum_{s=2}^{N-1} \left[\Phi(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] \Phi(\alpha_{s,1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) \\ &+ \left[1 - \Phi(\mu_{N-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] \Phi(\alpha_{N,1} - \mathbf{x}_{N,t}'\boldsymbol{\beta}_N);\end{aligned}$$

$$\begin{aligned}&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[\Phi(\alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) - \Phi(\alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \\ &+ \sum_{s=2}^{N-1} \left\{ \left[\Phi(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] [\Phi(\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) - \Phi(\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s)] \right\} \\ &+ \left[1 - \Phi(\mu_{N-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] [\Phi(\alpha_{N,j} - \mathbf{x}_{N,t}'\boldsymbol{\beta}_N) - \Phi(\alpha_{N,j-1} - \mathbf{x}_{N,t}'\boldsymbol{\beta}_N)] \text{ for } 2 \leq j \leq J-1;\end{aligned}$$

$$\begin{aligned}&\Pr(y_t = J|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[1 - \Phi(\alpha_{1,J-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \\ &+ \sum_{s=2}^{N-1} \left\{ \left[\Phi(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] [\Phi(\alpha_{s,J} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) - \Phi(\alpha_{s,J-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s)] \right\} \\ &+ \left[1 - \Phi(\mu_{N-1} - \mathbf{x}_t'\boldsymbol{\beta})\right] [1 - \Phi(\alpha_{N,J-1} - \mathbf{x}_{N,t}'\boldsymbol{\beta}_N)].\end{aligned}$$

## The exogenous Swopit-r model

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) \\
= &\sum_{s=1,3} \Pr(r_t = s|\mathbf{x}_t) \Pr(y_t = j|\mathbf{x}_{s,t}, r_t = s) + I_{j=q} \Pr(r_t = 2|\mathbf{x}_t) \\
= &\sum_{s=1,3} \left\{ [\Phi(\mu_s - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta})][\Phi(\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s) - \Phi(\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s)] \right\} \\
&+ I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})].
\end{aligned} \tag{8}
$$

These probabilities can be computed as

$$
\begin{aligned}
&\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) \\
&= \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})\Phi(\alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) + [1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})]\,\Phi(\alpha_{3,1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3);
\end{aligned}
$$

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[\Phi(\alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) - \Phi(\alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \\
&+ I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] \\
&+ [1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})]\,[\Phi(\alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3) - \Phi(\alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3)] \text{ for } 2 \leq j \leq J-1;
\end{aligned}
$$

$$
\begin{aligned}
&\Pr(y_t = J|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[1 - \Phi(\alpha_{1,J-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \\
&+ [1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})]\,[1 - \Phi(\alpha_{3,J-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3)].
\end{aligned}
$$

## The exogenous Swopit-z model

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = I_{j\leq q} \Pr(r_t = 1|\mathbf{x}_t) \Pr(y_t = j|\mathbf{x}_{s,t}, r_t = 1) \\
&+ I_{j=q} \Pr(r_t = 2|\mathbf{x}_t) + I_{j\geq q} \Pr(r_t = 3|\mathbf{x}_t) \Pr(y_t = j|\mathbf{x}_{s,t}, r_t = 3) \\
&= I_{j\leq q} \left\{ \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[\Phi(\alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) - \Phi(\alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \right\} \\
&+ I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] \\
&+ I_{j\geq q} \left\{ [1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})][\Phi(\alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3) - \Phi(\alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3)] \right\}.
\end{aligned} \tag{9}
$$

These probabilities can be computed as

$$
\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})\Phi(\alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1);
$$

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = I_{j\leq q}\Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})[\Phi(\alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) - \Phi(\alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1)] \\
&+ I_{j=q}[\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta})] \\
&+ I_{j\geq q}\,[1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})]\,[\Phi(\alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3) - \Phi(\alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3)] \text{ for } 2 \leq j \leq J-1;
\end{aligned}
$$

$$
\Pr(y_t = J|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = [1 - \Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta})]\,[1 - \Phi(\alpha_{3,J-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3)].
$$

## The exogenous Swopit models with EEVs

In the presence of EEVs, the probabilities to observe the outcome $j$ in (7) can be rewritten for the CF estimator of the exogenous $N$-regime Swopit model as

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{2,t}, ..., \mathbf{x}_{N,t}, \widetilde{\mathbf{w}}_t) \\
= &\sum_{s=1}^{N} \left\{ \left[ \Phi\left( \frac{\mu_s - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) - \Phi\left( \frac{\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1 - var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \right] \right. \\
&\left. \times \left[ \Phi\left( \frac{\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}} \right) - \Phi\left( \frac{\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1 - var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}} \right) \right] \right\}.
\end{aligned}
$$

The probabilities in (8) for the exogenous Swopit-r model with the EEVs can be rewritten for the CF estimator as

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}, \widetilde{\mathbf{w}}_t) \\
&= \sum_{s=1}^{N} \left\{ \left[ \Phi\left( \frac{\mu_s - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) - \Phi\left( \frac{\mu_{s-1} - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \right] \right. \\
&\left. \times \left[ \Phi\left( \frac{\alpha_{s,j} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1-var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}} \right) - \Phi\left( \frac{\alpha_{s,j-1} - \mathbf{x}_{s,t}'\boldsymbol{\beta}_s - \mathbf{w}_{s,t}\boldsymbol{\gamma}_s - \boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s}{\sqrt{1-var(\boldsymbol{\nu}_{s,t}\boldsymbol{\lambda}_s)}} \right) \right] \right\} \\
&+ I_{j=q} \left[ \Phi\left( \frac{\mu_2 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) - \Phi\left( \frac{\mu_1 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \right].
\end{aligned}
$$

The probabilities in (9) for the exogenous Swopit-z model with the EEVs can be rewritten for the CF estimator as

$$
\begin{aligned}
&\Pr(y_t = j|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}, \widetilde{\mathbf{w}}_t) \\
&= I_{j\leq q} \Phi\left( \frac{\mu_1 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \\
&\times \left[ \Phi\left( \frac{\alpha_{1,j} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1 - \mathbf{w}_{1,t}\boldsymbol{\gamma}_1 - \boldsymbol{\nu}_{1,t}\boldsymbol{\lambda}_1}{\sqrt{1-var(\boldsymbol{\nu}_{1,t}\boldsymbol{\lambda}_1)}} \right) - \Phi\left( \frac{\alpha_{1,j-1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1 - \mathbf{w}_{1,t}\boldsymbol{\gamma}_1 - \boldsymbol{\nu}_{1,t}\boldsymbol{\lambda}_1}{\sqrt{1-var(\boldsymbol{\nu}_{1,t}\boldsymbol{\lambda}_1)}} \right) \right] \\
&+ I_{j=q} \left[ \Phi\left( \frac{\mu_2 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) - \Phi\left( \frac{\mu_1 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \right] \\
&+ I_{j\geq q} \left[ 1 - \Phi\left( \frac{\mu_2 - \mathbf{x}_t'\boldsymbol{\beta} - \mathbf{w}_t\boldsymbol{\gamma} - \boldsymbol{\nu}_t\boldsymbol{\lambda}}{\sqrt{1-var(\boldsymbol{\nu}_t\boldsymbol{\lambda})}} \right) \right] \\
&\times \left[ \Phi\left( \frac{\alpha_{3,j} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3 - \mathbf{w}_{3,t}\boldsymbol{\gamma}_3 - \boldsymbol{\nu}_{3,t}\boldsymbol{\lambda}_3}{\sqrt{1-var(\boldsymbol{\nu}_{3,t}\boldsymbol{\lambda}_3)}} \right) - \Phi\left( \frac{\alpha_{3,j-1} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3 - \mathbf{w}_{3,t}\boldsymbol{\gamma}_3 - \boldsymbol{\nu}_{3,t}\boldsymbol{\lambda}_3}{\sqrt{1-var(\boldsymbol{\nu}_{3,t}\boldsymbol{\lambda}_3)}} \right) \right].
\end{aligned}
$$

# Appendix B. A special case when the Swopit-z model nests the MIOP model

In general, the Swopit-z and MIOP models (see the upper panel of Figure 1) are not nested in each other, but they are not strictly non-nested. They overlap if their slope coefficients are all fixed to zero, and only the thresholds are estimated. However, there is an interesting special case when the Swopit-z model does nest the MIOP model. The special case arises under certain parameter restrictions provided (i) there are only three outcome categories of the dependent variable, (ii) both $\mathbf{X}_1$ and $\mathbf{X}_3$ in the Swopit-z outcome equations contain all covariates in the MIOP regime equation, and (iii) $\mathbf{X}$ in the Swopit-z regime equation includes all covariates in the MIOP outcome equation.

To begin, I first describe the MIOP econometric framework. Then I explain under which conditions the Swopit-z model nests the MIOP model.

## The MIOP model

Let $t$ $(t = 1, 2, ..., T)$ be one of the available $T$ observations. Let $y_t$ be an observed dependent variable that can take on a finite number $J$ of ordinal values, coded by index $j$ $(j = 1, 2, ..., J,\ J > 2)$, among which a potentially heterogeneous ("inflated") and often predominant response is coded as $q$, $1 < q < J$. The observed outcome $y_t$ can be generated in any of two states, coded as 1 or 0 and interpreted as latent regimes in the time-series context or as latent segments of population in the cross-section context. The realized states (regimes) $r_t^m$ are only partially observed and of an ordinal nature. The regime switching decision is determined by the continuous latent variable $r_t^{m*}$, endogenously driven in response to the observed data and unobservables according to the binary probit regime equation. The correspondence between $r_t^{m*}$ and $r_t^m$ is determined by an unobserved threshold in the usual binary-response fashion. For each $t$, only one out of two potential realizations of $y_t$ is observed. Conditional on being in the regime $r_t^m = 1$, the observed outcome $y_t$ is determined (in the usual OP fashion) by a continuous latent variable $y_t^{m*}$, which is driven in response to the observed data and unobservables according to an outcome equation. Conditional on being in the regime $r_t^m = 0$, the observed outcome $y_t$ is always $q$.

To summarize, the MIOP model can be described by the following system

$$
\begin{array}{ll}
r_t^{m*} = \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m + \varepsilon_t^m & \text{(regime equation),} \\
r_t^m|\mathbf{x}_t^m = \begin{cases} 1 \text{ if } \mu^m < r_t^{m*}|\mathbf{x}_t^m \\ 0 \text{ if } r_t^{m*}|\mathbf{x}_t^m \le \mu^m \end{cases} & \text{(regime matching rule)} \\
y_{1,t}^{m*} = \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m + \varepsilon_{1,t}^m & \text{(outcome equation),} \\
y_t|(\mathbf{x}_{1t}^m, r_t^m = 1) = j \text{ if } \alpha_{1,j-1}^m < y_{1,t}^{m*}|\mathbf{x}_{1,t}^m \le \alpha_{1,j}^m,\ j = 1, 2, ..., J & \text{(outcome matching} \\
y_t|(r_t^m = 0) = q,\ 1 < q < J & \text{rules),} \\
\begin{bmatrix} \varepsilon_{1,t}^m \\ \varepsilon_t^m \end{bmatrix} \overset{iid}{\sim} Normal\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} 1 & \rho^m \\ \rho^m & 1 \end{bmatrix} \right) & \begin{array}{l}\text{(interdependence} \\ \text{between the regime} \\ \text{and outcome decisions),}\end{array}
\end{array}
$$

where $\mathbf{x}_t^m$ and $\mathbf{x}_{1,t}^m$ are the $t^{\text{th}}$ rows of the observed data matrices $\mathbf{X}^m$ and $\mathbf{X}_1^m$, respectively; $\boldsymbol{\beta}^m$ and $\boldsymbol{\beta}_1^m$ are the vectors of unknown slope parameters; $\varepsilon_t^m$ and $\varepsilon_{1,t}^m$ are the $t^{\text{th}}$

rows of the iid across $t$ disturbance terms $\boldsymbol{\varepsilon}^m$ and $\boldsymbol{\varepsilon}_1^m$; $\varepsilon_t^m$ is independent of $\varepsilon_{1,t}^m$ at leads and lags: $E(\varepsilon_t^m \varepsilon_{1,t+\tau}^m) = 0$ for $\forall \tau \neq 0$; $\mu^m$ and $-\infty \equiv \alpha_{1,0}^m \leq \alpha_{1,1}^m \leq ... \leq \alpha_{1,J}^m \equiv \infty$ are the unknown threshold parameters; the joint probability density between $\varepsilon_t^m$ and $\varepsilon_{1,t}^m$ is standardized bivariate normal with CDF $\Phi_2(\varepsilon^m;\varepsilon_1^m;\rho^m)$.

The probabilities of the outcome $j$ in the MIOP model are given by

$$
\begin{aligned}
&\Pr(y_t^m = j|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Pr(r_t^{m*}|\mathbf{x}_t^m > \mu^m \text{ and } \alpha_{1,j-1}^m < y_{1,t}^{m*}|\mathbf{x}_{1,t}^m \leq \alpha_{1,j}^m) \\
&+I_{j=q} \Pr(r_t^{m*}|\mathbf{x}_t^m \leq \mu^m) \\
&= \Pr(\varepsilon_t > \mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m \text{ and } \alpha_{1,j-1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m < \varepsilon_{1,t}^m \leq \alpha_{1,j}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) \\
&+I_{j=q} \Pr(\varepsilon_t^m \leq \mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m) \\
&= \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,j}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) \\
&-\Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,j-1} - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) + I_{j=q}\Phi(\mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m),
\end{aligned} \tag{10}
$$

where $I_{j=q}$ is an indicator function such that $I_{j=q} = 1$ if $j = q$, and $I_{j=q} = 0$ if $j \neq q$.

## A special case

Suppose a dependent variable $y_t^m$ takes only three discrete values $j$ coded as $\{1, 2, 3\}$, and an inflated response is coded as 2 ($q = 2$).

The probabilities of observing the outcome $j$ in the MIOP model are given according to (10) as

$$
\begin{aligned}
&\Pr(y_t = 1|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m); \\
&\Pr(y_t = 2|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi(\mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m) \\
&+\Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,2}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) \\
&-\Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m); \\
&\Pr(y_t = 3|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; -\alpha_{1,2}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m;\rho^m).
\end{aligned} \tag{11}
$$

The probabilities of observing the outcome $j$ in the Swopit-z model are given according to (5) as

$$
\begin{aligned}
&\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; \rho_1); \\
&\Pr(y_t = 2|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}; -\alpha_{1,1} + \mathbf{x}_{1,t}'\boldsymbol{\beta}_1; -\rho_1) \\
&+\Phi(\mu_2 - \mathbf{x}_t'\boldsymbol{\beta}) - \Phi(\mu_1 - \mathbf{x}_t'\boldsymbol{\beta}) + \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; \alpha_{3,2} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; -\rho_3); \\
&\Pr(y_t = 3|\mathbf{x}_t, \mathbf{x}_{1,t}, \mathbf{x}_{3,t}) = \Phi_2(-\mu_2 + \mathbf{x}_t'\boldsymbol{\beta}; -\alpha_{3,2} + \mathbf{x}_{3,t}'\boldsymbol{\beta}_3; \rho_3).
\end{aligned} \tag{12}
$$

Suppose now that the matrices $\mathbf{X}_1$ and $\mathbf{X}_3$ in the Swopit-z outcome equations are identical to the matrix $\mathbf{X}^m$ in the MIOP regime equation, and that the matrix $\mathbf{X}$ in the Swopit-z regime equation is identical to the matrix $\mathbf{X}_1^m$ in the MIOP outcome equation. Then $\mathbf{x}_{1,t} = \mathbf{x}_{3,t} = \mathbf{x}_t^m$ and $\mathbf{x}_t = \mathbf{x}_{1,t}^m$. Impose the following restrictions on the Swopit-z parameters:

$$\begin{aligned} \boldsymbol{\beta}_3 &= -\boldsymbol{\beta}_1 = \boldsymbol{\beta}^m,\ \boldsymbol{\beta} = \boldsymbol{\beta}_1^m,\ \mu_1 = \alpha_{1,1}^m,\ \mu_2 = \alpha_{1,2}^m, \\ \alpha_{3,2} &= -\alpha_{1,1} = \mu^m \text{ and } -\rho_1 = \rho_3 = \rho^m. \end{aligned} \tag{13}$$

Then, using that $\Phi(-\theta) = 1 - \Phi(\theta)$, the probabilities of observing the outcome $j$ in the Swopit-z model according to (12) can be written as

$$\Pr(y_t = 1|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m);$$

$$\begin{aligned}
&\Pr(y_t = 2|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi_2(\alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; \mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m;\rho^m) \\
&+\Phi(\alpha_{1,2}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) - \Phi(\alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) + \Phi_2(-\alpha_{1,2}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; \mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; -\rho^m) \\
&= 1 - \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) - \Phi(-\alpha_{1,1}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) \\
&+\Phi(\alpha_{1,2}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) - \Phi(\alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) \\
&+1 - \Phi(\alpha_{1,2}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m) - \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; -\alpha_{1,2}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; \rho^m) \\
&= 1 - \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) - \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; -\alpha_{1,2}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; \rho^m) \\
&= \Phi(\mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m) + \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,2}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m) \\
&-\Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; \alpha_{1,1}^m - \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m; -\rho^m);
\end{aligned}$$

$$\Pr(y_t = 3|\mathbf{x}_t^m, \mathbf{x}_{1,t}^m) = \Phi_2(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m; -\alpha_{1,2}^m + \mathbf{x}_{1,t}^{m\prime}\boldsymbol{\beta}_1^m;\rho^m),$$

which are identical to the probabilities in the MIOP model given by (11).

In more general cases, when $\mathbf{X}_1$ and $\mathbf{X}_3$ are not identical to the matrix $\mathbf{X}^m$, but contain all variables in $\mathbf{X}^m$, and when $\mathbf{X}$ is not identical to $\mathbf{Z}^m$, but contains all variables in $\mathbf{Z}^m$, we can use additional restrictions by fixing the values of the coefficients on all extra variables in $\mathbf{X}$, $\mathbf{X}_1$ and $\mathbf{X}_3$ to zero. With these additional parameter restrictions, the selected set of the explanatory variables in each Swopit-z outcome equation is identical to the set of the variables in the MIOP regime equation, the selected set of the explanatory variables in the Swopit-z regime equation is identical to the set of the variables in the MIOP outcome equation, and the probabilities in the MIOP and Swopit-z models are identical.

Notice that the restrictions in (13), namely $\boldsymbol{\beta}_3 = -\boldsymbol{\beta}_1 = \boldsymbol{\beta}^m$ and $\alpha_{3,2} = -\alpha_{1,1} = \mu^m$, impose a sort of symmetry on the policy reactions in the hawkish and dovish policy stances in the Swopit-z model, since they imply that the conditional probability of a hike in the hawkish regime $\Pr(y_t = 3|\mathbf{x}_t, \mathbf{x}_{3,t}, r_t = 3)$ is determined by the same mechanisms as the conditional probability of a cut in the dovish regime $\Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, r_t = 1)$:

$$\begin{aligned}
&\Pr(y_t = 3|\mathbf{x}_t, \mathbf{x}_{3,t}, r_t = 3) = 1 - \Phi(\alpha_{3,2} - \mathbf{x}_{3,t}'\boldsymbol{\beta}_3) = 1 - \Phi(\mu^m - \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m) \\
&= \Phi(-\mu^m + \mathbf{x}_t^{m\prime}\boldsymbol{\beta}^m) = \Phi(\alpha_{1,1} - \mathbf{x}_{1,t}'\boldsymbol{\beta}_1) = \Pr(y_t = 1|\mathbf{x}_t, \mathbf{x}_{1,t}, r_t = 1).
\end{aligned}$$

# Appendix C. Supporting material for Monte Carlo experiments

## Accuracy of slope parameter estimates in the Swopit-z model with exogenous switching

In this set of experiments, three vectors of covariates $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are drawn once (and held fixed in all simulations) as $\mathbf{v}_1 \overset{iid}{\sim} Normal(2,1)$, $\mathbf{v}_2 \overset{iid}{\sim} Normal(0,1)$ and $\mathbf{v}_3 \overset{iid}{\sim} Normal(0,1)$. The repeated samples with 250, 500 and 1000 observations are generated by the Swopit-z DGP with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} = (0.6,\ 0.4)'$, $\boldsymbol{\beta}_1 = (0.3,\ \text{-}0.8)'$, $\boldsymbol{\beta}_3 = (0.9,\ \text{-}0.3)'$, $\boldsymbol{\mu} = (0.91,\ 1.5)'$, $\boldsymbol{\alpha}_1 = (\text{-}0.95,\ 0.25)'$, $\boldsymbol{\alpha}_3 = (\text{-}0.13,\ 1.2)$, and $\rho_1 = \rho_2 = 0$. The samples are estimated using the Swopit-z model with the same specifications of the latent equations as in the above DGP.

Table 12. Monte Carlo results: the accuracy of estimated slope parameters $\beta_s$ in the Swopit-z model with $\rho_1$=$\rho_3$=0

| Sample size | 250 | 500 | 1000 |
|---|---|---|---|
| Bias of parameter estimates, % | 6.5 | 3.2 | 1.4 |
| RMSE of parameter estimates, % | 40.6 | 27.0 | 18.5 |
| Coverage probability of parameter estimates, % (at 95% level) | 93.4 | 94.5 | 94.9 |
| Bias of standard error estimates, % | 8.4 | 3.3 | 1.7 |

Notes. Bias is the absolute difference between the estimated and true values, devided by the true values, in percent. RMSE is the root mean square error relative to the true value, in percent. Coverage probability is the percentage of times the estimated asymptotic 95 percent confidence intervals cover the true values. Bias of s.e. estimates is the absolute difference between the average of estimated standard errors and standard deviation of parameter estimates in all iterations, devided by the standard deviation of parameter estimates, in percent. All above measures are averaged across all replications and for all slope parameters.

As Table 12 reports, the Swopit-z estimator is consistent and the asymptotic distribution is a good approximation of the small sample distribution of the slope parameters $\boldsymbol{\beta}$, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_3$. As the sample size grows, the biases and RMSE reduce substantially (both biases are less than 2% with 1000 observations), and the CP gets closer to the nominal level of 95% and reaches 94.9%.

## Accuracy of slope parameter estimates in the Swopit-z model with endogenous switching

In this set of experiments, three vectors of covariates $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are drawn once (and held fixed in all simulations) as $\mathbf{v}_1 \overset{iid}{\sim} Normal(2,1)$, $\mathbf{v}_2 \overset{iid}{\sim} Normal(0,1)$ and $\mathbf{v}_3 \overset{iid}{\sim} Normal(0,1)$. The repeated samples with 250, 500 and 1000 observations are generated by the Swopit-z DGP with $\mathbf{X} = (\mathbf{v}_1, \mathbf{v}_3)$, $\mathbf{X}_1 = \mathbf{X}_3 = (\mathbf{v}_2, \mathbf{v}_3)$, $\boldsymbol{\beta} = (0.6,\ 0.4)'$, $\boldsymbol{\beta}_1 = (0.3,\ -0.8)'$, $\boldsymbol{\beta}_3 = (0.9,\ -0.3)'$, $\boldsymbol{\mu} = (0.9,\ 1.5)'$, $\boldsymbol{\alpha}_1 = (-1.22,\ 0.08)'$, $\boldsymbol{\alpha}_3 = (0.3,\ 1.63)$, $\rho_1 = 0.3$, and $\rho_3 = 0.6$. The samples are estimated using the Swopit-z model with the same specifications of the latent equations as in the above DGP.

Table 13. Monte Carlo results: the accuracy of estimated slope parameters $\beta_s$ in the Swopit-z model with endogenous switching

| Sample size | 250 | 500 | 1000 |
|---|---|---|---|
| Bias of parameter estimates, % | 7.6 | 2.7 | 1.0 |
| RMSE of parameter estimates, % | 42.3 | 27.4 | 18.7 |
| Coverage probability of parameter estimates, % (at 95% level) | 92.5 | 94.2 | 94.6 |
| Bias of standard error estimates, % | 4.6 | 2.3 | 1.6 |

Notes. Bias is the absolute difference between the estimated and true values, devided by the true values, in percent. RMSE is the root mean square error relative to the true value, in percent. Coverage probability is the percentage of times the estimated asymptotic 95 percent confidence intervals cover the true values. Bias of s.e. estimates is the absolute difference between the average of estimated standard errors and standard deviation of parameter estimates in all iterations, devided by the standard deviation of parameter estimates, in percent. All above measures are averaged across all replications and for all slope parameters.

As Table 13 reports, the Swopit-z estimator is consistent and the asymptotic distribution is a good approximation of the small sample distribution of the slope parameters $\boldsymbol{\beta}$, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_3$. As the sample size grows, the biases and RMSE reduce substantially (both biases are less than 2% with 1000 observations), and the CP gets closer to the nominal level of 95% and reaches 94.6%.

# Appendix D. Supporting material for empirical application

Table 14. Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 07-Jul-87 | 0 | no change | 08-Jul-87 | -0.040 | 0 | 0 | 0.164 | 1.620 |
| 18-Aug-87 | 0 | no change | 19-Aug-87 | -0.020 | 0 | 0 | 0.228 | 1.611 |
| 27-Aug-87 | 0.125 | small hike | 27-Aug-87 | 0.040 | 1 | 0 | 0.260 | 1.611 |
| 03-Sep-87 | 0.5 | large hike | 3-Sep-87: 0.125<br>4-Sep-87: 0.375 | 0.090 | 1 | 0 | 0.426 | 1.611 |
| 22-Sep-87 | 0.0625 | no change | 24-Sep-87 | 0.010 | 1 | 0 | 0.426 | 1.582 |
| 03-Nov-87 | -0.5 | large cut | 04-Nov-87 | -0.130 | 0 | 0 | 0.008 | 1.669 |
| 16-Dec-87 | 0 | no change | 17-Dec-87 | -0.140 | 0 | 1 | 0.534 | 1.637 |
| 05-Jan-88 | 0 | no change | 05-Jan-88 | -0.050 | 0 | 0 | 0.130 | 1.637 |
| 28-Jan-88 | -0.1875 | small cut | 28-Jan-88 | -0.060 | 0 | 0 | 0.134 | 1.374 |
| 10-Feb-88 | -0.125 | small cut | 11-Feb-88 | 0.030 | 0 | 0 | 0.146 | 1.374 |
| 29-Mar-88 | 0.25 | small hike | 30-Mar-88 | 0.010 | 0 | 0 | 0.144 | 1.494 |
| 09-May-88 | 0.25 | small hike | 09-May-88 | 0.050 | 0 | 0 | 0.540 | 1.543 |
| 17-May-88 | 0 | no change | 18-May-88 | -0.040 | 0 | 0 | 0.110 | 1.543 |
| 25-May-88 | 0.25 | small hike | 25-May-88 | -0.020 | 1 | 0 | 0.482 | 1.561 |
| 22-Jun-88 | 0.1875 | small hike | 22-Jun-88 | -0.010 | 1 | 0 | -0.108 | 1.384 |
| 30-Jun-88 | 0.0625 | no change | 01-Jul-88 | -0.070 | 1 | 0 | -0.136 | 1.384 |
| 19-Jul-88 | 0.1875 | small hike | 19-Jul-88 | 0.000 | 1 | 0 | 0.028 | 1.454 |
| 08-Aug-88 | 0.4375 | large hike | 8-Aug-88: 0.0625<br>9-Aug-88: 0.375 | 0.090 | 1 | 0 | 0.100 | 1.454 |
| 16-Aug-88 | 0 | no change | 17-Aug-88 | -0.040 | 1 | 0 | 0.166 | 1.454 |
| 20-Sep-88 | 0 | no change | 21-Sep-88 | 0.020 | 1 | 0 | -0.110 | 1.489 |
| 01-Nov-88 | 0 | no change | 02-Nov-88 | 0.032 | 1 | 0 | -0.192 | 1.453 |
| 17-Nov-88 | 0.25 | small hike | 17-Nov-88: 0.0625<br>22-Nov-88: 0.1875 | -0.078 | 1 | 0 | 0.208 | 1.554 |
| 14-Dec-88 | 0.3125 | large hike | 15-Dec-88 | 0.018 | 1 | 0 | 0.450 | 1.554 |
| 05-Jan-89 | 0.3125 | large hike | 05-Jan-89 | -0.024 | 1 | 0 | -0.270 | 1.563 |
| 08-Feb-89 | 0.125 | small hike | 09-Feb-89 | 0.000 | 1 | 0 | 0.058 | 1.524 |
| 14-Feb-89 | 0.1875 | small hike | 14-Feb-89 | 0.020 | 1 | 0 | -0.070 | 1.524 |
| 24-Feb-89 | 0.4375 | large hike | 24-Feb-89 | 0.070 | 1 | 0 | -0.158 | 1.693 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 28-Mar-89 | 0 | no change | 29-Mar-89 | -0.070 | 1 | 0 | -0.194 | 1.498 |
| 16-May-89 | 0.0625 | no change | 17-May-89 | -0.022 | 1 | 0 | -0.744 | 1.361 |
| 05-Jun-89 | -0.25 | small cut | 05-Jun-89 | -0.036 | 0 | 0 | -1.086 | 1.361 |
| 06-Jul-89 | -0.25 | small cut | 07-Jul-89 | -0.026 | 0 | 0 | -1.446 | 1.311 |
| 26-Jul-89 | -0.25 | small cut | 26-Jul-89 | -0.062 | 0 | 0 | -1.224 | 1.400 |
| 22-Aug-89 | 0 | no change | 23-Aug-89 | 0.000 | 0 | 0 | -0.708 | 1.430 |
| 03-Oct-89 | 0 | no change | 04-Oct-89 | 0.034 | 0 | 1 | -0.720 | 1.353 |
| 16-Oct-89 | -0.3125 | large cut | 16-Oct-89 | -0.207 | 0 | 1 | -0.854 | 1.353 |
| 06-Nov-89 | -0.25 | small cut | 07-Nov-89 | -0.104 | 0 | 1 | -0.940 | 1.263 |
| 14-Nov-89 | 0 | no change | 15-Nov-89 | -0.020 | 0 | 1 | -0.652 | 1.263 |
| 19-Dec-89 | -0.25 | small cut | 20-Dec-89 | -0.169 | 0 | 1 | -0.828 | 1.361 |
| 07-Feb-90 | 0 | no change | 08-Feb-90 | -0.014 | 0 | 0 | -0.110 | 1.235 |
| 27-Mar-90 | 0 | no change | 28-Mar-90 | 0.000 | 0 | 0 | 0.034 | 1.477 |
| 15-May-90 | 0 | no change | 16-May-90 | 0.000 | 0 | 0 | 0.084 | 1.321 |
| 03-Jul-90 | 0 | no change | 04-Jul-90 | 0.000 | 0 | 0 | -0.248 | 1.207 |
| 13-Jul-90 | -0.25 | small cut | 13-Jul-90 | -0.138 | 0 | 1 | -0.118 | 1.207 |
| 21-Aug-90 | 0 | no change | 22-Aug-90 | 0.000 | 0 | 1 | -0.464 | 1.148 |
| 07-Sep-90 | 0 | no change | 07-Sep-90 | 0.039 | 0 | 1 | -0.594 | 1.148 |
| 17-Sep-90 | 0 | no change | 17-Sep-90 | -0.023 | 0 | 1 | -0.286 | 1.148 |
| 02-Oct-90 | 0 | no change | 03-Oct-90 | 0.021 | 0 | 1 | -0.480 | 1.127 |
| 29-Oct-90 | -0.25 | small cut | 29-Oct-90 | -0.020 | 0 | 1 | -0.472 | 1.135 |
| 13-Nov-90 | -0.25 | small cut | 16-Nov-90 | 0.000 | 0 | 1 | -0.484 | 1.135 |
| 07-Dec-90 | -0.25 | small cut | 07-Dec-90 | -0.271 | 0 | 1 | -0.274 | 1.041 |
| 18-Dec-90 | -0.25 | small cut | 19-Dec-90 | -0.233 | 0 | 1 | -0.206 | 1.041 |
| 08-Jan-91 | -0.25 | small cut | 08-Jan-91 | -0.175 | 0 | 1 | 0.122 | 1.129 |
| 01-Feb-91 | -0.5 | large cut | 01-Feb-91 | -0.259 | 0 | 1 | -0.918 | 0.987 |
| 06-Feb-91 | 0 | no change | 07-Feb-91 | 0.000 | 0 | 1 | -0.244 | 0.987 |
| 08-Mar-91 | -0.25 | small cut | 08-Mar-91 | -0.162 | 0 | 1 | 0.096 | 0.850 |
| 26-Mar-91 | 0 | no change | 27-Mar-91 | 0.000 | 0 | 1 | 0.170 | 0.989 |
| 12-Apr-91 | 0 | no change | 12-Apr-91 | -0.100 | 0 | 0 | 0.452 | 0.989 |
| 30-Apr-91 | -0.25 | small cut | 30-Apr-91 | -0.170 | 0 | 0 | 0.310 | 0.901 |
| 14-May-91 | 0 | no change | 15-May-91 | 0.019 | 0 | 0 | 0.374 | 0.901 |
| 03-Jul-91 | 0 | no change | 05-Jul-91 | 0.000 | 0 | 0 | 0.198 | 0.982 |
| 05-Aug-91 | -0.25 | small cut | 06-Aug-91 | -0.149 | 0 | 0 | 0.384 | 1.040 |
| 20-Aug-91 | 0 | no change | 21-Aug-91 | 0.124 | 0 | 0 | 0.052 | 1.070 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 13-Sep-91 | -0.25 | small cut | 13-Sep-91 | -0.053 | 0 | 1 | 0.070 | 1.070 |
| 01-Oct-91 | 0 | no change | 02-Oct-91 | -0.011 | 0 | 1 | 0.102 | 1.065 |
| 30-Oct-91 | -0.25 | small cut | 30-Oct-91 | -0.060 | 0 | 1 | 0.104 | 1.033 |
| 05-Nov-91 | -0.25 | small cut | 06-Nov-91 | -0.125 | 0 | 1 | 0.114 | 1.033 |
| 06-Dec-91 | -0.25 | small cut | 06-Dec-91 | -0.087 | 0 | 1 | -0.190 | 1.096 |
| 17-Dec-91 | -0.5 | large cut | 20-Dec-91 | -0.238 | 0 | 1 | -0.086 | 1.066 |
| 05-Feb-92 | 0 | no change | 06-Feb-92 | -0.013 | 0 | 0 | 0.190 | 1.103 |
| 31-Mar-92 | 0 | no change | 01-Apr-92 | 0.010 | 0 | 1 | 0.674 | 1.304 |
| 09-Apr-92 | -0.25 | small cut | 09-Apr-92 | -0.243 | 0 | 1 | 0.418 | 1.304 |
| 19-May-92 | 0 | no change | 20-May-92 | 0.000 | 0 | 1 | 0.120 | 1.115 |
| 01-Jul-92 | -0.5 | large cut | 02-Jul-92 | -0.363 | 0 | 0 | 0.144 | 1.230 |
| 18-Aug-92 | 0 | no change | 19-Aug-92 | 0.026 | 0 | 1 | 0.122 | 1.119 |
| 04-Sep-92 | -0.25 | small cut | 04-Sep-92 | -0.219 | 0 | 1 | 0.106 | 1.119 |
| 06-Oct-92 | 0 | no change | 07-Oct-92 | 0.052 | 0 | 1 | -0.528 | 1.237 |
| 17-Nov-92 | 0 | no change | 18-Nov-92 | -0.100 | 0 | 1 | 0.648 | 1.256 |
| 22-Dec-92 | 0 | no change | 23-Dec-92 | 0.039 | 0 | 1 | 0.732 | 1.242 |
| 03-Feb-93 | 0 | no change | 04-Feb-93 | -0.012 | 0 | 0 | 0.330 | 1.302 |
| 23-Mar-93 | 0 | no change | 24-Mar-93 | -0.044 | 0 | 0 | 0.290 | 1.208 |
| 18-May-93 | 0 | no change | 19-May-93 | -0.026 | 0 | 0 | 0.270 | 1.213 |
| 07-Jul-93 | 0 | no change | 08-Jul-93 | 0.027 | 1 | 0 | 0.146 | 1.244 |
| 17-Aug-93 | 0 | no change | 18-Aug-93 | 0.000 | 1 | 0 | 0.428 | 1.212 |
| 21-Sep-93 | 0 | no change | 22-Sep-93 | 0.000 | 0 | 0 | 0.228 | 1.323 |
| 16-Nov-93 | 0 | no change | 17-Nov-93 | 0.023 | 0 | 0 | 0.546 | 1.351 |
| 21-Dec-93 | 0 | no change | 22-Dec-93 | 0.000 | 0 | 0 | 0.604 | 1.432 |
| 04-Feb-94 | 0.25 | small hike | 04-Feb-94 | 0.117 | 0 | 0 | 0.324 | 1.540 |
| 28-Feb-94 | 0 | no change | 28-Feb-94 | -0.050 | 0 | 0 | 0.736 | 1.294 |
| 22-Mar-94 | 0.25 | small hike | 22-Mar-94 | -0.034 | 0 | 0 | 1.104 | 1.309 |
| 18-Apr-94 | 0.25 | small hike | 18-Apr-94 | 0.100 | 0 | 0 | 1.282 | 1.309 |
| 17-May-94 | 0.5 | large hike | 17-May-94 | 0.133 | 0 | 0 | 1.678 | 1.455 |
| 06-Jul-94 | 0 | no change | 06-Jul-94 | -0.050 | 0 | 0 | 0.986 | 1.510 |
| 16-Aug-94 | 0.5 | large hike | 16-Aug-94 | 0.145 | 1 | 0 | 1.340 | 1.415 |
| 27-Sep-94 | 0 | no change | 27-Sep-94 | -0.200 | 0 | 0 | 1.112 | 1.442 |
| 15-Nov-94 | 0.75 | large hike | 15-Nov-94 | 0.140 | 1 | 0 | 1.444 | 1.525 |
| 20-Dec-94 | 0 | no change | 20-Dec-94 | -0.169 | 0 | 0 | 1.758 | 1.540 |
| 01-Feb-95 | 0.5 | large hike | 01-Feb-95 | 0.052 | 1 | 0 | 1.208 | 1.529 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 28-Mar-95 | 0 | no change | 28-Mar-95 | 0.103 | 0 | 0 | 0.326 | 1.323 |
| 23-May-95 | 0 | no change | 23-May-95 | 0.000 | 1 | 0 | 0.012 | 1.236 |
| 06-Jul-95 | -0.25 | small cut | 06-Jul-95 | -0.012 | 0 | 0 | -0.610 | 1.239 |
| 22-Aug-95 | 0 | no change | 22-Aug-95 | 0.000 | 0 | 1 | 0.080 | 1.380 |
| 26-Sep-95 | 0 | no change | 26-Sep-95 | 0.000 | 0 | 0 | -0.122 | 1.398 |
| 15-Nov-95 | 0 | no change | 15-Nov-95 | 0.060 | 0 | 0 | -0.324 | 1.390 |
| 19-Dec-95 | -0.25 | small cut | 19-Dec-95 | -0.103 | 0 | 0 | -0.394 | 1.337 |
| 31-Jan-96 | -0.25 | small cut | 31-Jan-96 | -0.070 | 0 | 0 | -0.466 | 1.420 |
| 26-Mar-96 | 0 | no change | 26-Mar-96 | -0.031 | 0 | 0 | 0.198 | 1.490 |
| 21-May-96 | 0 | no change | 21-May-96 | 0.000 | 0 | 0 | 0.316 | 1.519 |
| 03-Jul-96 | 0 | no change | 03-Jul-96 | -0.050 | 0 | 0 | 0.060 | 1.434 |
| 20-Aug-96 | 0 | no change | 20-Aug-96 | -0.042 | 1 | 0 | 0.448 | 1.455 |
| 24-Sep-96 | 0 | no change | 24-Sep-96 | -0.125 | 1 | 0 | 0.666 | 1.525 |
| 13-Nov-96 | 0 | no change | 13-Nov-96 | 0.000 | 1 | 0 | 0.128 | 1.438 |
| 17-Dec-96 | 0 | no change | 17-Dec-96 | 0.011 | 1 | 0 | 0.182 | 1.514 |
| 05-Feb-97 | 0 | no change | 05-Feb-97 | -0.030 | 1 | 0 | 0.328 | 1.329 |
| 25-Mar-97 | 0.25 | small hike | 25-Mar-97 | 0.026 | 1 | 0 | 0.458 | 1.528 |
| 20-May-97 | 0 | no change | 20-May-97 | -0.113 | 0 | 0 | 0.354 | 1.473 |
| 02-Jul-97 | 0 | no change | 02-Jul-97 | -0.016 | 1 | 0 | -0.272 | 1.397 |
| 19-Aug-97 | 0 | no change | 19-Aug-97 | -0.013 | 1 | 0 | 0.008 | 1.447 |
| 30-Sep-97 | 0 | no change | 30-Sep-97 | 0.000 | 1 | 0 | -0.070 | 1.363 |
| 12-Nov-97 | 0 | no change | 12-Nov-97 | -0.042 | 1 | 0 | -0.126 | 1.500 |
| 16-Dec-97 | 0 | no change | 16-Dec-97 | -0.010 | 1 | 0 | -0.058 | 1.531 |
| 04-Feb-98 | 0 | no change | 04-Feb-98 | 0.000 | 0 | 0 | -0.310 | 1.519 |
| 31-Mar-98 | 0 | no change | 31-Mar-98 | 0.000 | 0 | 0 | -0.150 | 1.636 |
| 19-May-98 | 0 | no change | 19-May-98 | -0.026 | 1 | 0 | -0.150 | 1.538 |
| 01-Jul-98 | 0 | no change | 01-Jul-98 | -0.005 | 1 | 0 | -0.480 | 1.530 |
| 18-Aug-98 | 0 | no change | 18-Aug-98 | 0.012 | 1 | 0 | -0.370 | 1.615 |
| 29-Sep-98 | -0.25 | small cut | 29-Sep-98 | 0.060 | 0 | 0 | -0.896 | 1.613 |
| 15-Oct-98 | -0.25 | small cut | 16-Oct-98 | -0.262 | 0 | 1 | -0.906 | 1.613 |
| 17-Nov-98 | -0.25 | small cut | 17-Nov-98 | -0.058 | 0 | 1 | -0.542 | 1.576 |
| 22-Dec-98 | 0 | no change | 22-Dec-98 | -0.017 | 0 | 0 | -0.362 | 1.649 |
| 03-Feb-99 | 0 | no change | 03-Feb-99 | 0.000 | 0 | 0 | -0.182 | 1.720 |
| 30-Mar-99 | 0 | no change | 30-Mar-99 | 0.000 | 0 | 0 | -0.116 | 1.799 |
| 18-May-99 | 0 | no change | 18-May-99 | -0.036 | 0 | 0 | 0.018 | 1.574 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 30-Jun-99 | 0.25 | small hike | 30-Jun-99 | -0.040 | 1 | 0 | 0.284 | 1.676 |
| 24-Aug-99 | 0.25 | small hike | 24-Aug-99 | 0.022 | 1 | 0 | 0.232 | 1.661 |
| 05-Oct-99 | 0 | no change | 05-Oct-99 | -0.042 | 0 | 0 | -0.064 | 1.676 |
| 16-Nov-99 | 0.25 | small hike | 16-Nov-99 | 0.086 | 1 | 0 | 0.138 | 1.618 |
| 21-Dec-99 | 0 | no change | 21-Dec-99 | 0.016 | 0 | 0 | 0.426 | 1.600 |
| 02-Feb-00 | 0.25 | small hike | 02-Feb-00 | -0.054 | 0 | 0 | 0.564 | 1.712 |
| 21-Mar-00 | 0.25 | small hike | 21-Mar-00 | -0.031 | 1 | 0 | 0.398 | 1.781 |
| 16-May-00 | 0.5 | large hike | 16-May-00 | 0.052 | 1 | 0 | 0.322 | 1.663 |
| 28-Jun-00 | 0 | no change | 28-Jun-00 | -0.020 | 1 | 0 | -0.372 | 1.592 |
| 22-Aug-00 | 0 | no change | 22-Aug-00 | -0.017 | 1 | 0 | -0.294 | 1.512 |
| 03-Oct-00 | 0 | no change | 03-Oct-00 | 0.000 | 1 | 0 | -0.500 | 1.531 |
| 15-Nov-00 | 0 | no change | 15-Nov-00 | 0.000 | 1 | 0 | -0.394 | 1.530 |
| 19-Dec-00 | 0 | no change | 19-Dec-00 | 0.052 | 1 | 0 | -0.810 | 1.532 |
| 03-Jan-01 | -0.5 | large cut | 03-Jan-01 | -0.382 | 0 | 1 | -1.052 | 1.562 |
| 31-Jan-01 | -0.5 | large cut | 31-Jan-01 | 0.005 | 0 | 1 | -1.208 | 1.575 |
| 20-Mar-01 | -0.5 | large cut | 20-Mar-01 | 0.056 | 0 | 1 | -1.186 | 1.647 |
| 11-Apr-01 | 0 | no change | 11-Apr-01 | 0.016 | 0 | 1 | -1.014 | 1.647 |
| 18-Apr-01 | -0.5 | large cut | 18-Apr-01 | -0.425 | 0 | 1 | -0.852 | 1.613 |
| 15-May-01 | -0.5 | large cut | 15-May-01 | -0.078 | 0 | 1 | -0.680 | 1.613 |
| 27-Jun-01 | -0.25 | small cut | 27-Jun-01 | 0.050 | 0 | 1 | -0.472 | 1.622 |
| 21-Aug-01 | -0.25 | small cut | 21-Aug-01 | 0.016 | 0 | 1 | -0.290 | 1.672 |
| 17-Sep-01 | -0.5 | large cut | 17-Sep-01 | -0.323 | 0 | 1 | -0.306 | 1.672 |
| 02-Oct-01 | -0.5 | large cut | 02-Oct-01 | -0.069 | 0 | 1 | -0.508 | 1.527 |
| 06-Nov-01 | -0.5 | large cut | 06-Nov-01 | -0.100 | 0 | 1 | -0.460 | 1.574 |
| 11-Dec-01 | -0.25 | small cut | 11-Dec-01 | 0.000 | 0 | 1 | 0.322 | 1.552 |
| 30-Jan-02 | 0 | no change | 30-Jan-02 | 0.015 | 0 | 1 | 0.434 | 1.570 |
| 19-Mar-02 | 0 | no change | 19-Mar-02 | -0.026 | 0 | 1 | 0.846 | 1.678 |
| 07-May-02 | 0 | no change | 07-May-02 | 0.000 | 0 | 0 | 0.528 | 1.646 |
| 26-Jun-02 | 0 | no change | 26-Jun-02 | 0.000 | 0 | 0 | 0.378 | 1.733 |
| 13-Aug-02 | 0 | no change | 13-Aug-02 | 0.034 | 0 | 0 | -0.032 | 1.672 |
| 24-Sep-02 | 0 | no change | 24-Sep-02 | 0.025 | 0 | 1 | 0.006 | 1.609 |
| 06-Nov-02 | -0.5 | large cut | 06-Nov-02 | -0.194 | 0 | 1 | -0.252 | 1.843 |
| 10-Dec-02 | 0 | no change | 10-Dec-02 | 0.000 | 0 | 0 | 0.282 | 1.603 |
| 29-Jan-03 | 0 | no change | 29-Jan-03 | 0.000 | 0 | 0 | 0.066 | 1.835 |
| 18-Mar-03 | 0 | no change | 18-Mar-03 | 0.048 | 0 | 0 | -0.080 | 1.622 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 06-May-03 | 0 | no change | 06-May-03 | 0.037 | 0 | 0 | -0.028 | 1.780 |
| 25-Jun-03 | -0.25 | small cut | 25-Jun-03 | 0.150 | 0 | 1 | -0.280 | 1.732 |
| 12-Aug-03 | 0 | no change | 12-Aug-03 | 0.000 | 0 | 1 | 0.342 | 1.803 |
| 16-Sep-03 | 0 | no change | 16-Sep-03 | 0.000 | 0 | 1 | 0.204 | 1.872 |
| 28-Oct-03 | 0 | no change | 28-Oct-03 | 0.000 | 0 | 1 | 0.288 | 1.888 |
| 09-Dec-03 | 0 | no change | 09-Dec-03 | 0.000 | 0 | 1 | 0.374 | 1.960 |
| 28-Jan-04 | 0 | no change | 28-Jan-04 | 0.000 | 0 | 1 | 0.182 | 2.088 |
| 16-Mar-04 | 0 | no change | 16-Mar-04 | 0.000 | 0 | 1 | 0.158 | 1.855 |
| 04-May-04 | 0 | no change | 04-May-04 | -0.006 | 0 | 1 | 0.536 | 2.007 |
| 30-Jun-04 | 0.25 | small hike | 30-Jun-04 | -0.010 | 0 | 0 | 1.116 | 1.967 |
| 10-Aug-04 | 0.25 | small hike | 10-Aug-04 | 0.022 | 0 | 0 | 0.752 | 1.802 |
| 21-Sep-04 | 0.25 | small hike | 21-Sep-04 | 0.017 | 0 | 0 | 0.528 | 2.000 |
| 10-Nov-04 | 0.25 | small hike | 10-Nov-04 | 0.000 | 0 | 0 | 0.636 | 1.898 |
| 14-Dec-04 | 0.25 | small hike | 14-Dec-04 | 0.000 | 0 | 0 | 0.546 | 2.027 |
| 02-Feb-05 | 0.25 | small hike | 02-Feb-05 | 0.000 | 0 | 0 | 0.502 | 2.004 |
| 22-Mar-05 | 0.25 | small hike | 22-Mar-05 | 0.000 | 0 | 0 | 0.658 | 2.195 |
| 03-May-05 | 0.25 | small hike | 03-May-05 | 0.000 | 0 | 0 | 0.480 | 1.837 |
| 30-Jun-05 | 0.25 | small hike | 30-Jun-05 | 0.000 | 0 | 0 | 0.286 | 2.009 |
| 09-Aug-05 | 0.25 | small hike | 09-Aug-05 | 0.000 | 0 | 0 | 0.462 | 2.004 |
| 20-Sep-05 | 0.25 | small hike | 20-Sep-05 | 0.015 | 0 | 0 | 0.252 | 2.009 |
| 01-Nov-05 | 0.25 | small hike | 01-Nov-05 | 0.000 | 0 | 0 | 0.424 | 2.108 |
| 13-Dec-05 | 0.25 | small hike | 13-Dec-05 | 0.000 | 0 | 0 | 0.256 | 2.014 |
| 31-Jan-06 | 0.25 | small hike | 31-Jan-06 | 0.000 | 1 | 0 | 0.142 | 1.933 |
| 28-Mar-06 | 0.25 | small hike | 28-Mar-06 | 0.000 | 1 | 0 | 0.136 | 2.120 |
| 10-May-06 | 0.25 | small hike | 10-May-06 | -0.007 | 1 | 0 | 0.170 | 1.960 |
| 29-Jun-06 | 0.25 | small hike | 29-Jun-06 | -0.015 | 1 | 0 | 0.264 | 1.957 |
| 08-Aug-06 | 0 | no change | 08-Aug-06 | -0.040 | 1 | 0 | -0.150 | 1.850 |
| 20-Sep-06 | 0 | no change | 20-Sep-06 | 0.000 | 1 | 0 | -0.228 | 1.665 |
| 25-Oct-06 | 0 | no change | 25-Oct-06 | 0.000 | 1 | 0 | -0.170 | 1.772 |
| 12-Dec-06 | 0 | no change | 12-Dec-06 | 0.000 | 1 | 0 | -0.326 | 1.486 |
| 31-Jan-07 | 0 | no change | 31-Jan-07 | 0.000 | 1 | 0 | -0.156 | 1.642 |
| 21-Mar-07 | 0 | no change | 21-Mar-07 | 0.000 | 1 | 0 | -0.328 | 1.525 |
| 09-May-07 | 0 | no change | 09-May-07 | 0.000 | 1 | 0 | -0.314 | 1.518 |
| 28-Jun-07 | 0 | no change | 28-Jun-07 | 0.000 | 1 | 0 | -0.308 | 1.474 |
| 07-Aug-07 | 0 | no change | 07-Aug-07 | 0.026 | 1 | 0 | -0.460 | 1.467 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 10-Aug-07 | 0 | no change | 10-Aug-07 | 0.000 | 1 | 0 | -0.490 | 1.467 |
| 17-Aug-07 | 0 | no change | 17-Aug-07 | 0.155 | 1 | 0 | -0.202 | 1.381 |
| 18-Sep-07 | -0.5 | large cut | 18-Sep-07 | -0.150 | 0 | 0 | -1.008 | 1.381 |
| 31-Oct-07 | -0.25 | small cut | 31-Oct-07 | -0.020 | 0 | 0 | -0.848 | 1.191 |
| 11-Dec-07 | -0.25 | small cut | 11-Dec-07 | 0.008 | 0 | 0 | -1.246 | 1.229 |
| 22-Jan-08 | -0.75 | large cut | 22-Jan-08 | -0.741 | 0 | 0 | -1.394 | 1.006 |
| 30-Jan-08 | -0.5 | large cut | 30-Jan-08 | -0.095 | 0 | 1 | -1.182 | 1.006 |
| 18-Mar-08 | -0.75 | large cut | 18-Mar-08 | 0.167 | 0 | 1 | -1.420 | 1.065 |
| 30-Apr-08 | -0.25 | small cut | 30-Apr-08 | -0.055 | 0 | 1 | -0.324 | 0.947 |
| 25-Jun-08 | 0 | no change | 25-Jun-08 | -0.030 | 0 | 0 | 0.604 | 0.975 |
| 05-Aug-08 | 0 | no change | 05-Aug-08 | -0.006 | 0 | 0 | 0.260 | 1.066 |
| 16-Sep-08 | 0 | no change | 16-Sep-08 | 0.059 | 0 | 0 | -0.202 | 0.965 |
| 08-Oct-08 | -0.5 | large cut | 08-Oct-08 | -0.142 | 0 | 0 | -0.154 | 0.895 |
| 29-Oct-08 | -0.5 | large cut | 29-Oct-08 | -0.060 | 0 | 1 | 0.734 | 0.817 |
| 16-Dec-08 | -0.75 | large cut | 16-Dec-08 | -0.119 | 0 | 1 | 0.356 | 0.625 |
| 28-Jan-09 | 0 | no change | 28-Jan-09 | 0.000 | 0 | 1 | 0.252 | 0.550 |
| 18-Mar-09 | 0 | no change | 18-Mar-09 | -0.006 | 0 | 1 | 0.508 | 0.583 |
| 29-Apr-09 | 0 | no change | 29-Apr-09 | -0.005 | 0 | 1 | 0.342 | 0.510 |
| 24-Jun-09 | 0 | no change | 24-Jun-09 | -0.025 | 0 | 1 | 0.262 | 0.532 |
| 12-Aug-09 | 0 | no change | 12-Aug-09 | -0.008 | 0 | 1 | 0.324 | 0.582 |
| 23-Sep-09 | 0 | no change | 23-Sep-09 | 0.000 | 0 | 1 | 0.244 | 0.598 |
| 04-Nov-09 | 0 | no change | 04-Nov-09 | 0.000 | 0 | 1 | 0.270 | 0.590 |
| 16-Dec-09 | 0 | no change | 16-Dec-09 | -0.010 | 0 | 1 | 0.230 | 0.574 |
| 27-Jan-10 | 0 | no change | 27-Jan-10 | -0.019 | 0 | 1 | 0.188 | 0.557 |
| 16-Mar-10 | 0 | no change | 16-Mar-10 | 0.000 | 0 | 1 | 0.234 | 0.575 |
| 28-Apr-10 | 0 | no change | 28-Apr-10 | 0.000 | 0 | 1 | 0.248 | 0.626 |
| 23-Jun-10 | 0 | no change | 23-Jun-10 | 0.000 | 0 | 1 | 0.110 | 0.593 |
| 10-Aug-10 | 0 | no change | 10-Aug-10 | 0.000 | 0 | 1 | 0.080 | 0.549 |
| 21-Sep-10 | 0 | no change | 21-Sep-10 | 0.000 | 0 | 1 | 0.052 | 0.598 |
| 03-Nov-10 | 0 | no change | 03-Nov-10 | 0.003 | 0 | 1 | 0.028 | 0.610 |
| 14-Dec-10 | 0 | no change | 14-Dec-10 | 0.000 | 0 | 1 | 0.126 | 0.519 |
| 26-Jan-11 | 0 | no change | 26-Jan-11 | 0.000 | 0 | 1 | 0.096 | 0.529 |
| 15-Mar-11 | 0 | no change | 15-Mar-11 | 0.000 | 0 | 1 | 0.108 | 0.596 |
| 27-Apr-11 | 0 | no change | 27-Apr-11 | 0.000 | 0 | 1 | 0.132 | 0.549 |
| 22-Jun-11 | 0 | no change | 22-Jun-11 | -0.009 | 0 | 1 | 0.084 | 0.560 |

Table 14 (contd). Data used in estimations

| Date of FOMC decision | Change to federal funds rate target | Dependent variable $y_t$ | Effective date of FOMC action | $surprise_t$ | $tight_{t-1}$ | $ease_{t-1}$ | $spread_t$ | $house_t$ |
|---|---|---|---|---|---|---|---|---|
| 09-Aug-11 | 0 | no change | 09-Aug-11 | 0.000 | 0 | 1 | 0.024 | 0.629 |
| 21-Sep-11 | 0 | no change | 21-Sep-11 | 0.008 | 0 | 1 | 0.002 | 0.571 |
| 02-Nov-11 | 0 | no change | 02-Nov-11 | 0.000 | 0 | 1 | 0.052 | 0.658 |
| 13-Dec-11 | 0 | no change | 13-Dec-11 | -0.004 | 0 | 1 | 0.032 | 0.628 |
| 25-Jan-12 | 0 | no change | 25-Jan-12 | 0.000 | 0 | 1 | 0.022 | 0.657 |
| 13-Mar-12 | 0 | no change | 13-Mar-12 | 0.017 | 0 | 1 | 0.064 | 0.699 |
| 25-Apr-12 | 0 | no change | 25-Apr-12 | 0.000 | 0 | 1 | 0.042 | 0.654 |
| 20-Jun-12 | 0 | no change | 20-Jun-12 | 0.000 | 0 | 1 | 0.008 | 0.708 |
| 01-Aug-12 | 0 | no change | 01-Aug-12 | 0.005 | 0 | 1 | 0.032 | 0.760 |
| 13-Sep-12 | 0 | no change | 13-Sep-12 | 0.009 | 0 | 1 | 0.028 | 0.746 |
| 24-Oct-12 | 0 | no change | 24-Oct-12 | 0.000 | 0 | 1 | 0.030 | 0.872 |
| 12-Dec-12 | 0 | no change | 12-Dec-12 | 0.000 | 0 | 1 | 0.014 | 0.894 |
| 30-Jan-13 | 0 | no change | 30-Jan-13 | 0.000 | 0 | 1 | 0.016 | 0.954 |
| 20-Mar-13 | 0 | no change | 20-Mar-13 | 0.000 | 0 | 1 | -0.004 | 0.917 |
| 01-May-13 | 0 | no change | 01-May-13 | 0.000 | 0 | 1 | -0.012 | 1.036 |
| 19-Jun-13 | 0 | no change | 19-Jun-13 | 0.014 | 0 | 1 | 0.034 | 0.914 |
| 31-Jul-13 | 0 | no change | 31-Jul-13 | 0.000 | 0 | 1 | 0.026 | 0.836 |
| 18-Sep-13 | 0 | no change | 18-Sep-13 | 0.000 | 0 | 1 | 0.046 | 0.891 |
| 30-Oct-13 | 0 | no change | 30-Oct-13 | 0.000 | 0 | 1 | 0.032 | 0.891 |
| 18-Dec-13 | 0 | no change | 18-Dec-13 | 0.006 | 0 | 1 | 0.050 | 1.091 |
| 29-Jan-14 | 0 | no change | 29-Jan-14 | 0.000 | 0 | 1 | 0.040 | 0.999 |

Notes. The dependent variable $y_t$ is defined in Section 5.1; the explanatory variables $tight_{t-1}$, $ease_{t-1}$, $spread_t$ and $house_t$ are defined in Section 5.2; the monetary shock $surprise_t$ is defined in Section 5.6.

The dates when FOMC decisions became effective do not completely coinside with the dates when these decisions were made. Up to 1994, the FOMC decisions, made at scheduled meetings, became effective on the next day. Since February 1994, when the Fed began announcing its decision immediately after each FOMC scheduled meeting, the FOMC decisions became effective on the same day. The target changes made at unscheduled meetings were typically implemented by the Fed and recognized by futures market participants on the same day (with two exceptions for FOMC decisions on November 6, 1989 (technical factors delayed market reaction to the next day) and on October 15, 1998 (which was implemented after the close of futures market, so the market reacted on the next day). The employed chronology of FOMC actions is taken from the Board of Governors of the Fed (via ALFRED) with several adjustments suggested by Kuttner (2001, 2003): (i) 25 bp cut on June 5, 1989 instead of June 6, 1989 as in ALFRED; (ii) 25 bp cut on July 26, 1989 instead of July 27, 1989; (iii) 31.25 bp cut on October 16, 1989 instead of October 19, 1989; (iv) 25 bp cut on November 7, 1989 instead of November 6, 1989; (v) 25 bp cut on December 18, 1990 instead of December 19, 1990; (vi) 25 bp cut on January 8, 1991 instead of January 9, 1991; and (vii) 25 bp cut on October 30, 1991 instead of October 31, 1991.

Table 15. List of candidate explanatory variables

**Inflation measures:**

- Consumer price index (CPI) for all urban consumers: all items (Bureau of Labor Statistics)
- CPI (Blue Chip consensus forecasts for current quarter, and for one, two, and three quarters ahead)
- Core CPI (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Personal consumption expenditures (PCE) (Bureau of Economic Analysis)
- Real PCE (Bureau of Economic Analysis)
- Headline PCE inflation (CPI inflation before 2000) (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Real PCE, chain weight (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Price index for gross domestic product (GDP), chain weight (Blue Chip consensus forecasts for current quarter, and for one, two, and three quarters ahead)
- Price index for GDP (gross national product (GNP) before 1992Q1), chain weight (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Implicit price deflator: GNP (prior to 1991/12/03) – GDP (since 1991/12/04) (Bureau of Economic Analysis)
- Implicit price deflator: GNP (Bureau of Economic Analysis)
- Implicit price deflator: business sector (Bureau of Labor Statistics)
- Implicit price deflator: nonfarm business sector (Bureau of Labor Statistics)
- Implicit price deflator: private fixed investment (Bureau of Economic Analysis)
- Implicit price deflator: final sales of domestic product (Bureau of Economic Analysis)
- Output-price index (Bureau of Economic Analysis)
- Producer price index: finished goods (Bureau of Labor Statistics)
- Inflation expectation (University of Michigan survey)
- Diffusion indexes (Philadelphia Fed survey)

**Output, productivity and costs measures:**

- GDP (Bureau of Economic Analysis)
- Real GDP (real GNP prior to 1991/12/03) (Bureau of Economic Analysis)
- Real GNP (Bureau of Economic Analysis)
- Real GDP (real GNP until 12/10/1991) (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)

Table 15 (contd). List of candidate explanatory variables

- Real GDP (Blue Chip consensus forecasts for current quarter, and for one, two, and three quarters ahead)
- Output gap (the difference between actual and potential output) projection for current quarter, and for one, two, three, four and five quarters ahead (assumptions used by the staff of the Board of Governors of the Fed in constructing its Greenbook projection, source: Philadelphia Fed's real-time data set)
- Real potential GDP (U.S. Congressional Budget Office)
- Final sales of domestic product (Bureau of Economic Analysis)
- Real final sales of domestic product (Bureau of Economic Analysis)
- Nonfarm business sector: output per hour of all persons (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Real output per hour of all persons: business sector (Bureau of Labor Statistics)
- Real output per hour of all persons: nonfarm business sector (Bureau of Labor Statistics)
- Real output of all persons: business sector (Bureau of Labor Statistics)
- Real output of all persons: nonfarm business sector (Bureau of Labor Statistics)
- Real compensation per hour: business sector (Bureau of Labor Statistics)
- Real compensation per hour: nonfarm business sector (Bureau of Labor Statistics)
- Unit labor cost: business sector (Bureau of Labor Statistics)
- Unit labor cost: nonfarm business sector (Bureau of Labor Statistics)
- Industrial production index (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Average gross hourly earnings: total private industries (Bureau of Labor Statistics)
- Index of aggregate weekly hours: total private hours (Bureau of Labor Statistics)
- Industrial production index: total (Board of Governors of the Fed)
- Capacity utilization: total (Board of Governors of the Fed)
- Capacity utilization: manufacturing (Board of Governors of the Fed)

**Employment measures:**

- Unemployment rate: all civilian workers (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Unemployment rate: all civilian workers (Bureau of Labor Statistics)
- Total nonfarm payroll employment: all employees (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Total nonfarm payroll employment: all employees (Bureau of Labor Statistics)
- Index of help wanted advertising in newspapers (Conference Board)
- Diffusion indexes (Philadelphia Fed survey)

Table 15 (contd). List of candidate explanatory variables

**Other activity measures:**

- Real business fixed investment, chain weight (before 1996Q1: fixed weight) (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Real residential fixed investment, chain weight (before 1996Q1: fixed weight (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Housing starts: total number of new privately owned housing units started (Census Bureau)
- Housing starts: total number of new privately owned housing units started (Greenbook projection for current quarter, and for one, two, three and four quarters ahead, Board of Governors of the Fed)
- Balance on current account (Bureau of Economic Analysis)
- Personal saving (Bureau of Economic Analysis)
- Gross private domestic investment (Bureau of Economic Analysis)
- Real gross private domestic investment (Bureau of Economic Analysis)
- Fixed private investment (Bureau of Economic Analysis)
- Fixed private investment (Bureau of Economic Analysis)
- Real fixed private investment (Bureau of Economic Analysis)
- Disposable personal income (Bureau of Economic Analysis)
- Manufacturers' new orders: durable goods (Census Bureau)
- New private housing units authorized by building permit (Census Bureau)
- Personal income (Bureau of Economic Analysis)
- Real disposable personal income (Bureau of Economic Analysis)
- Retail sales: total excluding food services (Census Bureau)
- New one family houses for sale (Census Bureau)
- New one family houses sold (Census Bureau)
- Shipments: total manufacturing (Census Bureau)
- Diffusion indexes (Philadelphia Fed survey)

**Monetary aggregates:**

- M1 Money Stock (Board of Governors of the Fed)
- M2 Money Stock (Board of Governors of the Fed)
- M3 Money Stock (Board of Governors of the Fed)

Table 15 (contd). List of candidate explanatory variables

**Financial indicators:**

- Speads (differences) between the one-, two-, three-, five-, seven- and ten-year treasury constant maturity rate and effective federal funds rate (Board of Governors of the Fed)
- Speads (differences) between the one-, two-, three-, five-, seven- and ten-year treasury constant maturity rate and prime rate (Board of Governors of the Fed)
- Speads (differences) between the one-, two-, three-, five-, seven- and ten-year treasury constant maturity rate and secondary market rate on 1-month certificate of deposit (Board of Governors of the Fed)
- Speads (differences) between the one-, two-, three-, five-, seven- and ten-year treasury constant maturity rate and 1-month commercial paper rate (since 1997/09/01: 1-month AA financial commercial paper rate) (Board of Governors of the Fed)
- 3-month T-bill rate projection for current quarter, and for one, two, three, and four quarters ahead (assumptions used by the staff of the Board of Governors of the Fed in constructing its Greenbook projection, source: Philadelphia Fed's real-time data set)
- Mortgage rates (merged) projection for current quarter, and for one, two, three, and four quarters ahead (assumptions used by the staff of the Board of Governors of the Fed in constructing its Greenbook projection, source: Philadelphia Fed's real-time data set)

**Exchange rates:**

- Nominal exchange rate index: trade weighted, major currencies (Board of Governors of the Fed)
- Nominal exchange rate index (before 1998 – the G-10 mutlilateral trade-weighted index, since 1998 – the major currencies index) projection for current quarter, and for one, two, three, and four quarters ahead (assumptions used by the staff of the Board of Governors of the Fed in constructing its Greenbook projection, source: Philadelphia Fed's real-time data set)
- Real (price-adjusted) exchange rate index projection for current quarter, and for one, two, three, and four quarters ahead (assumptions used by the staff of the Board of Governors of the Fed in constructing its Greenbook projection, source: Philadelphia Fed's real-time data set)

**FOMC statements:**

- $tight_{t-1}$ and $ease_{t-1}$ — the two binary indicators derived from the "policy bias" statement or "balance-of-risks" assessment at the previous FOMC meeting, classified as "easing", "symmetrical" or "tightening": $tight_{t-1}$ that takes value one if the statement at the previous FOMC meeting was tightening, and zero otherwise; and $ease_{t-1}$ that takes value one if the statement was easing, and zero otherwise.

Table 16. Sample descriptive statistics

| Variable | Mean | Median | Standard deviation | Minimum | Maximum | First-order autocorrelation coefficient |
|---|---|---|---|---|---|---|
| $y_t$ | -0.04 | 0.00 | 0.92 | -0.50 | 0.50 | 0.50 |
| $tight_{t-1}$ | 0.26 | 0.00 | 0.44 | 0.00 | 1.00 | 0.63 |
| $ease_{t-1}$ | 0.32 | 0.00 | 0.47 | 0.00 | 1.00 | 0.73 |
| $spread_t$ | 0.05 | 0.11 | 0.54 | -1.45 | 1.76 | 0.82 |
| $house_t$ | 1.48 | 1.51 | 0.27 | 0.85 | 2.20 | 0.92 |
| $surprise_t$ | -0.03 | 0.00 | 0.09 | -0.43 | 0.15 | -0.02 |

Notes. Sample period: 7/1987–1/2006 (190 observations). For the definitions of the variables see Section 5.2.

Table 17. Tests for unit roots

| Variable | Sample period (and size) | The Augmented Dickey-Fuller (ADF) unit root tests | | | | |
|---|---|---|---|---|---|---|
| | | Data frequency | Deterministic terms* | Lag length | t-statistic | P-value** |
| $y_t$ | 7/87-1/06 (254 obs.) | FOMC decisions | C | 3 | -4.30 | 0.0005 |
| $tight_{t-1}$ | 7/87-1/06 (256 obs.) | FOMC decisions | C | 1 | -4.72 | 0.0001 |
| $ease_{t-1}$ | 7/87-1/06 (257 obs.) | FOMC decisions | C | 0 | -5.29 | 0.0000 |
| $spread_t$ | 1/62-2/17 (14341 obs.) | daily | C | 41 | -7.84 | 0.0000 |
| $house_t$ | 1/59-1/17 (680 obs.) | monthly | C, LT | 16 | -4.23 | 0.0043 |
| $surprise_t$ | 7/87-1/06 (254 obs.) | FOMC decisions | C | 3 | -6.44 | 0.0000 |

Notes. * C - constant, LT - linear trend; ** MacKinnon (1996) one-sided p-values. For the definitions of the variables see Section 5.2. The lag order of the lagged first differences of the dependent variable in the ADF tests is selected according to a criterion of no serial correlation among the ADF regression residuals.

Table 18. Modeling changes to federal funds rate target: the estimates of the Swopit-z model parameters without exclusion restrictions

| Variables | Regime equation | Outcome equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| $tight_{t-1}$ | 1.61 (0.38)*** | 5.25 (136.22) | 2.33 (1.00)** |
| $ease_{t-1}$ | -1.85 (0.68)*** | -0.36 (0.37) | -10.38 (297.67) |
| $spread_t$ | 2.06 (0.37)*** | 0.64 (0.39)* | 1.52 (0.74)** |
| $house_t$ | 5.06 (1.00)*** | -1.45 (0.82)* | -1.38 (1.96) |
| $threshold_1$ | 6.91 (1.67)*** | -3.32 (1.01)*** | -7.07 (172.75) |
| $threshold_2$ | 9.63 (1.68)*** | -1.91 (0.93)** | 0.83 (3.31) |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses. The explanatory variables are defined in Section 5.2.

Table 19. Modeling changes to federal funds rate target: the OP and MIOP estimates

| Variables | OP model | MIOP model | |
|---|---|---|---|
| | | Regime equation | Outcome equation |
| $tight_{t-1}$ | 1.41 (0.25)*** | -8.00 (201.73) | 2.77 (0.60)*** |
| $ease_{t-1}$ | -0.89 (0.23)*** | -5.18 (201.73) | -1.25 (0.28)*** |
| $spread_t$ | 1.56 (0.20)*** | 1.52 (0.55) | 1.74 (0.25)*** |
| $house_t$ | 1.55 (0.36)*** | 4.11 (1.36)*** | 1.70 (0.41)*** |
| $threshold_1$ | -0.11 (0.55) | -1.57 (201.73) | -0.11 (0.60) |
| $threshold_2$ | 1.09 (0.53)** | | 1.39 (0.61)** |
| $threshold_3$ | 3.91 (0.64)*** | | 4.06 (0.73)*** |
| $threshold_4$ | 5.29 (0.69)*** | | 6.29 (0.88)*** |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses. The explanatory variables are defined in Section 5.2.

Table 20. Modeling changes to federal funds rate target: the estimates of the Swopit-r model parameters

| Variables | Regime equation | Outcome equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| *tight* $_{t-1}$ | 1.67 (0.35)*** | | 2.69 (0.98)*** |
| *ease* $_{t-1}$ | -3.07 (0.79)*** | -0.40 (0.35) | |
| *spread* $_{t}$ | 1.96 (0.35)*** | 1.04 (0.29)*** | 1.95 (0.75)*** |
| *house* $_{t}$ | 5.60 (1.03)*** | | |
| *threshold* $_{1}$ | 7.43 (1.58)*** | -1.63 (0.35)*** | -0.94 (0.61) |
| *threshold* $_{2}$ | 10.46 (1.72)*** | -0.26 (0.34) | -0.94 (0.61) |
| *threshold* $_{3}$ | | 5.90 (n/e) | -0.94 (0.61) |
| *threshold* $_{4}$ | | 31.18 (n/e) | 3.49 (1.07)*** |

Notes. Sample period: 7/1987–1/2006 (190 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses. The explanatory variables are defined in Section 5.2.

Table 21. Modeling changes to federal funds rate target: the estimates of the exogenous Swopit-z model parameters

| Variables | Regime equation | Outcome equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| *tight* $_{t-1}$ | 1.38 (0.31)*** | | 2.38 (0.90)*** |
| *ease* $_{t-1}$ | -1.07 (0.39)*** | -0.01 (0.30) | |
| *spread* $_{t}$ | 1.99 (0.33)*** | 1.38 (0.23)*** | 1.72 (0.71)** |
| *house* $_{t}$ | 3.05 (0.57)*** | | |
| *threshold* $_{1}$ | 4.33 (0.97)*** | -1.51 (0.29)*** | -2.26 (5.22) |
| *threshold2* | 6.38 (0.99)*** | -0.58 (0.29)** | 3.26 (0.98)*** |

Notes. Sample period: 7/1987–12/2016 (281 observations). ***/**/* denote the statistical significance at the 1/5/10 percent level. The asymptotic standard errors are shown in parentheses. The explanatory variables are defined in Section 5.2.

Table 22. Modeling changes to federal funds rate target: the parameter estimates of the endogenous Swopit-z model

| Variables | Regime equation | Outcome equations | |
|---|---|---|---|
| | | Dovish regime | Hawkish regime |
| $tight_{t-1}$ | 1.35 | | 2.29 |
| $ease_{t-1}$ | -0.97 | 0.00 | |
| $spread_t$ | 1.97 | 1.44 | 1.71 |
| $house_t$ | 3.23 | | |
| $threshold_1$ | 3.82 | -1.59 | -2.27 |
| $threshold2$ | 6.74 | -0.58 | 3.39 |
| | Correlation coefficients: | $\rho_1 = 0.90$ | $\rho_3 = 0.36$ |

Notes. Sample period: 7/1987–12/2016 (281 observations). The explanatory variables are defined in Section 5.2. The Hessian matrix fails to invert – the s.e. of parameters are not estimated.