

# Measuring Innovation with Patents when Patenting is Strategic

Jonathan F. Lee\*

October 2017

## JOB MARKET PAPER

### Abstract

I model a firm's decision to create an invention and, separately, her decision to protect that invention with intellectual property (IP). Because external forces, such as industry characteristics or policy regimes, can affect the innovation and protection decisions differently, the model predicts that innovation measures based on IP usage, such as patent counts, may not correlate with innovative effort. For example, the threat of competition generally has an inverse-U shaped relationship with observed patenting but has a normal-U relationship with innovative effort. In this case, the average quality of a firm's patent portfolio is a better proxy for innovation. I derive general conditions under which various patent statistics, such as quality-adjusted patenting or average patent quality, are useful proxies for how innovation responds to external influences.

JEL-Classification: K11, L24, O31, O34

Keywords: innovation, intellectual property, patents, trade secrets, market structure, competition

---

\*Queen's University, Dept. of Economics. Contact: [leejf@econ.queensu.ca](mailto:leejf@econ.queensu.ca).

I am grateful for helpful comments from Ján Zábajník, Marie-Louise Vierø, Veikko Theile, Jean de Bettignies, Olena Ivus, Corinne Langinier, Yossi Spiegel, Francisco Ruiz-Aliseda, and Robert Clark, as well as seminar participants at Queen's University, the Canadian Economics Association 2016 Annual Meeting, the International Industrial Organization 2016 Conference, and the Searle Center Innovation Economics 2017 conference.

Innovation is generally understood as the key to economic growth and prosperity, but measuring the innovative activity in an economy is problematic. Spending on research and development, one possible measure, is only sometimes recorded by firms, and it is unclear to what extent that spending increases innovative output, as opposed to basic scientific knowledge (Nagaoka et al., 2010). The Solow residual, the classic measure of aggregate technological change, comprises the empirical leftovers after accounting for whatever we *can* measure in our data (Solow, 1957). The most common measure of innovation in wide usage is quality-adjusted patenting: a researcher counts the flow of new patents granted to an entity, weights them by some quality metric (usually citations from future patents), and uses that weighted sum as measured innovation. These patenting measures are far from exact: Griliches (1990) writes, “Not all inventions are patentable, not all inventions are patented,” but nevertheless “patent statistics remain a unique resource for the analysis of the process of technical change.”

Adjusting patent counts by quality can alleviate some of the problems with using patenting as a measure of innovation, but other issues remain. Quality adjustments intend to adjust for the variation in the amount of innovation embodied by a patent, but one must assume that a marginal increase in observed patenting activity is driven by at least *some* increase in innovative activity. In this paper, I show that this assumption only holds for certain empirical settings and is not true generally: external forces that increase a firm’s innovative activity can nevertheless decrease her incentive to protect her innovations from competitors with IP such as patents. In these cases, average patent quality is a valid measure of innovation instead: as innovation increases and patenting decreases, firms retain only their highest-quality patents.

In this paper, I present a two-firm model of innovation, intellectual property (IP) protection, and market competition. One firm, the Leader, exerts innovative effort to create a new product with multiple components. If she is successful, she then decides which of those components to protect with IP and what forms of IP to use. I consider patents and trade secrets here, though these methods can stand in respectively for any formal or informal method of IP protection more broadly. Trade secrets determine the likelihood that a potential competitor, the Follower, is able to compete with the Leader in the market. Patents determine the expected value of patent litigation and damages awarded if the Follower enters the market and infringes upon the Leader’s patents. If these expected damages are large, the Follower declines

to compete and the Leader secures a monopoly. If the expected damages are small, however, the Follower competes and the patents facilitate a transfer of profit from the Follower to the Leader. The model is agnostic about the actual form of market competition that ensues; I simply specify how much producer surplus is competed away through entry and how the remaining surplus is divided between the two firms.

One of the model’s core assumptions is that products are “complex”; a product comprises multiple components, each of which may be covered by its own IP. Thus a single product can be protected by many patents and many secrets, and firms must decide how many of which kinds of IP to use. Holding innovation fixed, firms will change overall IP usage or substitute between IP methods in response to external influences. The firm’s incentives to modify her IP portfolio are a core mechanism of the model, and they are independent of the direct incentives to innovate. These complex products exist in a wide variety of industries: Arora et al. (2003) estimate the average number of patents per innovation ranges from 2 in pharmaceuticals to 6.6 in industrial chemicals to 8.8 in rubber products, with an overall average of 5.6 patents per innovation. Premarin, an estrogen medication, is particularly illustrative. This pharmaceutical has been on the market since 1942, and the drug is still actively marketed and sold today. Long after patent expiration, no generic competitor has been cleared by the US FDA. The process used to extract the active ingredient in Premarin is protected by trade secret, even though the chemical makeup of the ingredient is commonly known and the relevant patents have expired. This secrecy has led to the extended absence of competition. The chemical compound, extraction process, and manufacturing methods can be viewed as separate components of a single product (Premarin), and each of these products could have been patented or kept secret at the time of invention. Thus Premarin’s manufacturer has reaped profits due to its mixed portfolio of IP. (Noonan, 2011; Lobel, 2013)

This paper uses patents and trade secrets to study the trade-offs and complementarities of formal and informal IP rights more broadly, and predicts when a firm will prefer a mixed basket of IP. By including alternative protection methods in the firm’s choice set, the model differentiates between forces that encourage protection generally and those that encourage patenting specifically. Thus firms will use both methods simultaneously because incentives to protect are high, not because of any inherent complementarity between the two methods. In the model, simultaneous usage will be most likely when incentives to protect generally are moderate, but the benefits of

lead time (referred to below as “incumbency advantage”) are low.

The model yields three key insights into the patenting–innovation relationship. First, changes in the observed patenting behavior of a firm occur through two channels: the innovation effect, which captures the incentive to innovate and earn market profits from the innovation, and the protection effect, which captures the incentive to protect those market profits from competitors. Second, these two effects can reinforce or counteract each other, so the observed response of patenting to a given external factor depends on the relative magnitudes of each channel. Third, patent counts (quality–adjusted or otherwise) are a valid metric for innovation when the two effects coincide, and average patent quality is valid when the two effects counteract.

As a specific example of these general results, I show how changing competitive pressure in an industry decreases a firm’s incentives to innovate while simultaneously increasing her incentives to protect the innovations that do occur: the propensity to invent decreases while the propensity to patent increases. Taken together, these two forces imply that observed patenting only matches underlying innovative output as a special case: in general, observed patenting is not a valid measure of innovation when the competitive nature of an industry is changing. Quality–adjusted patenting performs slightly better but remains invalid generally. However, the firm’s average patent quality moves with underlying innovative effort in all cases.

Generally, the choice of appropriate innovation metric depends on the fundamentals of the model in question. The model and its implications suggest some guidelines for how to choose the most appropriate metric to capture the response of innovation to an exogenous factor. Does the factor influence incentives to use IP generally, or does it elicit a response in one form of IP (*e.g.*, patents) specifically? Is IP protection generally used to appropriate profit gains or to defend against profit losses? Using one’s prior beliefs and existing evidence, the answer to these questions leads to a suggested innovation metric for the context at hand.

## Relevant Literature

The paper contributes to three related bodies of research: the measurement of innovative activity, the relationship between innovation and competition, and the choice between various forms of IP. I will briefly outline each here, but the interested reader should consult the more exhaustive review articles cited below.

First and foremost, the paper refines our understanding of how patent statistics relate to the underlying rate of innovation that we wish to measure. Seminal work includes Pakes and Griliches (1980), Griliches (1981), and Pakes (1985), which relate a firm’s patenting to its research and development (R&D) spending and its market value. Scholars took stock of the contemporary knowledge in Griliches et al. (1987) and Griliches (1990). The latter guided research in the following decades, noting that citation-weighted patent data were a promising avenue of future work and identifying Trajtenberg (1990) as a precursor. Citation-weighted patent counts became a standard way of weighting a patent by the amount or quality of innovation inherent within the patent, and further work refined the concept. Notably, Hall et al. (2005) validated the measure by relating citation-weighted patents to a firm’s market value. The authors also developed a method of correcting for “truncation bias”, to correct for the likelihood of unobserved future citations. For a more complete treatment of this topic, see Hall et al. (2010) and Nagaoka et al. (2010).

The measurement literature begins with the assertion that patents are a fundamentally sound proxy for innovative activity, and the bulk of research has attempted to refine and improve this measure. In this paper, I urge caution by detailing when that fundamental assertion is not valid: observed patenting can move in the opposite direction from innovation and would thus not be a useful proxy. I propose a different measure, average patent quality, which should be a useful alternative in the researcher’s toolbox.

A second broader literature explores the relationship between innovation and competition. One major strand of this research, usually attributed to Arrow (1962), argues that a monopolist would have less incentive to improve her product, since she lacks any market rivals. The other strand, beginning with Schumpeter (1942), argues that competitive firms would not make enough profit to recoup their innovation costs, so only firms with sufficient market power would innovate. Schumpeterian growth models arise from this line of thinking (Aghion and Howitt, 1992; Aghion et al., 2013). Recent work joins these opposing strands into one cohesive theory and predicts that competition and innovation have an inverse-U relationship (Aghion et al., 2005). Others have observed that Arrow’s argument focuses on competition *ex ante*, while Schumpeter’s concerns competition *ex post*, so the apparent contradiction is a bit of a misunderstanding (Whinston, 2011). For thorough reviews of this large body of work, see Gilbert (2006), Shapiro (2011), and Peneder (2012).

By considering the decision to innovate and the decision to use IP simultaneously, the model below presents a more nuanced view of the innovation–competition relationship. Here, competitive forces at the industry level influence a firm’s innovative output and, separately, its IP usage. Success in innovation then determines whether a new market is created, while IP usage determines the competitiveness of that new market.<sup>1</sup> In other words, observed market competition is not directly causal for innovation, and competitive industries can generate noncompetitive markets in IP-intensive sectors. It directly follows that measures of market power (such as the Lerner index) are outcomes, not determinants, of the innovation–IP process, and would likely only predict future innovative output through financial channels (*e.g.*, profits can be invested in future R&D) or through secular trends in industry-level competition.<sup>2</sup>

In this literature, the paper closest to the current one is de Bettignies et al. (2016), which considers an upstream firm’s optimal innovative effort and IP strategy (specifically, licensing agreements) in response to the intensity of downstream firm competition. Their paper finds, as mine does, that intensity of competition in the industry has a U-shaped effect on the firm’s innovative output, and also finds that the innovator’s optimal IP strategy elicits downstream entry when intensity of competition is low and downstream monopoly when intensity is high. These results mirror the predictions of this paper, in which the innovating firm guarantees itself a monopoly when competitive pressure is high and allows rival firm entry when competitive pressure is low. However, de Bettignies et al. study the effects of downstream competition on an upstream innovator’s licensing strategy, while the model of this paper studies how a firm’s innovative effort and IP portfolio depend on competitive forces in its own industry. Further, I use this model to determine when patent counts are valid or invalid as a measure of innovation, and I propose an alternative measure that can be used when conventional measures fail. Issues related to measurement of innovation are outside the scope of de Bettignies et al. (2016).

A third literature examines the patent system as just one of many meth-

---

<sup>1</sup>Here, I identify an industry as a collection of related product markets, wherein firms in that industry could enter the markets relatively easily. Automotive firms could more easily enter the consumer electric car market than household appliance manufacturers could, for instance.

<sup>2</sup>Instrumental variables are sometimes used to purge this endogeneity; see Aghion et al. (2005) and de Bettignies et al. (2016) as examples.

ods to encourage innovation and to appropriate its returns. In most cases, patents are seen as substitutes for other means of appropriation. Alternatives include trade secrecy (Friedman et al., 1991; Arora, 1997; Zájbojník, 2002; Denicolò and Franzoni, 2004; Hussinger, 2006; Moser, 2012; Zhang, 2012; Png, 2015), product complexity (Henry and Ruiz-Aliseda, 2016), and lead time (Cohen et al., 2000). However, survey evidence suggests that patents may be used in tandem with these and other methods (Levin et al., 1987; Cohen et al., 2000), and some work investigates the simultaneous use of multiple protection methods (Anton and Yao, 2004; Graham, 2004; Ottoz and Cugno, 2008; Belleflamme and Bloch, 2013; Graham and Hegde, 2014). The distinction between IP and innovation, crucial to the results of this paper, features in many of the papers cited above: *e.g.* Moser (2012) finds that many inventors who cannot patent effectively will nevertheless invent and rely on informal protection methods. Insights from the IP choice literature shape policy recommendations; see Hall (2007) and Shapiro (2011). For an extensive discussion of the choice between formal and informal intellectual property, see Hall et al. (2014).

The patent system is not perfect, and clever use of patents allows for more than just a straightforward guarantee of monopoly. Often the mere threat of a patent lawsuit is enough to deter competitor entry, even if the validity of the patent is suspect (Lerner, 1995; Anton and Yao, 2004; Anton et al., 2006). Other strategic complications abound, such as defensive patenting, patent pools, or patent thickets, each with its own body of work. These interactions, while important, are not the focus of the current paper. Instead, I focus on how strategic patenting for entry deterrence and revenue extraction influences how we should measure underlying innovative activity, improves our understanding of the innovation–competition relationship, and clarifies the relationships between various forms of IP rights.

The paper proceeds as follows. Section 1 outlines the theoretical model, Section 2 solves for equilibrium, and Section 3 presents the important results of the paper. Section 4 concludes. Appendix A presents a technical foundation for the main model, and proofs can be found in Appendix B.

## 1 Theoretical Environment

Consider a model in which a firm (the “Leader”, feminine pronouns) attempts to create a new product. If successful, she then decides how to protect it from

competition. Once protection is established, a competitor (the “Follower”, masculine pronouns) may be able to enter the market. To enter, the Follower must gain sufficient knowledge of the product; only then can he enter the market if he wishes. Competition (or monopoly) ensues, and both the Leader and Follower receive payoffs.

The game occurs in three stages: Creation, Protection, and Entry. Figure 1 depicts each stage in sequence, with market profits given at each terminus. In the Creation stage, the Leader attempts to invent a new product that consists of a continuum of measure  $N$  of components.<sup>3</sup> She exerts innovation effort  $e \geq 0$  at cost  $c \cdot e$  and successfully creates a new product with probability  $\phi(e)$ . I assume  $\phi(e)$  is an increasing, smooth, and concave function with  $\phi(0) = 0$  and  $\lim_{e \rightarrow \infty} \phi(e) \leq 1$ . If the Leader employs more resources in the innovative process, she is more likely to succeed at creating a new product. Note that her effort only improves the likelihood of creation; the quality and characteristics of the created product are taken as exogenous features of the product’s technology.

If the Leader fails to innovate, the game ends. If she is successful, the Protection stage follows, in which the Leader then chooses  $n_p$  of the product’s  $N$  components to patent,  $n_s$  to keep secret, and leaves the remaining  $N - n_p - n_s$  unprotected and fully disclosed. Patents enable litigation but cost  $f$  each in filing fees, attorney retainers, etc. A larger collection of patents increases the value of legal action: in expectation, an entrant will pay the Leader damages (or negotiated licensing revenues) equal to  $g(n_p)$  times the Leader’s profits that are lost due to entry (a “lost profits” approach).<sup>4</sup> I assume that  $g(n_p)$  is an increasing, continuous, smooth, and concave function defined on  $[0, N]$ , with  $g(0) = 0$ .

One might reasonably assume that  $g(n_p)$  should simply be an indicator function for  $n_p > 0$ : the existence of a single patent should be enough to win a court case. Instead, I model  $g(n_p)$  as a smoothly increasing function between zero and one, and I do so for two reasons. First, I allow for “weak patents” in the sense that the validity and breadth of any one patent is functionally uncertain until it has been tried in court, so that possessing more patents increases the probability of at least one patent being found valid. Second,

---

<sup>3</sup>I use a continuum of components for simplicity; discretizing the number of components adds unnecessary complexity and leads to similar results.

<sup>4</sup>Using a “reasonable royalties” approach, in which the court sets the royalty rate when lost profits cannot be determined, yields qualitatively similar, but less clean, results.

a patent on a single component of a complex product does not protect the entire product, and therefore cannot protect the entirety of lost profits due to entry.<sup>5</sup> I model patent protection accordingly.

Trade secrets reduce the likelihood that the Follower will learn enough to enter the market later in the game. The Leader pays  $s$  per component that she keeps secret, which represents the cost of non-disclosure agreements, the loss of efficiency from maintaining a “need-to-know basis” policy within a firm, or any costs associated with security.<sup>6</sup> The Follower learns all patented and unprotected components with certainty but may not learn all  $n_s$  secrets, which he must do in order to enter the market. The probability that the Follower learns all knowledge in a portfolio of secrets of size  $n_s$  (and can therefore enter the market if desired) equals  $\ell(n_s)$ .<sup>7</sup> I assume that  $\ell(n_s)$  is a decreasing, continuous, smooth, and convex function defined on  $[0, N]$ , with  $\ell(0) = 1$ .

The Protection stage begins as the Leader chooses an IP portfolio  $(n_p, n_s)$  and ends with the Follower’s learning process. If the Follower does not learn enough to enter the market, the Leader earns monopoly profits  $\pi_M$  and the game ends. If the Follower does learn enough to enter, the game moves to the Entry stage and the Follower decides whether or not to enter the market. If he chooses not to enter, the game ends, the Leader earns monopoly profits  $\pi_M$  and the Follower’s payoff is zero (the normalized value of his outside option). If the Follower does enter, however, then the two firms compete for shares of the available producer surplus. I define  $\Delta$  as the loss in total market

---

<sup>5</sup>Indeed, the US Supreme Court established in *Garretson v. Clark* (1884) the precedent that patent infringement damages “must reflect the value attributable to the infringing features of the product, and no more,” a precedent that has been upheld by a variety of Federal court cases since (Schenk and Plumpe, 2015). This rule applies to both utility and design patents, as has been demonstrated recently in *Samsung Electronics Co. v. Apple, Inc.* (2016): “the relevant article...for arriving at a damages award...need not be the end product sold to the consumer but may only be a component of that product.”

<sup>6</sup>By law under 18 U.S.C. § 1839 (3)(A), a firm must have “taken reasonable measures” to keep the information secret in order for it to meet the legal definition of a trade secret, and these measures certainly have a cost. If the definition is met, the owner is afforded legal protection against corporate espionage, but not independent invention or reverse engineering. Similar laws exist elsewhere, *e.g.* the Uniform Trade Secrets Act in Canada. For a micro-founded model of the costs of trade secrecy through non-disclosure agreements and restriction of knowledge within the hierarchy of a firm, see Zájbojník (2002).

<sup>7</sup>I assume that knowledge of all components is necessary for entry, but all that is really required for the model is that the probability of learning enough to successfully enter is decreasing in  $n_s$ , which  $\ell(n_s)$  accomplishes.

profits due to entry and  $\lambda \in [0, 1]$  as the Leader's share of the market profits that remain.<sup>8</sup> Thus the Leader's market profits equal  $\lambda(1 - \Delta)\pi_M$  and those of the Follower equal  $(1 - \lambda)(1 - \Delta)\pi_M$ .<sup>9</sup> Patent litigation or licensing then yields transfers in expectation from the Follower to the Leader, and expected payoffs from entry equal

$$\begin{aligned}\Pi_L &= \lambda(1 - \Delta)\pi_M + g(n_p)(1 - \lambda(1 - \Delta))\pi_M \\ \Pi_F &= (1 - \lambda)(1 - \Delta)\pi_M - g(n_p)(1 - \lambda(1 - \Delta))\pi_M\end{aligned}\tag{1}$$

since  $(1 - \lambda(1 - \Delta))\pi_M$  is the size of the Leader's lost profits.<sup>10</sup>

The above framework is concise, but it makes an implicit assumption about the nature of the product and its components. Specifically, I assume *component independence*: the patent or secrecy status of one component does not influence the protection that could be afforded to another component via patent or secret.

**Assumption 1.** *For a given  $n_s$ ,  $\ell(n_s)$  is constant for all  $n_p$ . For a given  $n_p$ ,  $g(n_p)$  is constant for all  $n_s$ .*

This assumption could be violated in two ways. First, there could be direct knowledge spillovers between components: disclosing knowledge of one component could make other components easier to discover. Second, a secret component could have higher patent potential than some patented components, and removing this secret would increase  $g(n_p)$  because the Leader would swap it with a less effective patent. Assumption 1 asserts that protection is additive, addressing the first violation; and that patent potential is inversely related to secrecy potential across components, addressing the second. For the interested reader, I formally micro-found these two notions in Appendix A and show that they imply Assumption 1 and the shapes of  $g(n_p)$  and  $\ell(n_s)$  (concavity, etc.). For the body of the paper, though, I simply take  $g(n_p)$  and  $\ell(n_s)$  to be primitives of the model.

---

<sup>8</sup>Note that I do not assume  $\Delta > 0$ . Though it seems likely that entry will reduce aggregate profits (*e.g.* Cournot), it is not inevitable (*e.g.* differentiated goods markets).

<sup>9</sup>This parameterization is flexible and can accommodate many different assumptions about the market's structure. For instance, Cournot duopoly with linear demand and constant marginal costs is captured by  $\Delta = \frac{1}{9}$  and  $\lambda = \frac{1}{2}$ .

<sup>10</sup>Here, both firms are risk-neutral, so the expected litigation outcome  $g(n_p)$  serves as a credible threat point for the Leader in licensing negotiations. The model thus captures both licensing and litigation outcomes.

I assume that  $g(N) < 1$ , which implies that the Leader cannot be awarded aggregate damages higher than her lost profits, and that  $|g''(n_p)| > \frac{g'(n_p)^2}{1-g(n_p)}$  and  $\ell''(n_s) > \frac{\ell'(n_s)^2}{\ell(n_s)}$ , which state that components are sufficiently heterogeneous in patent and secrecy potential and also act as sufficient-curvature assumptions, ensuring the Leader's problem is strictly concave globally.

## 2 Analysis

I solve the model by backwards induction. I first consider the entry decision of the Follower, conditional on full learning and a given patent portfolio  $n_p$ . I then consider the portfolio of IP chosen by the Leader and, lastly, the consequences of her choices for incentives to innovate.

### Entry

Conditional on learning all components, the Follower observed  $n_p$  and enters the market if his expected profit is strictly greater than zero, which is true when

$$\Pi_F = (1-\lambda)(1-\Delta)\pi_M - g(n_p)(1-\lambda(1-\Delta))\pi_M > 0 \rightarrow g(n_p) < \frac{(1-\lambda)(1-\Delta)}{1-\lambda(1-\Delta)}$$

So the Follower enters when the number of patents is smaller than some threshold  $\bar{n}_p$ . If such a threshold exists, it is defined by  $g(\bar{n}_p) = \frac{(1-\lambda)(1-\Delta)}{1-\lambda(1-\Delta)}$ . Otherwise,  $\bar{n}_p$  is taken to be infinite, or at least strictly greater than  $N$ .<sup>11</sup>

**Lemma 1.** *The Follower enters the market if  $n_p < \bar{n}_p$ . The Leader enjoys a monopoly if  $n_p \geq \bar{n}_p$ .*

Note that if  $\frac{(1-\lambda)(1-\Delta)}{1-\lambda(1-\Delta)} > g(N)$ , then exclusion is impossible. And since  $g(N) < 1$ , a necessary condition for the ability to exclude is  $\Delta > 0$ ; i.e., if total profits shrink due to entry:

**Lemma 2.** *Exclusion of the Follower is possible only if the firms' total surplus shrinks due to entry.*

---

<sup>11</sup>One could envision a model where the Follower must pay some fixed entry cost  $F$ , say to innovate or build production capacity. In this case,  $g(\bar{n}_p) = \frac{(1-\lambda)(1-\Delta)}{1-\lambda(1-\Delta)} - (1-\lambda(1-\Delta))\frac{F}{\pi_M}$  and the rest of the analysis is unchanged.

This result only follows because of the assumption that  $g(N) < 1$ : the Leader can never recoup all of her lost profits through the legal system. While this is probably a reasonable assumption, there could be legal environments in which the Leader is able to recover more than what was lost. Further, environments exist in which entry increases surplus (*i.e.*,  $\Delta < 0$ ), such as markets with sufficiently differentiated products. In these markets, if  $g(N) \geq 1$  the Leader would never exclude, regardless of whether  $\Delta$  is positive or negative.<sup>12</sup> The  $g(N) < 1$  case is more strategically rich for the Leader, so I retain that assumption below.

Incorporating the learning mechanism (and accounting for protection costs and Lemma 1), the Leader's expected payoff from an IP portfolio  $(n_p, n_s)$  equals

$$\Pi_L(n_p, n_s) = \begin{cases} \ell(n_s) (\lambda(1 - \Delta)\pi_M + g(n_p)(1 - \lambda(1 - \Delta))\pi_M) \\ \quad + (1 - \ell(n_s))\pi_M - n_p f - n_s s, & n_p < \bar{n}_p \\ \pi_M - n_p f - n_s s, & n_p \geq \bar{n}_p \end{cases}$$

In the Protection stage, the Leader chooses  $n_p$  and  $n_s$  to maximize  $\Pi_L$ .

## Protection

Note that for any choice of  $n_p > \bar{n}_p$ , the optimal  $n_s$  is 0 and a choice of  $\bar{n}_p$  is preferred. Thus if the Leader chooses to exclude, she does so optimally by choosing  $(n_p, n_s) = (\bar{n}_p, 0)$ . If the Leader instead decides to allow the possibility of entry, she chooses  $(n_p, n_s)$  to solve

$$\max_{n_p, n_s} \pi_M - \ell(n_s)(1 - g(n_p))(1 - \lambda(1 - \Delta))\pi_M - n_p f - n_s s$$

$$\text{subject to } \begin{array}{ll} n_p \geq 0 & n_p < \bar{n}_p \\ n_s \geq 0 & n_p + n_s \leq N \end{array}$$

Note that here, the Leader is effectively choosing a fraction  $(1 - \ell(n_s)(1 - g(n_p))(1 - \lambda(1 - \Delta)))$  of profits to protect; call this the protection level. Given  $g(N) < 1$  and the sufficient curvature assumptions, the problem is strictly

---

<sup>12</sup>Using  $n_p = \bar{n}_p$ , a Leader will retain  $\pi_M$ , but using  $n_p = \bar{n}_p - \epsilon$  will yield almost the whole surplus, which is larger than  $\pi_M$  here by assumption.

concave globally and the first-order conditions characterize an optimum. Rearranged, the first-order conditions yield

$$\frac{s}{f} = \frac{-\ell'(n_s^*)}{\ell(n_s^*)} \frac{1 - g(n_p^*)}{g'(n_p^*)} = \frac{\text{MP}_s}{\text{MP}_p} \quad (\text{Ratio})$$

$$\begin{aligned} s + f &= [\ell(n_s^*)g'(n_p^*) - \ell'(n_s^*)(1 - g(n_p^*))] (1 - \lambda(1 - \Delta))\pi_M \quad (\text{Sum}) \\ &= \text{MP}_p + \text{MP}_s \end{aligned}$$

where  $\text{MP}_{p,s}$  is the marginal increase in the protection level from one additional patent or secret:

$$\begin{aligned} \text{MP}_p &= \ell(n_s)g'(n_p)(1 - \lambda(1 - \Delta))\pi_M \\ \text{MP}_s &= \ell'(n_s)(1 - g(n_p))(1 - \lambda(1 - \Delta))\pi_M \end{aligned}$$

Equations (Ratio) and (Sum) define equilibrium, assuming entry is not forbidden. The Leader compares the payoff from this optimal strategy to  $\pi_M - \bar{n}_p f$ , the payoff from exclusion, and then chooses the profit-maximizing strategy. Incorporating equations (Ratio) and (Sum) into this comparison yields the set of equilibrium portfolios with entry  $(n_p^*, n_s^*)$  that are preferred to Exclusion, which are the ones that satisfy

$$\bar{n}_p f \geq \ell(n_s^*)(1 - g(n_p^*))(1 - \lambda(1 - \Delta))\pi_M + n_p^* f + n_s^* s \quad (\text{Entry})$$

Thus expressions (Ratio), (Sum), and (Entry) fully characterize the Leader's problem, which is depicted graphically in Figure 2. Under entry, the optimal portfolio is given by the intersection of (Ratio) and (Sum), the dashed lines. If this point lies to the left of the solid curve, then (Entry) is satisfied and entry occurs, as it does in Figure 2. Otherwise, the Leader would exclude the Follower. In either case, conditions (Ratio), (Sum), and (Entry) characterize the unique equilibrium portfolio chosen by the Leader.

The condition that  $n_p + n_s \leq N$  can be depicted in Figure 2 by drawing a line from  $(N, 0)$  to  $(0, N)$ . If the solution to equations (Ratio) and (Sum) lies beyond this boundary, then the Leader will choose some portfolio along  $n_p + n_s = N$ , but it is straightforward to show that the qualitative conclusions of the model will be the same.<sup>13</sup> For simplicity, I make the following simplifying

---

<sup>13</sup>Solving the Leader's constrained optimization problem yields two first-order conditions. When  $n_p + n_s \leq N$  is slack, the conditions are identical to (Ratio) and (Sum). When the constraint does bind, one condition is a more general form of (Ratio) and the other is simply  $n_s = N - n_p$ , which slopes downward just as (Sum) does. Condition (Entry) is unchanged, so the model makes the same qualitative predictions as the unconstrained version considered in the body of the paper.

assumption:

**Assumption 2.** *The parameters of the model are such that  $n_p + n_s \leq N$  never binds. In other words,  $N$  is sufficiently large.*

## Creation

Substituting the Leader's optimal portfolio choice into her expected payoff function  $\Pi_L$  yields her ex ante expected profit from an innovation:

$$\Pi_L^* = \max \left\{ \frac{\pi_M - \ell(n_s^*)(1 - g(n_p^*))(1 - \lambda(1 - \Delta))\pi_M - n_p^*f - n_s^*s}{\pi_M - \bar{n}_p f} \right\} \quad (2)$$

which the Leader takes as constant in the Creation stage. She chooses optimal innovation effort  $e^*$  to equate her marginal expected profit from effort with its marginal cost:

$$\phi'(e^*)\Pi_L^* = c \quad (3)$$

which immediately implies that innovation is increasing in  $\Pi_L^*$ .

## 3 Results

With the model solved, I now determine how industry characteristics determine the Leader's choice of IP portfolio, how they influence her incentive to innovate, and the consequences for IP-based measures of innovation. I determine when different classes of measures are valid generally, then demonstrate these effects in the context of industry competition. Finally, I use the results to construct a guide for choosing a valid measure of innovation in a variety of models.

### 3.1 IP Portfolio Choice

Equations (Ratio), (Sum), and (Entry) map industry parameters into IP portfolio choices. These portfolios can be broadly classified as Empty (no protection is used), Licensing (only patents are used, but fewer than  $\bar{n}_p$ ), Concealment (only secrets are used), Mixed (both patents and secrets are used), and Exclusion ( $\bar{n}_p$  patents are used). In other words, the Leader chooses whether to exclude a potential entrant or to allow entry, and then how to protect her innovation should entry occur. I will principally consider the

effects of two industry characteristics, competitive pressure and incumbency advantage (defined presently).

Since  $\Delta$  equals the loss in profits due to entry, I interpret it as a measure of the competitive pressure within an industry. If  $\Delta$  is close to zero, for example, then market entry will leave firms' profits mostly intact. If instead  $\Delta$  is close to one, then market entry will eliminate most of the firms' profits and market participants face increasing pressure to deter competitor entry. Similarly,  $\lambda$  measures the importance of incumbency advantage in the industry, whether it be due to brand loyalty, a lead time in production, etc. If  $\lambda$  is low, then potential entrants can destroy an incumbent's profits easily. If  $\lambda$  is high, then competitors stand to gain very little from entering a market and the incumbent has less to fear. Thus,  $\lambda$  also captures aspects of the competitive nature of an industry.

Proposition 1 details how the industry parameters  $\Delta$  and  $\lambda$  shape the Leader's optimal IP portfolio.

**Proposition 1.** *There exist thresholds functions  $\lambda_1(\Delta)$ ,  $\lambda_2(\Delta)$  and  $\Delta_{Bd}(\lambda)$  such that for a given  $\lambda$  and  $\Delta$ :*

1. *If  $\Delta \leq \Delta_{Bd}(\lambda)$ , then*
  - (a) *if  $\lambda < \lambda_1(\Delta)$ , the Leader chooses  $n_p \in (0, \bar{n}_p)$  and  $n_s > 0$  (Mixed);*
  - (b) *if  $\lambda \in [\lambda_1(\Delta), \lambda_2(\Delta))$ , the Leader chooses  $n_p = 0$  and  $n_s > 0$  (Concealment) when  $\frac{s}{f} < \frac{-\ell'(0)}{g'(0)}$ , and she chooses  $n_p \in (0, \bar{n}_p)$  and  $n_s = 0$  (Licensing) when  $\frac{s}{f} > \frac{-\ell'(0)}{g'(0)}$ ;*
  - (c) *if  $\lambda \geq \lambda_2(\Delta)$ , the Leader chooses  $n_p = 0$  and  $n_s = 0$  (Empty).*
2. *If  $\Delta > \Delta_{Bd}(\lambda)$ , the Leader chooses  $n_p = \bar{n}_p$  (Exclusion).*

*Further,*

3.  *$\Delta_{Bd}(\lambda)$  is a decreasing function with  $\Delta_{Bd}(0) \in [0, 1]$  and  $\Delta_{Bd}(1) = 0$ ;*
4.  *$\lambda_1(\Delta)$  and  $\lambda_2(\Delta)$  are increasing functions with  $\lambda_1(\Delta) \leq \lambda_2(\Delta)$ .*

Figure 3 depicts the regions described by Proposition 1. The  $\Delta_{Bd}(\lambda)$  line divides the  $\Delta$ - $\lambda$  plane into Entry and Exclusion regions (a boundary, hence "Bd"). Entry occurs for lower  $\Delta$  or  $\lambda$ , and Exclusion occurs for higher values. Under Entry, the  $\lambda_1(\Delta)$  and  $\lambda_2(\Delta)$  lines distinguish between different kinds

of portfolios: below  $\lambda_1(\Delta)$ , the Leader uses both patents and secrets (Mixed); above  $\lambda_2(\Delta)$ , the Leader does not protect at all (Empty), and in between, the Leader uses only one kind of IP (Licensing or Concealment).

The competitive nature of the industry determines the Leader's portfolio choice. For industries with relatively little surplus loss from entry, protection is relatively weak, especially if there is a large incumbency advantage. But if entry destroys more surplus, protection increases as well. Eventually, the loss from entry is so great that the Leader will always exclude the Follower if able.

Note that, *ceteris paribus*, Licensing portfolios and Exclusion portfolios are qualitatively different in their characteristics and effects, even though both are comprised solely of patents. Obviously,  $n_p < \bar{n}_p$  in any Entry portfolio since a portfolio with  $\bar{n}_p$  patents will exclude the Follower (recall Lemma 1). But condition (Entry) implies a stronger statement: under entry, the magnitude of  $\bar{n}_p - n_p$  *cannot* be made arbitrarily small by adjusting the parameters  $s, f, \Delta$ , or  $\lambda$ , so IP portfolios will not smoothly transition between Entry and Exclusion.<sup>14</sup>:

**Corollary 1.** *For a given set of functions  $g(\cdot)$  and  $\ell(\cdot)$ , there exists  $\epsilon > 0$  such that  $\bar{n}_p - n_p > \epsilon$  for any Licensing or Mixed portfolio. In particular, the difference cannot be made arbitrarily small by varying the parameters  $s, f, \Delta$ , and  $\lambda$ .*

Proposition 1 describes the Leader's choice in the Protection stage, conditional on having successfully innovated. These choices will determine the expected profit from innovation and the incentives to do so in the Creation stage of the game. Industry characteristics will therefore shape market outcomes through two channels: by altering IP portfolio choice should innovation occur, and by changing the incentives to innovate in the first place. Since these two channels operate simultaneously, care must be taken when using one (IP) to measure the other (innovation). I explore the ramifications of this simultaneity below.

---

<sup>14</sup>This result follows from the clear threshold created by Lemma 1. If instead,  $\Delta$  and  $\lambda$  are sufficiently uncertain to the Leader, then  $\bar{n}_p$  would be as well, and the comparative statics would smooth out.

## 3.2 Measuring Innovation with Observed IP Usage

Proposition 1 describes how the fundamentals of an industry directly influence a firm’s intellectual property usage. However, these fundamentals also influence that firm’s incentives to innovate, and therefore IP usage will only be a useful proxy for innovative output if changing fundamentals move both IP and innovation in the same direction. In general, such a relationship does not hold: innovation and IP usage can move in opposite directions.

### 3.2.1 The Problem with Weighted Patent Counts

The Leader’s expected innovation output  $\phi(e^*)$  is not generally observable, so we resort to measurements based on the Leader’s patenting  $n_p$ . However,  $n_p$  is not generally observable either: we only observe IP usage when innovation is successful. The expected value of observed patenting is therefore equal to  $\phi(e^*) \cdot n_p$ , which I denote  $n_p^o$ .<sup>15</sup> If the goal is to capture the effect of some external factor  $x$  on innovation, using observed patenting as a proxy, then innovation and observed patenting must move in the same direction as  $x$  changes. In other words, we require that

$$\text{sign} \left( \frac{\partial \phi(e^*)}{\partial x} \right) = \text{sign} \left( \frac{\partial n_p^o}{\partial x} \right) \quad (4)$$

When does (4) hold? Consider how observed patenting changes with respect to some industry or policy fundamental  $x$ :

$$\begin{aligned} \frac{\partial n_p^o}{\partial x} &= \phi(e^*) \frac{\partial n_p}{\partial x} + \frac{\partial \phi(e^*)}{\partial x} n_p \\ \varepsilon_{n_p^o}^x &= \underbrace{\varepsilon_{n_p}^x}_{\text{Protection}} + \underbrace{\varepsilon_{\Pi_L^*}^x \left( |\phi''| \frac{\phi}{\phi'^2} \right)}_{\text{Innovation}} \end{aligned} \quad (5)$$

where  $\varepsilon_i^x$  is the elasticity of quantity  $i$  with respect to  $x$ . For observed patenting to accurately capture changes in innovation, it must be that the first term either coincides with or is sufficiently small relative to the second

---

<sup>15</sup>In the remainder of the paper, I use the term “observed patenting” to refer to the expected value of patenting, even though patenting will either equal zero or  $n_p$  for any given product ex post.

term: changes in ex-post patent protection cannot outweigh changes in the underlying incentives to innovate. In other words, the Innovation effect must either coincide with or dominate the Protection effect.

**Proposition 2.** *As some external factor  $x$  changes, observed patenting moves with innovative effort iff*

1.  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  have the same sign; or
2.  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  have opposite signs and  $\left(|\phi''| \frac{\phi}{\phi'^2}\right) < \left|\varepsilon_{\Pi_L^*}^x\right| / \left|\varepsilon_{n_p}^x\right|$ .

So for observed patenting to be a valid proxy for innovative effort, the exogenous change must either influence patenting and expected profits in the same direction, or the influence on expected profits must be large relative to the influence on patenting. Furthermore, if diminishing returns to innovative effort are severe (*i.e.*, if  $\phi(e)$  is highly concave), the profit effect must be even larger. Examples of each case are readily apparent: an increase in the cost of patenting (larger  $f$ ) discourages patenting ( $\varepsilon_{n_p}^f < 0$ ) and increases the cost of appropriating returns to innovation ( $\varepsilon_{\Pi_L^*}^f < 0$ ), so  $n_p^o$  is a valid proxy. However, an increase in the cost of keeping secrets (larger  $s$ ) encourages a substitution towards patenting ( $\varepsilon_{n_p}^s > 0$ ) and increases the cost of appropriating returns to innovation ( $\varepsilon_{\Pi_L^*}^s < 0$ ), so  $n_p^o$  is an invalid proxy here; after a change in nondisclosure agreement law, for example, increased patenting rates would actually indicate a decrease in innovation. Here, another measure for innovation is needed.

### 3.2.2 Determining a Valid Metric for Innovation

Define a monotonic, nonnegative function  $\mu(n_p)$  as a patent-based metric, some function of the total number of patents a firm obtains to protect its innovation; this function need not be increasing. As above, the observed value of this metric equals  $\phi(e^*) \cdot \mu(n_p)$ , which I denote  $\mu^o(n_p)$ . Then  $\mu^o(n_p)$  will be a valid measure of innovation when both move in the same direction in response to a change in some  $x$ :

**Definition.**  $\mu(n_p)$  is a *valid innovation metric for  $x$*  if

$$\text{sign}\left(\frac{\partial \phi(e^*)}{\partial x}\right) = \text{sign}\left(\frac{\partial \mu^o(n_p)}{\partial x}\right)$$

for a given value or range of values of an exogenous variable  $x$ .

So under what circumstances would a given  $\mu(n_p)$  be a valid metric? Again, consider how its observed value  $\mu^o(n_p)$  changes with  $x$ :

$$\begin{aligned} \frac{\partial \mu^o(n_p)}{\partial x} &= \phi(e^*) \frac{\partial \mu(n_p)}{\partial x} + \frac{\partial \phi(e^*)}{\partial x} \mu(n_p) \\ \varepsilon_{\mu^o}^x &= \underbrace{\varepsilon_{n_p}^x}_{\text{Protection}} \cdot \underbrace{\varepsilon_{\mu}^{n_p}}_{\text{Metric}} + \underbrace{\varepsilon_{\Pi_L^*}^x \left/ \left( |\phi''| \frac{\phi}{\phi'^2} \right) \right.}_{\text{Innovation}} \end{aligned} \quad (6)$$

The innovation effect is unchanged from (5), but now the protection effect is modified by how the metric changes with patenting behavior (the “metric effect”). Applying (6) to Proposition 2 gives conditions on  $\varepsilon_{\mu}^{n_p}$  for which  $\mu(n_p)$  is a valid innovation metric:

**Proposition 3.**  *$\mu(n_p)$  is a valid innovation metric for  $x$  if either:*

1.  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  have the same sign and  $\varepsilon_{\mu}^{n_p} \geq 0$ ; or
2.  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  have opposite signs and  $\varepsilon_{\mu}^{n_p} \leq 0$ .

In other words, if the protection effect does not coincide with the innovation effect, simply use a metric that reverses the protection effect. The innovation and protection effects are primitives of the model, but the metric effect is the choice of the econometrician. Thus if the signs of the innovation and protection effects are known, one can always use a metric whose derivative is of the appropriate sign.<sup>16</sup>

What are some possible metrics for the econometrician to consider? The simplest metric is unadjusted patent counts:  $\mu(n_p) = n_p$ , with  $\varepsilon_{\mu}^{n_p} = \varepsilon_{n_p}^{n_p} = 1$ . However, the metric in most common usage is quality-adjusted patent counts: since innovations vary in their quality and value, researchers usually try to weight patents by some measure of their quality, such as citations by future patents. Since  $g(n_p)$  measures the value of a set of patents to the Leader, and each patent in that set covers some innovative component of the product,  $g(n_p)$  can be taken as a quality-adjusted measure of the innovation embodied

---

<sup>16</sup>Given (6), it is tempting to say that a metric’s accuracy is maximized if  $\varepsilon_{\mu}^{n_p} = 0$ ; that is true in the context of this model, but not generally. The model captures extensive changes in innovative output, not intensive changes in innovative quality, so it may still be advisable to have a metric with  $|\varepsilon_{\mu}^{n_p}| > 0$ .

in a patent portfolio.<sup>17</sup> Thus we can represent quality-adjusted patent counts as a metric with  $\mu(n_p) = g(n_p)$ . Then  $\varepsilon_\mu^{n_p} = \varepsilon_{g(n_p)}^{n_p} = \frac{g'(n_p) \cdot n_p}{g(n_p)} \in [0, 1]$ . Since  $\varepsilon_{g(n_p)}^{n_p} \leq \varepsilon_{n_p}^{n_p}$ , quality-adjustment can mitigate the protection effect, but this may not be sufficient to make it a valid metric.

One other possible metric is average patent quality. Because patenting is costly, the Leader will choose to patent the most valuable components and will forgo lower-quality patents whose expected benefits do not outweigh the cost. Thus the quality of the average patent decreases as the Leader's patent propensity rises. Define average patent quality as  $\mu(n_p) = \frac{g(n_p)}{n_p} \equiv q(n_p)$ ; then  $\varepsilon_\mu^{n_p} = \varepsilon_q^{n_p} = \frac{g'(n_p)}{g(n_p)/n_p} - 1 \in [-1, 0]$ . Average patent quality is a valid innovation metric precisely when quality-adjusted patenting is not.

The framework proposed here requires knowledge of the signs of  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  for the context in question, and these values may not be readily apparent. Here, economic theory and intuition must serve as a guide. How does  $x$  impact the effectiveness or cost of a patent or its substitute (*e.g.*, trade secrecy)? Do we generally observe firms using strategies of Exclusion, in which IP is forcefully applied to eliminate entry, or do we see Entry strategies, in which IP is licensed or litigated and competition ensues? It is up to the researcher to determine which metric is likely to be appropriate for the context under study. Below, I provide guidelines to choose the proper innovation metric in a general research setting, but first I demonstrate the consequences of Propositions 2 and 3 in context, using competitive pressure  $\Delta$  as an example.

### 3.3 Measurement in Context: Competitive Pressure $\Delta$

To demonstrate the above results, consider the effects of competitive pressure  $\Delta$  on the incentives to innovate. Does the threat of competition foster innovation or stifle it? As noted in the introduction, there is no strong theoretical consensus; the question is an empirical one, and the answer relies on how innovation is measured in practice. In the context of this model, I will show that competitive pressure and innovative effort have a V-shaped relationship, but that the *measured* relationship will resemble an inverse-U when

<sup>17</sup>Strictly speaking,  $g(n_p)$  measures the share of the product's market value attributable to the patented components. Market value is not necessarily equivalent to technological value, but it is the closest proxy available in the model. A sizable body of work lends credence to this approach: see *e.g.* Hall et al. (2005) and Trajtenberg (1990).

using the most common patent-based measure of innovation. Using an alternative measure, average patent quality, accurately captures the V-shaped relationship from the model.

### 3.3.1 Competitive Pressure, IP Usage, and Innovation

First consider how competitive pressure affects the Leader's ex-post IP portfolio choice. Conditional on successful innovation, competitive pressure on the Leader is an incentive to protect that innovation. But that pressure also discourages Follower entry, since entry shrinks the potential surplus. Therefore, higher  $\Delta$  leads to more use of both kinds of IP until Exclusion becomes the more viable option, and further increases in  $\Delta$  simply make Exclusion feasible with a smaller portfolio.

**Proposition 4.** *Let  $\Delta_{Bd} = \Delta_{Bd}(\lambda) \in [0, 1]$ . Then:*

1. *For  $\Delta < \Delta_{Bd}$ , the Leader uses an Entry portfolio and  $n_p$  and  $n_s$  are increasing in  $\Delta$ ; and*
2. *For  $\Delta \geq \Delta_{Bd}$ , the Leader uses an Exclusion portfolio and  $n_p = \bar{n}_p$  is decreasing in  $\Delta$ .*

Next consider the effects of competitive pressure  $\Delta$  on the Leader's ex-ante expected profit  $\Pi_L^*$ , innovative effort  $e^*$ , and innovative output  $\phi(e^*)$ . With more competitive pressure  $\Delta$ , entry destroys more profit and innovation is less lucrative, but protecting successful innovations with IP becomes more important. Once Exclusion occurs, more competitive pressure makes that Exclusion portfolio cheaper, increasing profits and the incentives to innovate accordingly.

**Proposition 5.** *For  $\Delta < \Delta_{Bd}$ ,  $\Pi_L^*$ ,  $e^*$ , and  $\phi(e^*)$  are decreasing in  $\Delta$ . For  $\Delta \geq \Delta_{Bd}$ ,  $\Pi_L^*$ ,  $e^*$ , and  $\phi(e^*)$  are increasing in  $\Delta$ .*

Propositions 4 and 5 cleanly illustrate the fundamental intuitions of the model. Suppose  $\Delta < \Delta_{Bd}$ ; competitive forces in the Leader's industry are fairly weak. Here, an increase in competitive pressure (*i.e.*,  $\Delta$ ) increases the profits destroyed by Follower entry, which has two effects: first, it increases the marginal return to blocking that entry through secrecy  $n_s$ , and second, it increases the amount of lost-profits damages awarded in a successful patent infringement lawsuit, which increases the marginal value of patenting  $n_p$ .

Thus increased competitive pressure increases the Leader's usage of both forms of IP: secrecy to decrease the likelihood that the profit pie shrinks, and patenting to retain a larger share of that pie if secrecy fails. However, because IP protection is imperfect here (*i.e.*,  $\ell(n_s) > 0$  and  $g(n_p) < 1$ ), increased competitive pressure still decreases the Leader's net profits from an innovation. Thus the marginal returns to innovative effort decrease as well, and the Leader innovates less.

Now suppose  $\Delta > \Delta_{Bd}$ ; competitive pressure in the industry is relatively high. Now, the threat of competition is so large that the Leader is willing to pay  $\bar{n}_p f$  to perfectly guarantee a monopoly. At the same time, the Follower's market payoffs are shrinking as well, so his entry is more easily deterred. Further increases in competitive pressure reduce the Follower's incentives to enter, making deterrence cheaper and decreasing patenting  $\bar{n}_p$ . Since the Leader's payoffs equal monopoly profits minus the cost of deterrence  $\bar{n}_p f$ , and since that cost is shrinking, increased competitive pressure now increases the Leader's net profits from an innovation and therefore her innovative effort.

These forces highlight a key insight of the model: the strength and mechanics of IP protection are endogenous, a choice of the Leader. In this example, if  $\Delta < \Delta_{Bd}$  then IP protection is imperfect: secrecy does not guarantee exclusion, and patenting does not fully recoup the Leader's losses. If instead  $\Delta \geq \Delta_{Bd}$ , then IP protection is perfect: patenting fully protects the Leader from competitors for a fixed cost. Patent protection can function in two ways, and the Leader chooses between them: if  $\Delta < \Delta_{Bd}$ , then patents are extractive, and if  $\Delta \geq \Delta_{Bd}$ , patents are exclusionary. Thus in making her IP portfolio decision, the Leader is consciously choosing between imperfect and perfect protection, between extractive and exclusionary patents.<sup>18</sup>

Figure 4 depicts Propositions 4 and 5 together, graphing both innovation  $\phi(e^*)$  and ex-post patenting  $n_p$  as functions of  $\Delta$ . The curvature of  $\phi(\cdot)$  gives innovation a V-shape, while ex-post patenting rises, jumps up, and then falls. The discontinuity occurs at  $\Delta_{Bd}$ , the value of  $\Delta$  for which (Entry) is exactly satisfied.

---

<sup>18</sup>These effects outline other interesting implications of the model. For example, intensely competitive industries should comprise monopolized markets when firms can make use of IP. Direct lessons for our understanding of the innovation-competition relationship will be the subject of future work.

### 3.3.2 Competitive Pressure and Measured Innovation

Propositions 4 and 5 show that  $\varepsilon_{n_p}^\Delta \geq 0$  and  $\varepsilon_{\Pi_L^*}^\Delta \leq 0$  for  $\Delta < \Delta_{Bd}$  and that  $\varepsilon_{n_p}^\Delta \leq 0$  and  $\varepsilon_{\Pi_L^*}^\Delta \geq 0$  for  $\Delta \geq \Delta_{Bd}$ ; in other words,  $\varepsilon_{n_p}$  and  $\varepsilon_{\Pi_L^*}$  are of opposite sign for all values of  $\Delta$ . Proposition 3 thus implies that patent counts, quality-adjusted or otherwise, will not be valid metrics in general.

Proposition 6 below details the specific relationship between observed patenting  $n_p^o$  and competitive pressure  $\Delta$ . Here, an additional set of assumptions on  $g(n_p)$ ,  $\ell(n_s)$ , and  $\phi(e)$  will prove useful for clarity in exposition.

**Assumption 3.** *The curvatures of  $g(n_p)$ ,  $\ell(n_s)$ , and  $\phi(e)$  are increasing:  $g'''(n_p) < 0$ ,  $\ell'''(n_s) > 0$ , and  $\phi'''(e) < 0$ .*

The economic content of Assumption 3 is that the diminishing returns to patenting, secrecy, and innovative effort are nontrivial even at higher  $n_p$ ,  $n_s$ , or  $e$ . These assumptions are not required for other results in the paper; they only serve to simplify the results for  $\Delta$ . I will not impose Assumption 3 unless explicitly noted and will detail their implications when used.

Given Assumption 3, we can apply Proposition 2 to the  $x = \Delta$  case. The results here will depend on the magnitude of the innovation effect. In particular, the condition that

$$|\phi''| \frac{\phi}{\phi'^2} < \left| \frac{\varepsilon_{\Pi_L^*}^\Delta}{\varepsilon_{n_p}^\Delta} \right| \quad (7)$$

must hold if patent counts are to be a valid innovation metric for  $\Delta$ :

**Proposition 6.** *Let Assumption 3 hold. Assuming an Entry portfolio,*

1. *if (7) holds at  $\Delta = \Delta_{Bd}$ , then there is a  $\Delta_{En} < \Delta_{Bd}$  such that patenting  $n_p$  is an invalid metric for  $\Delta < \Delta_{En}$  and a valid metric for  $\Delta \in [\Delta_{En}, \Delta_{Bd})$ ;*
2. *if (7) does not hold at  $\Delta = \Delta_{Bd}$ , patenting  $n_p$  is an invalid metric for all  $\Delta < \Delta_{Bd}$ .*

*Assuming an Exclusion portfolio,*

3. *if (7) holds at  $\Delta = \Delta_{Bd}$ , then there is a  $\Delta_{Ex} > \Delta_{Bd}$  such that patenting  $n_p$  is a valid metric for  $\Delta \in [\Delta_{Bd}, \Delta_{Ex}]$  and an invalid metric for  $\Delta > \Delta_{Ex}$ ;*

4. if (7) does not hold at  $\Delta = \Delta_{Bd}$ , patenting  $n_p$  is an invalid metric for all  $\Delta \geq \Delta_{Bd}$ .

In words, observed patenting is only a valid innovation metric when  $\Delta$  is close to the Exclusion threshold, and even then only if innovative output is sufficiently responsive to additional effort (recall Proposition 2). Next, I show how quality-adjusted patent counts perform. The key condition on the innovation effect, the analogue of (7), becomes

$$|\phi''| \frac{\phi}{\phi'^2} < \left| \frac{\varepsilon_{\Pi_L^*}^{\Delta}}{\varepsilon_{n_p}^{\Delta} \varepsilon_{g(n_p)}^{n_p}} \right| \quad (8)$$

Note that (7) is a stronger condition than (8); quality-adjusted patent counts improve performance, but the fundamental flaw persists.

**Corollary 2.** *Quality-adjusted patent counts  $g(n_p)$  are a valid metric for more values of  $\Delta$  than unadjusted patent counts  $n_p$ .*

*Specifically, let Assumption 3 hold. Assuming an Entry portfolio,*

1. *if (8) holds at  $\Delta = \Delta_{Bd}$ , then there is a  $\Delta_{En}^{Adj} < \Delta_{Bd}$  such that quality-adjusted patenting  $g(n_p)$  is an invalid metric for  $\Delta < \Delta_{En}^{Adj}$  and a valid metric for  $\Delta \in [\Delta_{En}^{Adj}, \Delta_{Bd}]$ ;*
2. *if (8) does not hold at  $\Delta = \Delta_{Bd}$ , quality-adjusted patenting  $g(n_p)$  is an invalid metric for all  $\Delta < \Delta_{Bd}$ .*

*Assuming an Exclusion portfolio,*

3. *if (8) holds at  $\Delta = \Delta_{Bd}$ , then there is a  $\Delta_{Ex}^{Adj} > \Delta_{Bd}$  such that quality-adjusted patenting  $g(n_p)$  is a valid metric for  $\Delta \in [\Delta_{Bd}, \Delta_{Ex}^{Adj}]$  and an invalid metric for  $\Delta > \Delta_{Ex}^{Adj}$ ;*
4. *if (8) does not hold at  $\Delta = \Delta_{Bd}$ , quality-adjusted patenting  $g(n_p)$  is an invalid metric for all  $\Delta \geq \Delta_{Bd}$ .*

*Further,*

5. *if  $\Delta_{En}$  exists, then  $\Delta_{En}^{Adj}$  exists and  $\Delta_{En}^{Adj} < \Delta_{En}$ ; and*
6. *if  $\Delta_{Ex}$  exists, then  $\Delta_{Ex}^{Adj}$  exists and  $\Delta_{Ex}^{Adj} > \Delta_{En}$ .*

In words, quality-adjustment can expand the valid range, but it cannot overcome the basic problems inherent in Proposition 3. Proposition 6 and Corollary 2 are verbose, so in Figure 5 I demonstrate the performance of unadjusted and quality-adjusted patent counts as innovation metrics here. Competitive pressure has a V-shaped relationship with underlying innovation, but observed patent counts only move in the same direction as innovation for a small range of  $\Delta$  around  $\Delta_{Bd}$ .<sup>19</sup>

Assumption 3 ensures that  $\Delta_{En}$  and  $\Delta_{Ex}$  are unique if they exist. Without these assumptions,  $n_p^o$  could have multiple local maxima and minima. In such a case, patenting would be a valid measure of innovation only on certain segments of the function. Further, since Assumption 3 is a sufficient condition; the negation of these assumptions is necessary but is by no means sufficient to show that patenting is a valid measure of innovation for all values of  $\Delta$ .

Both unadjusted and quality-adjusted patent counts are metrics with  $\varepsilon_\mu^{n_p} > 0$ :  $\varepsilon_{n_p}^{n_p} = 1$  and  $\varepsilon_{g(n_p)}^{n_p} = \frac{g'(n_p)}{g(n_p)/n_p} \in [0, 1]$ . Proposition 3 predicts that a metric with  $\varepsilon_\mu^{n_p} < 0$ , such as average patent quality  $q(n_p) \equiv \frac{g(n_p)}{n_p}$  whose elasticity is  $\varepsilon_\mu^{q(n_p)} = \frac{g'(n_p)}{g(n_p)/n_p} - 1 \in [-1, 0]$ , will be valid here.

**Proposition 7.** *Average patent quality  $q(n_p) \equiv \frac{g(n_p)}{n_p}$  is a valid innovation metric for all  $\Delta$ .*

Figure 6 compares innovation and the three innovation metrics for various levels of  $\Delta$ . The practical effect of this metric is to reverse the protection effect from equation 6 exactly when it contradicts the innovation effect. Average patent quality  $q(n_p)$  has a V-shaped relationship with  $\Delta$ , similar to movements in  $\phi(e^*)$ . There is a discontinuity at  $\Delta_{Bd}$ , but it is systematically smaller than the discontinuity in the patent counts  $n_p$  and  $g(n_p)$ .

### 3.4 Measurement in Other Contexts

In Section 3.3, I focused specifically on competitive pressure  $\Delta$ , its effects on unobserved innovative effort, and how those differ from the effects on innovation as typically measured. I chose to highlight  $\Delta$  because the competition-innovation relationship is economically important and because it cleanly il-

---

<sup>19</sup>Here,  $\phi(e)$  is not too concave, so  $\Delta_{En}$  and  $\Delta_{Ex}$  exist, but this is not always the case, as Proposition 6 and Corollary 2 state.

illustrates when and how traditional innovation measures fail. However, traditional methods are still valid in certain situations; the proper choice of innovation measure must be informed by the economic context in which it will be used. If one can be reasonably certain of the signs of the innovation and protection effects in equation 6, then one can choose the appropriate innovation metric.

In fact, Proposition 3 implies an intuitive method for determining a valid metric for a particular context (*i.e.*, a given  $x$ ) under consideration. If one can reasonably answer two general questions about the context under study, one can choose an appropriate innovation metric for that context:

1. Does  $x$  systematically affect the cost–effectiveness of patenting relative to other protection methods (*e.g.*, trade secrecy)?
2. As  $x$  changes, does intellectual property serve to appropriate profit gains or defend against profit losses?

First, consider the relative cost–effectiveness of patenting:

$$\frac{\frac{\partial \Pi_L}{\partial n_p} / f}{\frac{\partial \Pi_L}{\partial n_s} / s} = \frac{\ell(n_s)g'(n_p)/f}{-\ell'(n_s)(1 - g(n_p))/s} = \frac{MP_P/f}{MP_S/s} \quad (9)$$

If (9) increases as  $x$  rises for a given  $n_p$  and  $n_s$ , then the Leader will substitute towards patenting and away from secrecy in equilibrium, implying  $n_p^*$  will increase. This change can be compared against the direct effect of  $x$  on the Leader’s profits  $\Pi_L^*$  to determine the appropriate innovation metric via Proposition 3.

Next, consider whether IP serves to appropriate profit gains or defend against profit losses. In other words, are IP and profits complements or substitutes?

In an Entry equilibrium, the Leader’s IP choices retain a share  $\rho$  of the market profits she would have lost without IP protection, where

$$\rho \equiv 1 - \ell(n_s^*)(1 - g(n_p^*)) \quad (10)$$

and her revenue (gross of IP costs) equals

$$V(\rho, x) = \pi_M - (1 - \rho)(1 - \lambda(1 - \Delta))\pi_M \quad (11)$$

If IP serves to appropriate profit gains, then  $\rho$  and  $x$  will be complementary for a profit-increasing  $x$ ; *i.e.*,  $V(\rho, x)$  will be supermodular if IP appropriates profit gains and  $x$  increases the Leader's profits. Similarly,  $V(\rho, x)$  will be submodular if IP defends against losses in profits. Proposition 8 formalizes these claims:

**Proposition 8.** *Assume an Entry equilibrium and that  $\frac{\partial \Pi_L^*}{\partial x} \geq 0$ ; *i.e.*, that  $x$  is profit-increasing. Then*

1. *If  $x$  increases (decreases) the relative cost-effectiveness of patenting  $\left(\frac{\partial \rho}{\partial n_p}/f\right) / \left(\frac{\partial \rho}{\partial n_s}/s\right)$ , then a metric with  $\varepsilon_\mu^{n_p} > (<)0$  is valid; and*
2. *If  $x$  does not change the relative cost effectiveness of patenting and  $V(\rho, x)$  is supermodular (submodular), then a metric with  $\varepsilon_\mu^{n_p} > (<)0$  is valid.*

*The inequalities are reversed if  $x$  is profit-decreasing; *i.e.*, if  $\frac{\partial \Pi_L^*}{\partial x} \leq 0$ .*

Proposition 8 provides a framework for choosing a valid innovation metric for  $x$ . Suppose  $x$  is profit-increasing, and that we have two available metrics: quality-adjusted patent counts, which has a positive metric effect, and average patent quality, which has a negative metric effect. The intuitive content of Proposition 8 is given by the following questions for any exogenous variable  $x$ :

1. How does a profit-increasing  $x$  affect the relative cost-effectiveness of patenting?
  - Increases: use quality-adjusted patent counts.
  - Decreases: use average patent quality.
2. What role for IP does  $x$  emphasize?
  - Appropriation of profits: use quality-adjusted patent counts.
  - Defense against profit losses: use average patent quality.

These points are straightforward implications of Proposition 8. Since  $x$  is assumed to be profit-increasing, a positive-effect metric is appropriate when  $x$  increases patenting and vice-versa; if  $x$  is profit-decreasing, simply invert the metric recommendations.. The last two points follow from the intuition

behind supermodularity: if  $x$  increases profits and elicits an increase in IP usage, the IP is by definition used to more effectively appropriate those increased profits; and if  $x$  elicits a decrease in IP usage, it is because the need for IP to defend against losses is no longer as compelling.<sup>20</sup>

This framework still requires a prior understanding or beliefs about the setting in question. One must rely on existing evidence and economic intuition to choose the best innovation metric for their setting, and this choice is tantamount to a modeling assumption in one’s work. Because the firm’s IP strategy is a question of incentives, *any* use of a patent-based measure of innovation represents a fundamental modeling assumption about the nature and magnitudes of those incentives.

## 4 Conclusions

Innovators must solve a two-faceted problem. How does one innovate? How should one protect that innovation from imitators and competitors? External forces and incentives shape the innovator’s answers to these two questions in related but distinct ways. Traditionally, however, these questions are conflated: the existence of an IP protection automatically implies an innovation exists, so changes in IP usage must correlate with changes in innovative output. In the extreme, patents are treated as a textual waste product of the innovation production process.

But patenting is a strategic decision, influenced by the same contexts and forces that shape the decision to innovate. The model presented here distinguishes these two decisions and clarifies their relationship. Increased incentives to innovate will necessarily increase IP usage as well (the “innovation effect”), but this increase could be mitigated or offset completely if the post-innovation need for IP protection decreases (the “protection effect”). These two effects characterize the relationships between innovation, IP usage, and external forces such as industry competitiveness or economic policy.

---

<sup>20</sup>Proposition 8 assumes Entry; but the same intuitive framework continues to apply. Here, the Leader acquires perfect protection at cost  $\bar{\pi}_p f$  and revenue equals  $\pi_M$ . Any increase in patenting  $\bar{\pi}_p$  is tantamount to a decrease in its cost-effectiveness: more patenting is required to achieve the same level of protection. Thus if  $x$  affects patenting, refer to 1b. If  $x$  does not affect patenting, the protection effect of (6) equals zero and so any increase in profits immediately accrues to the Leader: 2a is a perfectly suitable decision rule.

The model highlights a concern with patent-based measures of innovation: patenting rates can only be a good proxy for innovation if the protection effect is small or coincides with the innovation effect. This caveat is particularly apparent when studying how innovative effort and IP usage relate to the threat of competition in an industry. Here, observed patenting will only correlate with innovation if competitive pressure is moderate and the returns to R&D do not diminish significantly at high levels of investment.

Fortunately, the model also suggests an alternative patent statistic that correlates better with innovation exactly when more conventional measures fail. Since a firm's marginal patent will be of lower-than-average quality, their average patent quality correlates with their innovative effort when the protection and innovation effects counteract each other. If the researcher has a strong reason to believe the innovation and protection effects counteract in the setting under study, they should consider using a product's average patent quality as a measure of innovative activity. The paper provides a general framework to guide the researcher's choice in a variety of empirical contexts.

Researchers and policymakers should be cautious when using patent statistics as a measure of innovation. One should be sure that the incentives to protect profits align with the incentive to innovate in the setting under study, and one should also consider the suitability of alternative measures such as average patent quality. When the nuances of the innovation-IP relationship are properly acknowledged, we can be more confident in patent-based measurements of innovation and the conclusions reached by their use.

## A Foundations of $g(n_p)$ and $\ell(n_s)$

In this section I derive conditions on the characteristics of a products components that guarantee the existence of the well-behaved  $g(n_p)$  and  $\ell(n_s)$  functions used in the body of the paper.

The Leader's product still consists of a measure  $N$  of components, but now suppose that a lawsuit concerning component  $i$  is won by the Leader with probability  $p_i$  and awards a share  $\alpha_i$  of lost profits due to entry.<sup>21</sup> The expected compensation from a lawsuit is thus  $p_i\alpha_i \equiv g_i$  times lost profits, and the expected litigation value of a portfolio of patents  $\mathcal{P}$  is therefore  $\int_{i \in \mathcal{P}} g_i di$  multiplied by lost profits.

Also suppose that components differ in their secrecy potential  $\ell_i$ , which are additive: the probability that the Follower fails to learn all of the secrets in a secrecy portfolio  $\mathcal{S}$  equals  $\int_{i \in \mathcal{S}} \ell_i di \leq 1$ .<sup>22</sup>

In principle, the values of  $g_i$  and  $\ell_i$  need not have any particular relationship for a given component  $i$ . But to obtain useful  $g(n_p)$  and  $\ell(n_s)$ , the following assumption on their relationship is needed.

**Assumption 4.** *There exists an ordering over  $i$  such that  $g_i$  is weakly decreasing,  $\ell_i$  is weakly increasing, and both are continuous and differentiable over  $[0, N]$ .*

This assumption implies that components with high patent potential necessarily have low secrecy potential and vice-versa. Assumption 4 reflects the idea that less complicated components are both easier to reverse-engineer (low  $\ell_i$ ) and to demonstrate equivalence to a component in a competitor's product (high  $g_i$ ).

Imposing Assumption 4 reduces the dimensionality of the Leader's portfolio choice problem considerably:

**Lemma 3.** *If Assumption 4 holds, then  $\int_{i \in \mathcal{P}} g_i di = \int_{i=0}^{|\mathcal{P}|} g_i di$  and  $\int_{i \in \mathcal{S}} \ell_i di = \int_{i=N-|\mathcal{S}|}^N \ell_i di$  for any optimally-chosen portfolios  $\mathcal{P}$  and  $\mathcal{S}$ : the protection afforded by an optimal portfolio is fully described by its size.*

<sup>21</sup>Thus even if  $\alpha_i = 1$  for all  $i$  (patents are strong), there will be dispersion in  $g_i$  from  $\alpha_i$ .

<sup>22</sup>It is tempting to instead define  $\ell_i$  as the independent probability that the Follower learns component  $i$ . Then the probability of learning all secrets would be the product of all the  $\ell_i$ , which would be zero for any secrecy portfolio of nonzero measure unless one resorts to product integrals or other exotic beasts. In any case, the temptation is not likely to lead to a tractable model.

Without Assumption 4, changing the number of secret components could change the cost of a given level of total patent protection  $\int_{i \in \mathcal{P}} g_i \, di$ : if the patent potential of a secret component is higher than that of any patented components, reducing the number of secrets will allow the Leader to achieve the same amount of aggregate litigation value with fewer total patents (a violation of the “component independence” assumption discussed in Section 1). With this assumption, however, aggregate patent potential and secret potential can be represented by separate functions. I therefore define

$$g(n_p) \equiv \int_{i=0}^{n_p} g_i \, di \text{ and } \ell(n_s) = 1 - \int_{i=N-n_s}^N \ell_i \, di$$

These definitions of  $g(n_p)$  and  $\ell(n_s)$  satisfy the assumptions made in the body of the paper:

**Corollary 3.** *If Assumption 4 holds, then:*

1.  $g(n_p)$  and  $\ell(n_s)$  satisfy Assumption 1;
2.  $g(n_p)$  is continuous, twice differentiable, increasing, and weakly concave, with  $g(0) = 0$ ; and
3.  $\ell(n_s)$  is continuous, twice differentiable, decreasing, and weakly convex, with  $\ell(0) = 1$ .

The assumptions on  $g_i$  and  $\ell_i$  imply that  $g(n_p)$  and  $\ell(n_s)$  are continuous and twice differentiable. It also follows that  $g(n_p)$  is weakly concave with  $g(0) = 0$  and  $\ell(n_s)$  is weakly convex with  $\ell(0) = 1$ . Note, however, that the assumptions of  $g(N) < 1$  and sufficient concavity of  $g(n_p)$  and  $\ell(n_s)$ , which were made in Section 1, are independent of the results of this appendix; they represent additional conditions under which the main results of the paper hold.

## B Proofs

*Proof of Lemma 1.* Given in the body of the paper. □

*Proof of Lemma 2.* Since  $g(N) < 1$ , it is necessary that

$$\frac{(1-\lambda)(1-\Delta)}{1-\lambda(1-\Delta)} < 1 \quad \rightarrow \quad 0 < \Delta$$

□

*Proof of Proposition 1.* I first suppose (Entry) is satisfied so that Exclusion does not occur and determine when the Leader chooses a Mixed, Licensing, Concealment, or Empty portfolio. Then I compare each of those outcomes to Exclusion to see when the Leader chooses it instead of allowing entry.

Consider a Mixed portfolio and increase  $s + f$ , holding  $\frac{s}{f}$  fixed. Doing so decreases  $n_p$  and  $n_s$ . If  $n_p$  decreases to zero before  $n_s$  does, then (Ratio) becomes  $\frac{s}{f} = \frac{-\ell'(n_s)}{\ell(n_s)} \frac{1}{g'(0)}$ . If  $n_s$  goes to zero first, then (Ratio) becomes  $\frac{s}{f} = \frac{-\ell'(0)}{1} \frac{1-g(n_p)}{g'(n_p)}$ . Since  $\frac{-\ell'(n_s)}{\ell(n_s)} \frac{1}{g'(0)} \leq \frac{-\ell'(0)}{g'(0)} \leq -\ell'(0) \frac{1-g(n_p)}{g'(n_p)}$  by the sufficient concavity assumptions, if the Leader only uses one method of protection, it will be secrecy if  $\frac{s}{f} < \frac{-\ell'(0)}{g'(0)}$  and patenting if  $\frac{s}{f} > \frac{-\ell'(0)}{g'(0)}$ . For the remainder of the proof, assume  $\frac{s}{f} > \frac{-\ell'(0)}{g'(0)}$ . Identical arguments will hold for the other case.

Suppose the Leader is indifferent between Empty and Licensing portfolios; that is, the first-order necessary condition of her maximization problem for  $n_p$  holds with equality at  $n_p = 0$ , and that of  $n_s$  is slack. Then  $g'(0)(1 - \lambda(1 - \Delta))\pi_M = f$ , and she would prefer Empty if  $f$  increased slightly. Rearranging yields  $\lambda > \left(1 - \frac{f}{g'(0)\pi_M}\right) \frac{1}{1-\Delta}$  as the condition for when Empty is preferred over Licensing. Then  $\lambda_2(\Delta) \equiv \left(1 - \frac{f}{g'(0)\pi_M}\right) \frac{1}{1-\Delta}$ , which is increasing in  $\Delta$ .

Now suppose the Leader is indifferent between Licensing and Mixed: both first-order conditions hold exactly and the one for  $n_s$  holds at  $n_s = 0$ . Increasing  $s$  slightly would give  $-\ell'(0)(1 - g(n_p))(1 - \lambda(1 - \Delta))\pi_M < s$ , which when combined with the first-order condition for  $n_p$  and rearranged gives  $\lambda > \left(1 - \frac{s+f}{\pi_M(g'(n_p) - \ell'(0)(1-g(n_p)))}\right) \frac{1}{1-\Delta}$  as the condition for when Licensing is preferred to Mixed. Then  $\lambda_1(\Delta) \equiv \left(1 - \frac{s+f}{\pi_M(g'(n_p) - \ell'(0)(1-g(n_p)))}\right) \frac{1}{1-\Delta}$ , which is increasing in  $\Delta$ .

Now it remains to consider when Exclusion is preferred to each of Empty, Licensing, and Mixed. This occurs precisely when (Entry) is satisfied. which can be expressed as

$$\bar{n}_p \geq \frac{1 - g(n_p)}{g'(n_p)} + n_p + n_s \frac{-\ell'(n_s)}{\ell(n_s)} \frac{1 - g(n_p)}{g'(n_p)}. \quad (12)$$

To compare Empty to Exclusion, (12) becomes  $\bar{n}_p \geq \frac{1}{g'(0)}$ . Substituting  $\bar{n}_p = g^{-1}\left(1 - \frac{\Delta}{1-\lambda(1-\Delta)}\right)$  (from the definition of  $\bar{n}_p$ ) yields  $\lambda \leq \frac{1-g\left(\frac{1}{g'(0)}\right) \frac{1}{1-\Delta}}{1-g\left(\frac{1}{g'(0)}\right)}$ ,

which defines the portion of  $\Delta_{Bd}(\lambda)$  directly below which Empty is the preferred portfolio under entry.

To compare Licensing to Exclusion, (12) becomes  $\bar{n}_p \geq \frac{1-g(n_p)}{g'(n_p)+n_p}$ . Using the same substitutions and rearrangements as the previous case, (12) becomes  $\lambda \leq \frac{1-g\left(\frac{1-g(n_p)}{g'(n_p)+n_p}\right)\frac{1}{1-\Delta}}{1-g\left(\frac{1-g(n_p)}{g'(n_p)+n_p}\right)}$ , which defines the portion of  $\Delta_{Bd}(\lambda)$  directly below which Licensing is the preferred portfolio under entry.

Finally, to compare Mixed to Exclusion, (12) becomes  $\bar{n}_p \geq \frac{1-g(n_p)}{g'(n_p)+n_p+n_s\frac{s}{f}}$ . Using the same substitutions and rearrangements as the previous cases, (12) becomes  $\lambda \leq \frac{1-g\left(\frac{1-g(n_p)}{g'(n_p)+n_p+n_s\frac{s}{f}}\right)\frac{1}{1-\Delta}}{1-g\left(\frac{1-g(n_p)}{g'(n_p)+n_p+n_s\frac{s}{f}}\right)}$ , which defines the portion of  $\Delta_{Bd}(\lambda)$  directly below which Licensing is the preferred portfolio under entry.

Note that all of the segments of  $\Delta_{Bd}(\lambda)$  derived above are decreasing functions of  $\lambda$ , and join at the Empty/Licensing and Licensing/Mixed boundaries to form a decreasing, continuous, piecewise function  $\Delta_{Bd}(\lambda)$ . The remaining claims are immediate.  $\square$

*Proof of Corollary 1.* This is seen directly from (12). One can choose parameters  $(s, f, \Delta, \lambda)$  such that (12) binds and  $n_s = 0$ . Varying these parameters to increase  $n_p$  will further decrease the difference.

Suppose, by way of contradiction, there exists a sequence of tuples  $(s_i, f_i, \Delta_i, \lambda_i)$  that elicit Entry portfolios such that  $\bar{n}_p - n_p(s_i, f_i, \Delta_i, \lambda_i) \rightarrow 0$ . Then (12) approaches  $0 \geq \frac{1-g(\bar{n}_p)}{g'(\bar{n}_p)}$ . But this is positive, so there must be some  $i$  in the sequence for which (12) did not hold.

However, note that if  $\Delta \geq 0$  and  $g(N) > 1$ , this outcome is possible since  $\frac{1-g(\bar{n}_p)}{g'(\bar{n}_p)}$  may take negative values.  $\square$

*Proof of Proposition 2.* Begin with equation (5) and note that  $\left(|\phi''|\frac{\phi}{\phi'^2}\right)$  is positive by definition.

To prove necessity, suppose observed patenting moves with innovative effort:  $\text{sign}(\varepsilon_{n_p^o}^x) = \text{sign}(\varepsilon_{\phi(e^*)}^x)$ . The signs of  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  must either agree or disagree. The only case which could invalidate the claim that patenting and innovative effort move together is if they disagree. Assume without loss

of generality that  $\varepsilon_{\Pi_L^*}^x > 0$ . Then the claim is true when  $\varepsilon_{n_p^o}^x > 0$ ; *i.e.*, when

$$\begin{aligned} \varepsilon_{\Pi_L^*}^x / \left( |\phi''| \frac{\phi}{\phi'^2} \right) &> |\varepsilon_{n_p}^x| \\ \varepsilon_{\Pi_L^*}^x / |\varepsilon_{n_p}^x| &> \left( |\phi''| \frac{\phi}{\phi'^2} \right) \end{aligned}$$

as claimed. The case where  $\varepsilon_{\Pi_L^*}^x < 0$  yields

$$|\varepsilon_{\Pi_L^*}^x| / \varepsilon_{n_p}^x > \left( |\phi''| \frac{\phi}{\phi'^2} \right)$$

so that the cases together provide the claim.

A proof of sufficiency is immediate from equation (5).  $\square$

*Proof of Proposition 3.* The proposition claims sufficiency, so the result is immediate from equation (6). A “stronger” version of the proposition claiming necessity would add a third case in which  $\varepsilon_{n_p}^x$  and  $\varepsilon_{\Pi_L^*}^x$  have opposite signs,  $\varepsilon_{\mu}^{n_p} \geq 0$ , and  $\left( |\phi''| \frac{\phi}{\phi'^2} \right)$  is small. But since  $\varepsilon_{\mu}^{n_p}$  can be freely chosen, the proposition is more relevant as written.  $\square$

*Proof of Proposition 4.* The Leader’s first-order necessary conditions for maximization of  $\Pi_L$  are given by

$$\ell(n_s)g'(n_p)(1 - \lambda(1 - \Delta))\pi_M - f \leq 0 \quad (13)$$

$$-\ell'(n_s)(1 - g(n_p))(1 - \lambda(1 - \Delta))\pi_M - s \leq 0 \quad (14)$$

which I will assume hold with equality and lead to a Mixed portfolio; comparative statics for an Empty portfolio are trivial and those for Licensing or Concealment are simplified versions of what is to follow. The proposition makes claims about  $\frac{\partial n_p}{\partial \Delta}$ , among others. To proceed, totally differentiate both (13) and (14) with respect to  $n_p$ ,  $n_s$ , and  $\Delta$ . Stacking as a matrix equation and rearranging leads to

$$\begin{bmatrix} dn_p \\ dn_s \end{bmatrix} = \frac{1}{-g''(1 - g)\ell''\ell - g'^2\ell'^2} \cdot \begin{bmatrix} g'(1 - g)(\ell''\ell - \ell'^2) \\ -\ell'\ell(|g|''(1 - g) - g'^2) \end{bmatrix} \cdot \frac{\lambda}{1 - \lambda(1 - \Delta)} \cdot d\Delta \quad (15)$$

Note that I have suppressed the  $n_p$  and  $n_s$  in parentheticals for clarity. Dividing by  $d\Delta$  yields expressions for  $\frac{\partial n_p}{\partial \Delta}$  and  $\frac{\partial n_s}{\partial \Delta}$ , both of which are positive

thanks to the sufficient concavity assumptions made (note that these assumptions also imply the entire maximization problem is concave; the first denominator above will be positive, which is equivalent to the second-order condition that the determinant of the Hessian be positive).

For the Exclusion claim of the Proposition, note that the Leader will use exactly  $\bar{n}_p$  patents and zero secrets, so the quantity of interest is  $\frac{\partial \bar{n}_p}{\partial \Delta}$ . Differentiating the definition of  $\bar{n}_p$  leads to  $\frac{\partial \bar{n}_p}{\partial \Delta} = \frac{g(\bar{n}_p)}{g'(\bar{n}_p)} \frac{-1}{(1-\Delta)(1-\lambda(1-\Delta))}$ , which is negative as claimed.

Finally, Proposition 1 shows that for a given  $\lambda$ , the Leader will transition between Entry and Exclusion portfolios at  $\Delta_{Bd}(\lambda)$ , which was shown to be in  $[0, 1]$ .  $\square$

*Proof of Proposition 5.* Assume Entry. First,  $\frac{\partial \Pi_L^*}{\partial \Delta} = -\ell(1-g)\lambda\pi_M$  by the Envelope Theorem; this is negative. Next, recall that  $\Pi_L^*$ ,  $e^*$ , and  $\phi(e^*)$  are related through (3). The chain rule implies  $\frac{\partial e^*}{\partial \Delta} = \frac{\phi'^2}{-\phi'' \cdot c^2} \cdot \frac{\partial \Pi_L^*}{\partial \Delta}$ , where the  $e^*$  in parenteticals have been suppressed for clarity. Since  $\phi'' < 0$ ,  $\frac{\partial e^*}{\partial \Delta}$  is negative as well. Since  $\phi$  is monotone, then,  $\frac{\partial \phi(e^*)}{\partial \Delta}$  is also negative.

Now assume Exclusion. Now  $\Pi_L^* = \pi_M - \bar{n}_p f$ . Since  $\frac{\partial \bar{n}_p}{\partial \Delta}$  is negative (Proposition 4), all of  $\Pi_L^*$ ,  $e^*$ , and  $\phi(e^*)$  are increasing in  $\Delta$  as claimed.  $\square$

*Proof of Proposition 6.* First assume an Entry portfolio. Then equation 5 becomes

$$\frac{\partial n_p^o}{\partial \Delta} = \frac{\partial \phi}{\partial \Delta} n_p + \phi \frac{\partial n_p}{\partial \Delta} = \frac{\phi'^2}{|\phi''|} \frac{1}{\Pi_L^*} \frac{\partial \Pi_L^*}{\partial \Delta} n_p + \phi \frac{\partial n_p}{\partial \Delta} \quad (16)$$

Therefore,  $\frac{\partial n_p^o}{\partial \Delta} \geq 0$  iff

$$n_p \leq \underbrace{\left( |\phi''| \frac{\phi}{\phi'^2} \right)}_A \cdot \underbrace{\left( \frac{\ell'' \ell - \ell'^2}{\frac{|g''|}{g'} \ell'' \ell - \frac{g'}{1-g} \ell'^2} \right)}_B \cdot \underbrace{\left( \frac{\Pi_L^*}{\ell(1-g)(1-\lambda(1-\Delta))\pi_M} \right)}_C \quad (17)$$

Since  $n_p$  is increasing in  $\Delta$ , if the right-hand side is collectively decreasing in  $\Delta$ , then there will be some unique  $\Delta_{En}$  such that  $n_p^o$  is increasing for  $\Delta < \Delta_{En}$  and decreasing for  $\Delta > \Delta_{En}$ . By equation (5), this will be exactly when  $|\phi''| \frac{\phi}{\phi'^2} = \left| \varepsilon_{\Pi_L^*}^{\Delta} / \varepsilon_{n_p}^{\Delta} \right|$ , which implies the two claims of the proof under Entry: if  $\Delta_{En} < \Delta_{Bd}$ , the first claim holds, and if  $\Delta_{En} \geq \Delta_{Bd}$ , the second claim is true.

I will proceed by showing that  $A$ ,  $B$ , and  $C$  are each decreasing in  $\Delta$ , which is a sufficient condition for the proof.

First consider  $A = |\phi''| \frac{\phi}{\phi'^2}$ . Noting that  $\frac{\partial A}{\partial \Delta} = \frac{\partial A}{\partial e} \frac{\partial e}{\partial \Delta}$ , it will suffice to prove that  $\frac{\partial A}{\partial e}$  is positive since  $\frac{\partial e}{\partial \Delta} < 0$ .

$$\begin{aligned} \frac{\partial A}{\partial e} &= -\phi''' \frac{\phi}{\phi'^2} + \frac{2|\phi''|^2 \phi}{\phi'^3} + \frac{|\phi''|}{\phi'} > 0 \\ \phi''' &< \frac{2|\phi''|^2 \phi + |\phi''| \phi'^2}{\phi' \phi} > 0 \end{aligned}$$

which is true under the assumption that  $\phi''' < 0$ .

Now consider  $B = \frac{\ell'' \ell - \ell'^2}{\frac{|g''|}{g'} \ell' \ell - \frac{g'}{1-g} \ell'^2}$ . Note that the assumptions of  $g'''(n_p) < 0$  and  $\ell'''(n_s) > 0$  imply respectively that  $\ell'' \ell - \ell'^2$  and  $|g''|(1-g) - g'^2$  are increasing:

$$\begin{aligned} \frac{\partial}{\partial \Delta} [\ell'' \ell - \ell'^2] &= \ell''' \ell - \ell'' \ell' > 0 \rightarrow \ell''' > \frac{\ell'' \ell'}{\ell} < 0 \\ \frac{\partial}{\partial \Delta} [|g''|(1-g) - g'^2] &= -g'''(1-g) + |g''|g' > 0 \rightarrow g''' < \frac{|g''|g'}{1-g} > 0 \end{aligned}$$

Therefore the numerator of  $B$  is increasing but the denominator is increasing more quickly;  $B$  is decreasing with respect to  $\Delta$ .

Finally consider  $C = \frac{\Pi_L^*}{\ell(1-g)(1-\lambda(1-\Delta))\pi_M} = \frac{\Pi_L^*}{\pi_M - n_p f - n_s s - \Pi_L^*}$ . The numerator is decreasing in  $\Delta$  by Proposition 5. Then  $C$  will be decreasing in  $\Delta$  if the denominator is increasing in  $\Delta$ :

$$-f \frac{\partial n_p}{\partial \Delta} - s \frac{\partial n_s}{\partial \Delta} - \frac{\partial \Pi_L^*}{\partial \Delta} > 0 \quad (18)$$

$$-\frac{\partial \Pi_L^*}{\partial \Delta} > f \frac{\partial n_p}{\partial \Delta} + s \frac{\partial n_s}{\partial \Delta} \quad (19)$$

Assume without loss of generality that  $(n_p, n_s)$  are interior (if not, their derivative with respect to  $\Delta$  is zero anyways) so that the first-order conditions (13) and (14) hold with equality. Substituting these for  $s$  and  $f$ , and computing the derivatives, leads to

$$|g''|(1-g)\ell''\ell - g'^2\ell'^2 > g'^2(\ell''\ell - \ell'^2) + \ell'^2(|g''|(1-g) - g'^2) \quad (20)$$

$$|g''|(1-g)\ell''\ell - g'^2\ell'^2 > \left[ \frac{g'^2}{|g''|(1-g)} + \frac{\ell'^2}{\ell''\ell} \right] \cdot |g''|(1-g)\ell''\ell - 2 \cdot g'^2\ell'^2 \quad (21)$$

which is true by the sufficient concavity assumptions: each term in square brackets is less than 1, and it can be shown that  $1+1=2$ , so the inequality holds. Thus  $C$  is decreasing in  $\Delta$ , and therefore the two claims are proven for Entry.

Now consider an Exclusion portfolio. Equation 5 implies that  $\frac{\partial n_p^o}{\partial \Delta} \geq 0$  iff

$$\bar{n}_p \geq \left( |\phi''| \frac{\phi}{\phi'^2} \right) \frac{\Pi_L^*}{f} \quad (22)$$

since now  $\frac{\partial \Pi_L}{\partial \Delta} = -f \frac{\partial \bar{n}_p}{\partial \Delta}$ . Since  $\bar{n}_p$  is decreasing in  $\Delta$ , and the right-hand side of (22) is increasing in  $\Delta$  (the first term by assumption of  $\phi''' < 0$ , the second by Proposition 4), there exists a  $\Delta_{Ex}$  where (22) holds with equality. If  $\Delta_{Ex} > \Delta_{Bd}$ , the first claim holds; if  $\Delta_{Ex} < \Delta_{Bd}$ , the second claim holds.  $\square$

*Proof of Corollary 2.* The results are a straightforward application of the metric effect. Equation 6 becomes

$$\frac{\partial g(n_p)^o}{\partial \Delta} = \frac{\partial \phi}{\partial \Delta} g(n_p) + \phi \frac{\partial n_p}{\partial \Delta} = \frac{\phi'^2}{|\phi''|} \frac{1}{\Pi_L^*} \frac{\partial \Pi_L^*}{\partial \Delta} g(n_p) + \phi \frac{\partial g(n_p)}{\partial \Delta} \quad (23)$$

so that  $\frac{\partial g(n_p)^o}{\partial \Delta} \geq 0$  iff

$$\frac{g(n_p)}{g'(n_p)} \leq \underbrace{\left( |\phi''| \frac{\phi}{\phi'^2} \right)}_A \cdot \underbrace{\left( \frac{\ell'' \ell - \ell'^2}{\frac{|g''|}{g'} \ell'' \ell - \frac{g'}{1-g} \ell'^2} \right)}_B \cdot \underbrace{\left( \frac{\Pi_L^*}{\ell(1-g)(1-\lambda(1-\Delta))\pi_M} \right)}_C \quad (24)$$

The left-hand side is increasing in  $\Delta$ , and the proof that the right-hand side is decreasing is identical to the proof for Proposition 5. Further, the inequality will change sign at a lower  $\Delta$  if

$$\frac{g(n_p)}{g'(n_p)} > n_p \rightarrow \frac{g(n_p)}{n_p} > g'(n_p) \quad (25)$$

*i.e.*, if the average is greater than the marginal, which is always true since  $g(\cdot)$  is concave, increasing, and starts at zero. Thus assuming Entry,  $\Delta_{En}^{Adj} < \Delta_{En}$ .

The proof for the Exclusion case follows in a similar fashion.  $\square$

*Proof of Proposition 7.* The result follows immediately from Propositions 3, 4, and 5, since  $\varepsilon_{n_p}^\Delta$  and  $\varepsilon_{\Pi_L^*}^\Delta$  have opposite signs for all  $\Delta$  and  $\varepsilon_\mu^{q(n_p)} = \frac{g'(n_p)}{g(n_p)/n_p} - 1 \in [-1, 0]$ . The metric effectively negates the protection effect from equation (6).  $\square$

*Proof of Proposition 8.* Without loss of generality, I will assume that  $\frac{\partial \Pi_L^*}{\partial x} \geq 0$  (i.e., that  $x$  is profit-increasing). The proof for  $\frac{\partial \Pi_L^*}{\partial x} \leq 0$  is identical in execution.

First, assume an Entry portfolio and consider the relative cost-effectiveness of patenting

$$\frac{\frac{\partial \rho}{\partial n_p} / f}{\frac{\partial \rho}{\partial n_s} / s} = \frac{\ell(n_s)g'(n_p)/f}{-\ell'(n_s)(1 - g(n_p))/s} \quad (26)$$

An increase (decrease) in this quantity violates (Ratio), and in equilibrium  $n_p$  will rise and  $n_s$  will fall. Thus  $\varepsilon_{n_p}^x > 0$  and a metric with  $\varepsilon_{\mu}^{n_p} > (<) 0$  will be valid, as claimed.

If cost-effectiveness is unchanged, then (Ratio) is unchanged and thus any effect of  $x$  must affect  $n_p^*$  and  $n_s^*$  in the same direction; otherwise  $\frac{MP_p}{MP_s}$  would not be unchanged. Since both  $n_p$  and  $n_s$  move  $\rho = 1 - \ell(n_s^*)(1 - g(n_p^*))$  in the same direction, then, we only need to analyze the comparative statics of  $x$  on  $\rho$  to understand the effects of  $x$  on  $n_p$ .

Since  $\frac{\partial^2 V(\rho, x)}{\partial \rho x} = \frac{\partial^2 \Pi_L^*}{\partial \rho x}$ , applying Topkis's Theorem to  $V(\rho, x)$  thus immediately implies the claimed result.  $\square$

*Proof of Lemma 3.* Consider an optimally-chosen  $\mathcal{P}$  and  $\mathcal{S}$ .  $\int_{i \in \mathcal{P}} g_i \, di = \int_{i=0}^{|\mathcal{P}|} g_i \, di$  if the first  $|\mathcal{P}|$  components are patented. We proceed by contradiction: suppose instead that there is at least one component  $j < |\mathcal{P}|$  that is not patented, and is either therefore secret or disclosed. Let  $k$  be the index of the component with the lowest  $g_i$  in  $\mathcal{P}$ . If component  $j$  is disclosed, the Leader can patent it and disclose component  $k$ . This leaves her cost unchanged but increases  $\int_{i \in \mathcal{P}} g_i \, di$ , since  $g_i$  is non-increasing and  $j < k$  by definition. If component  $j$  is secret, the Leader can patent it and make component  $k$  secret. This again leaves her cost unchanged but increases both  $\int_{i \in \mathcal{P}} g_i \, di$  and  $\int_{i \in \mathcal{S}} \ell_i \, di$  since  $g_j > g_k$  and  $\ell_j < \ell_k$  (since  $j < k$ ). In either case, the supposition is contradicted since the portfolio is demonstrably sub-optimal, and the claim follows.

The proof for  $\int_{i \in \mathcal{S}} \ell_i \, di = \int_{i=N-|\mathcal{S}|}^N \ell_i \, di$  is identical.  $\square$

*Proof of Corollary 3.* Assumption 1 follows immediately from the definitions of  $g(n_p)$  and  $\ell(n_s)$ .

$g(n_p)$  is continuous since  $g_i$  is defined over all  $n_p$ .  $g(n_p)$  is twice differentiable since  $g_i$  is continuous and differentiable.  $g(n_p)$  is weakly concave since

$g_i$  is weakly decreasing.  $g(0) = 0$  since the integral of a set of zero measure equals zero.

Results for  $\ell(n_s)$  are proven similarly.  $\square$

## References

- Aghion, Philippe, Ufuk Akcigit, and Peter Howitt (2013), “What do we learn from Schumpeterian growth theory?” Technical report, National Bureau of Economic Research.
- Aghion, Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt (2005), “Competition and innovation: an inverted-U relationship.” *The Quarterly Journal of Economics*, 120, 701–728.
- Aghion, Philippe and Peter Howitt (1992), “A Model of Growth through Creative Destruction.” *Econometrica*, 60, 323–51, URL <https://ideas.repec.org/a/ecm/emetrp/v60y1992i2p323-51.html>.
- Anton, James, Hillary Greene, and Dennis Yao (2006), “Policy implications of weak patent rights.” In *Innovation Policy and the Economy, Volume 6*, 1–26, The MIT Press.
- Anton, James J and Dennis A Yao (2004), “Little patents and big secrets: managing intellectual property.” *RAND Journal of Economics*, 1–22.
- Arora, Ashish (1997), “Patents, licensing, and market structure in the chemical industry.” *Research policy*, 26, 391–403.
- Arora, Ashish, Marco Ceccagnoli, and Wesley M. Cohen (2003), “R&d and the patent premium.” Working Paper 9431, National Bureau of Economic Research, URL <http://www.nber.org/papers/w9431>.
- Arrow, Kenneth (1962), “Economic welfare and the allocation of resources for invention.” In *The rate and direction of inventive activity: Economic and social factors*, 609–626, Princeton University Press.
- Belleflamme, Paul and Francis Bloch (2013), “Dynamic protection of innovations through patents and trade secrets.” *Available at SSRN 2537248*.
- Cohen, Wesley M, Richard R Nelson, and John P Walsh (2000), “Protecting their intellectual assets: Appropriability conditions and why US manufacturing firms patent (or not).” Technical report, National Bureau of Economic Research.

- de Bettignies, Jean-Etienne, Bulat Gainullin, Huafang Liu, and David T. Robinson (2016), “Competition, licensing, and innovation strategy.” URL <https://ssrn.com/abstract=2400839>.
- Denicolò, Vincenzo and Luigi Alberto Franzoni (2004), “Patents, secrets, and the first-inventor defense.” *Journal of Economics & Management Strategy*, 13, 517–538.
- Friedman, David D, William M Landes, and Richard A Posner (1991), “Some economics of trade secret law.” *The Journal of Economic Perspectives*, 61–72.
- Gilbert, Richard (2006), “Looking for Mr. Schumpeter: Where are we in the competition–innovation debate?” In *Innovation Policy and the Economy, Volume 6*, 159–215, The MIT Press.
- Graham, Stuart JH (2004), “Hiding in the patents shadow: Firms uses of secrecy to capture value from new discoveries.” *GaTech TI:GER Working Paper Series*.
- Graham, Stuart JH and Deepak Hegde (2014), “Do inventors value secrecy in patenting? Evidence from the American Inventor’s Protection Act of 1999.” URL <http://ssrn.com/abstract=2170555>.
- Griliches, Zvi (1981), “Market value, R&D, and patents.” *Economics letters*, 7, 183–187.
- Griliches, Zvi (1990), “Patent statistics as economic indicators: A survey.” *Journal of Economic Literature*, 28, 1661.
- Griliches, Zvi, Ariel Pakes, and Bronwyn H Hall (1987), “The value of patents as indicators of inventive activity.” In *Economic Policy and Technological Performance*, Citeseer.
- Hall, Bronwyn, Christian Helmers, Mark Rogers, and Vania Sena (2014), “The choice between formal and informal intellectual property: a review.” *Journal of Economic Literature*, 52, 375–423.
- Hall, Bronwyn H (2007), “Patents and patent policy.” *Oxford Review of Economic Policy*, 23, 568–587.

- Hall, Bronwyn H, Adam Jaffe, and Manuel Trajtenberg (2005), “Market value and patent citations.” *RAND Journal of economics*, 16–38.
- Hall, Bronwyn H, Jacques Mairesse, and Pierre Mohnen (2010), “Measuring the returns to R&D.” *Handbook of the Economics of Innovation*, 2, 1033–1082.
- Henry, Emeric and Francisco Ruiz-Aliseda (2016), “Keeping secrets: the economics of access deterrence.” *American Economic Journal: Microeconomics*, 8, 95–118.
- Hussinger, Katrin (2006), “Is silence golden? Patents versus secrecy at the firm level.” *Economics of Innovation and New Technology*, 15, 735–752.
- Lerner, Josh (1995), “Patenting in the shadow of competitors.” *Journal of law and economics*, 463–495.
- Levin, Richard C, Alvin K Klevorick, Richard R Nelson, Sidney G Winter, Richard Gilbert, and Zvi Griliches (1987), “Appropriating the returns from industrial research and development.” *Brookings papers on economic activity*, 783–831.
- Lobel, Orly (2013), “Filing for a patent versus keeping your invention a trade secret.” *Harvard Business Review*.
- Moser, Petra (2012), “Innovation without patents: Evidence from World’s Fairs.” *Journal of Law and Economics*, 55, 43–74.
- Nagaoka, Sadao, Kazuyuki Motohashi, and Akira Goto (2010), “Patent statistics as an innovation indicator.” *Handbook of the Economics of Innovation*, 2, 1083–1127.
- Noonan, Kevin E. (2011), “While not right for every invention, trade secret protection has its appeal.” Technical report, McDonnell, Boehnen, Hulbert, & Berghoff, LLP.
- Ottoz, Elisabetta and Franco Cugno (2008), “Patent–secret mix in complex product firms.” *American Law and Economics Review*, 10, 142–158.
- Pakes, Ariel (1985), “On patents, R&D, and the stock market rate of return.” *Journal of political economy*, 93, 390–409.

- Pakes, Ariel and Zvi Griliches (1980), "Patents and R&D at the firm level: A first report." *Economics letters*, 5, 377–381.
- Peneder, Michael (2012), "Competition and innovation: Revisiting the inverted-U relationship." *Journal of Industry, Competition and Trade*, 12, 1–5.
- Png, I.P.L. (2015), "Secrecy and patents: Evidence from the uniform trade secrets act." *Working Paper*, URL <https://ssrn.com/abstract=2617266>.
- Schenk, Kimberly J. and John G. Plumpe (2015), "Federal circuit provides guidance on jury instructions on apportionment of patent damages." Technical report, New York State Bar Association.
- Schumpeter, Joseph Alois (1942), *Socialism, capitalism and democracy*. Harper and Brothers.
- Shapiro, Carl (2011), "Competition and innovation: Did Arrow hit the bull's eye?" In *The Rate and Direction of Inventive Activity Revisited*, 361–404, University of Chicago Press.
- Solow, Robert M (1957), "Technical change and the aggregate production function." *The review of Economics and Statistics*, 312–320.
- Trajtenberg, Manuel (1990), "A penny for your quotes: patent citations and the value of innovations." *The Rand Journal of Economics*, 172–187.
- Whinston, Michael D (2011), "Comment on "Competition and innovation: Did Arrow hit the bull's eye?"." In *The Rate and Direction of Inventive Activity Revisited*, 404–410, University of Chicago Press.
- Zábojník, Ján (2002), "A theory of trade secrets in firms." *International Economic Review*, 43, 831–855.
- Zhang, Tianle (2012), "Patenting in the shadow of independent discoveries by rivals." *International Journal of Industrial Organization*, 30, 41–49.

## Figures

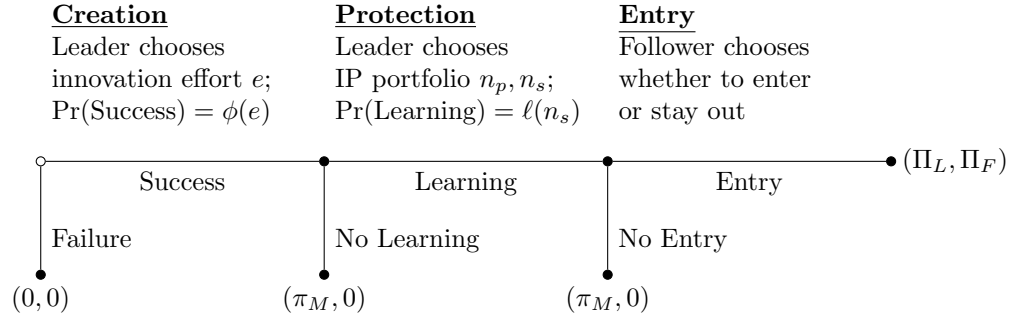


Figure 1: The model environment.

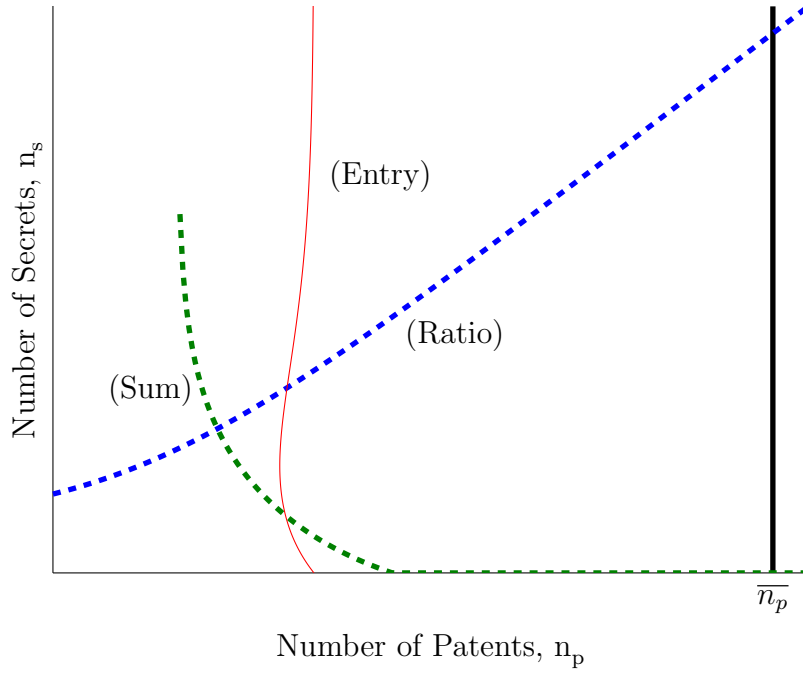


Figure 2: A graphical depiction of equilibrium as given by (Ratio), (Sum), and (Entry).

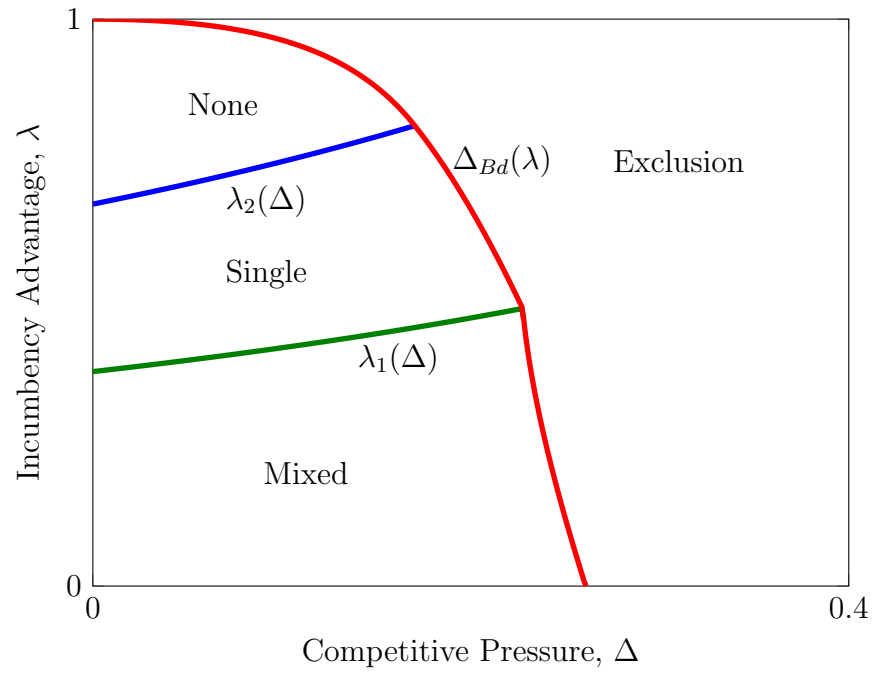


Figure 3: A graphical depiction of Proposition 1.

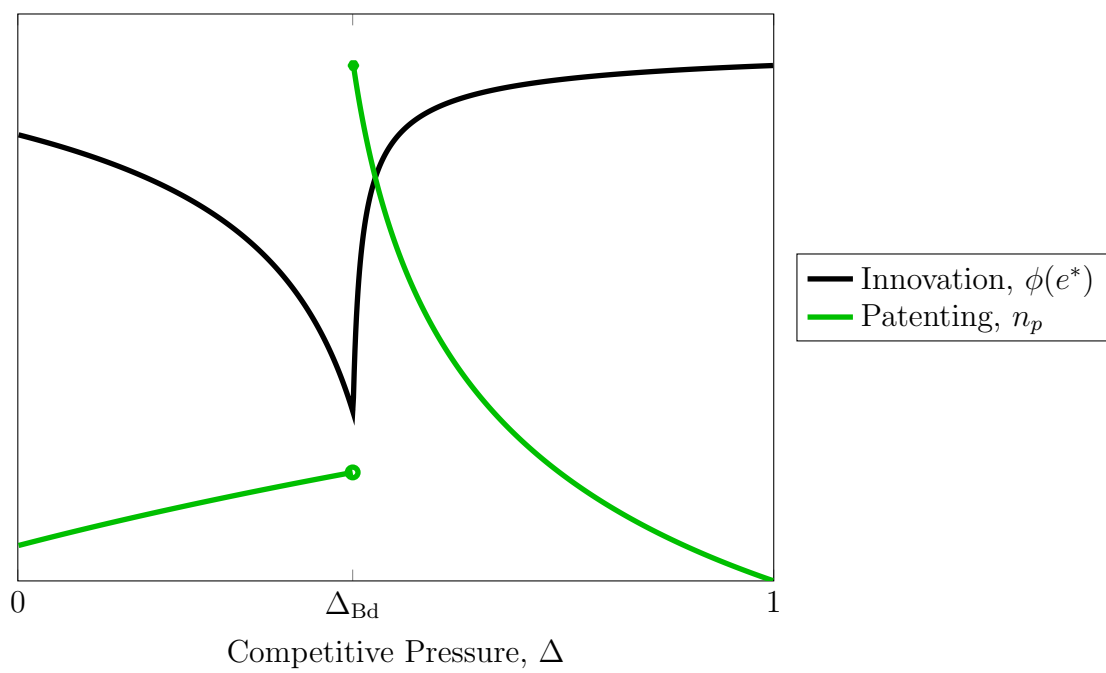


Figure 4: Ex-post patenting as a function of  $\Delta$ .

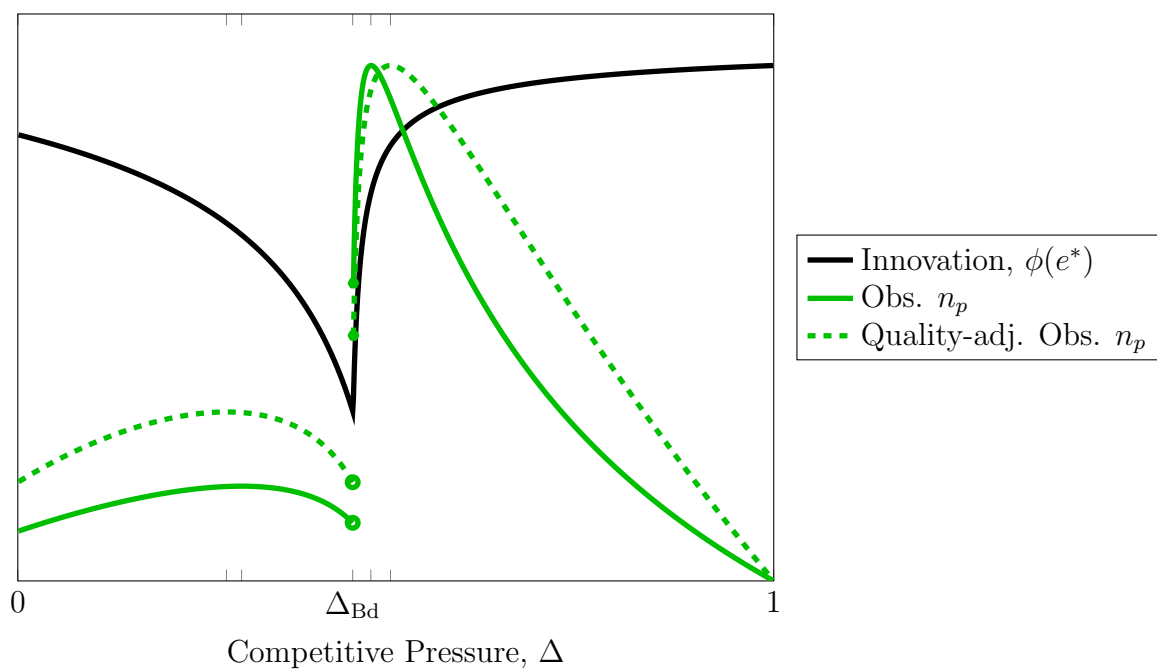


Figure 5: Patenting measures as functions of  $\Delta$ .

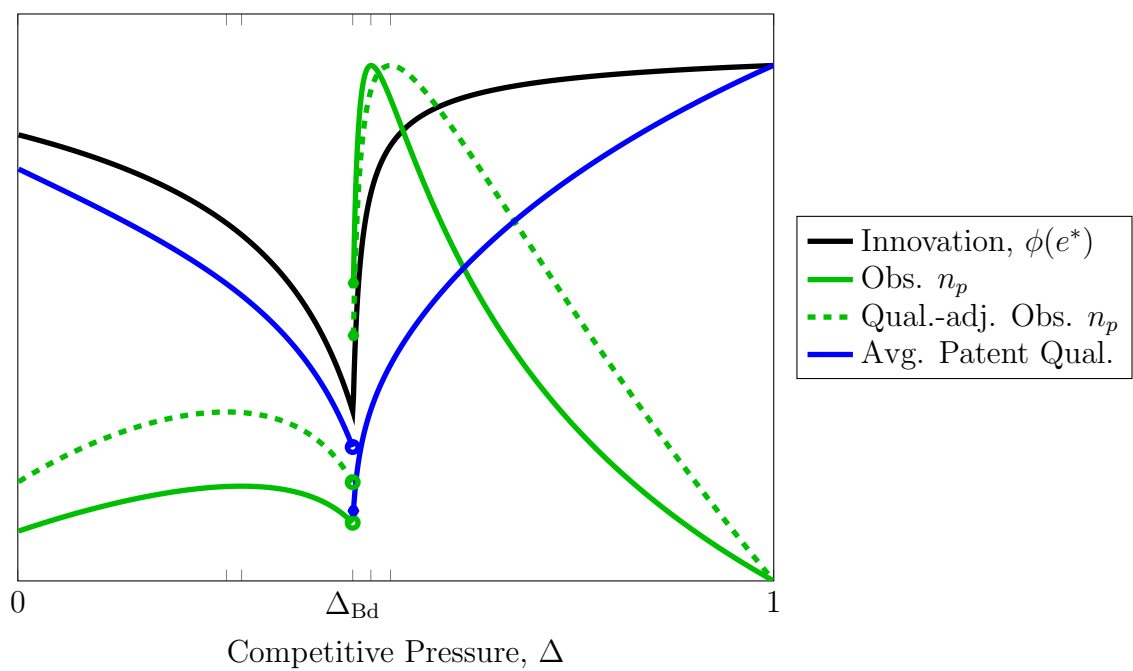


Figure 6: Patenting measures as functions of  $\Delta$ .