Optimal Trade Policy, Equilibrium Unemployment and Labor Market Inefficiency*

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Abstract

Why do politicians advocate trade protections to save domestic jobs when neoclassical trade models suggest that small open economies should implement free trade? The novel insight of this paper is that trade protections can be rationalized as a second-best policy that improves the domestic welfare when the equilibrium unemployment is different from the constrain-efficient unemployment. To understand the puzzle, I incorporate a Diamond-Mortensen-Pissarides frictional labor market into the standard Heckscher-Ohlin model of international trade. The model offers four main findings. First, when the relative price of the labor (capital)-intensive good increases, equilibrium unemployment decreases (increases). Second, a labor market in a competitive equilibrium is constrained-efficient when the Hosios condition is satisfied. Third, a capital-abundant country with inefficiently high unemployment may experience welfare losses from trade. Conditional on having the same observed trade share, a labor-abundant country with inefficiently high unemployment have extra welfare gains from international trade. Finally and importantly, when the labor market in a small open economy generates inefficiently high equilibrium unemployment, the optimal trade policy is to raise the domestic price of its labor-intensive goods (an import tariff in a capital-abundant country and an export subsidy in a labor-abundant country). Free trade is optimal only when a labor market is initially efficient. The model predictions are supported by patterns of tariffs in WTO member countries.

Keywords: Optimal Trade Policy, Employment, Gains from Trade, Heckscher-Ohlin.
JEL classification numbers: F11, F13, F16, F66, J64

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1 Introduction

> *"... Let us protect American jobs. A 40 percent tariff will protect our domestic industry and provide security to American families. ..."* Max Sandlin, Democratic Congressman from Texas (2002)
>
> *"Protectionism happens to be Congress's job."* Ernest F. Hollings, Democratic U.S. Senator from South Carolina (2004)

"Jobs are lost!" has always been a political criticism of trade liberalization. Governments are often pressured to protect jobs in import-competing sectors. Economists respond by arguing that an expansion of export sectors would create enough new jobs to compensate for job losses in import-competing sectors; all displaced workers only have to relocate from the contracting import-competing sectors to the expanding export sectors. The traditional trade models find that trade protections in a small open economy always generate distortions and decrease total welfare. This creates a puzzle: why do politicians advocate trade policies to prevent job losses despite the fact that trade protections are not economically sensible in neoclassical trade models?

This paper aims to understand this puzzle. I develop a tractable model that introduces a Diamond-Mortensen-Pissarides frictional labor market into the standard Heckscher-Ohlin model. In my model, recruiting workers involves costly search frictions: firms must pay an upfront cost to post vacancies before the open positions are randomly filled. I reduce the Dutt et al. (2009) model to a static, rather than dynamic, model and then introduce vacancy production.[1] This new general equilibrium model allows us to explicitly address labor market efficiency and the effect of trade policies on unemployment.

I first solve for a competitive equilibrium and a social planner's problem, and summarize the sufficient conditions for efficiency of a competitive equilibrium. Subsequently, I use this model to study the impact of a price change on equilibrium employment and welfare. Motivated by the threat of job losses due to international trade, I then investigate whether there is an economic rationale for using trade policies to protect workers at home. Initially, I study the optimal trade policy of a small open economy: abstracting from terms-of-trade externalities highlights the novel incentive for protectionism from the labor market. I then study the optimal trade policy in a large open economy, where the terms-of-trade externality exists and interacts with the labor-market motive.

I establish two main findings: one positive and one normative. First, total employment increases when the relative price of the labor-intensive good rises and vice versa. Intuitively, an increase in the price of the labor-intensive good raises the value of the marginal product of labor, inducing firms to search for workers more intensively. Unemployed workers have a greater chance of randomly receiving a job offer from firms and total employment increases accordingly. Labor income in my model is more sensitive to price changes than in a traditional Heckscher-Ohlin model,

[1] I show that the main results hold true in a dynamic model studied in Section 9.2.

because the impact of price changes on labor income is amplified by an additional fluctuation in total employment. I emphasize that if the combined effect of the fall in wage and the decrease in employment is large enough, then a capital-abundant country with inefficiently high unemployment can potentially be made worse off by international trade. In addition, conditional on having the same trade share, a labor-abundant country with inefficiently high unemployment enjoys an extra gain from international trade that is not present in traditional models that do not allow for unemployment.

Second, the main contribution of this paper is to explore why the benevolent government of a small open economy uses trade policies. The model rationalizes the use of trade policies as a second-best instrument to correct labor market inefficiency. In other words, implementing free trade is optimal only if the labor market is initially efficient. More specifically, when unemployment is inefficiently high, the country should use a trade policy to protect its labor-intensive sectors, because labor-intensive sectors are where the demand for labor is mainly created; a capital-abundant country would use inward-oriented policies, while a labor-abundant country would use outward-oriented policies.

One policy implication of this model is that protecting an import-competing industry is not always an effective way to prevent aggregate job losses. For example, when unemployment is inefficiently high, protecting the capital-intensive import-competing sector actually increases demand for capital and releases more workers into the labor market than the labor-intensive sector can absorb. Thus, aggregate unemployment rises.

In some scenarios, trade intervention improves world welfare. For example, if a labor-abundant country uses an export subsidy to increase domestic employment, then the policy intervention creates a positive externality for a capital-abundant trading partner, through a terms-of-trade externality. Tariffs are not necessarily always good for the world economy. Rather, small countries may have an employment incentive to support domestic production and, hence, world free trade may not be socially optimal after all. Even if governments maximize aggregate social welfare and have no explicit objective regarding employment, in my model, employment still plays a significant role in a trade policy decision. These results arise without any particular political economy motives.

According to traditional argument in the literature, a country prefers either an export tax or an import tariff if it can use its trade policy to manipulate world prices; a small country that cannot affect world prices prefers free trade. However, all countries (except Macao and Hong Kong) use positive tariffs. Despite the fact that many countries are small compared to the world, it is clear that nearly all have an incentive to protect themselves. Grossman and Helpman (1994) give one possible explanation for a small open economy: lobbyists pay the government in exchange for tariff protection in some sectors.

This paper proposes an alternative explanation, that the use of trade policies is related to the domestic employment situation; the labor-market motivation of a trade policy is directly linked to the inefficiency of the labor market. A government has an incentive to adjust an inefficient employment level to the efficient employment level. There are two sources of inefficiency in this model: a hold-up problem and a congestion externality. The hold-up problem arises because wage bargaining between firms and workers excludes the upfront cost firm owners pay to post the vacancy. Firms are only willing to post a job vacancy if they will receive a sufficiently large share of the surplus to cover their sunk cost, and hence post too few vacancies. In contrast, the congestion externality arises because firms post additional vacancies without recognizing their social cost on other firms' expected profits. Adding a vacancy causes congestion in the matching process and all firms have a smaller chance of filling their open positions. Thus, without internalizing the congestion externality, firms post too many vacancies. Depending on the magnitude of each effect, total employment may be either too low or too high. The sufficient condition for efficiency, called the Hosios efficiency condition (Hosios 1990), states a labor market is constrained-efficient only if the wage bargaining power of firms is equal to the elasticity of an exogenous job-matching function with respect to the total number of vacancies. In other words, the labor market is constrained-efficient when the two sources of inefficiency completely offset each other. When the Hosios condition holds, a labor market is constrained-efficient and the government need not interfere with markets.

Why do countries not eliminate inefficiency in a labor market by using a direct instrument to correct the inefficiency at the source? As pointed out by Bhagwati (1971), a trade policy is normally a second-best policy, as the first-best policy is likely a purely domestic policy aimed directly at the inefficiency. I interpret my exercise as demonstrating trade policy being a practical employment-inducing policy when the first-best policy, a direct tax-cum-subsidy on the vacancy posting, is impractical. In this sense, this model best describes developing economies whose informal sector is large and whose labor market is underdeveloped.[2] It is extremely difficult to implement a direct non-distortive tax-cum-subsidy on vacancy postings in countries where most firms are unregistered or can avoid labor regulation and other government or institutional regulations.

Although it may be less obvious, my results also apply to developed countries. The first-best policy requires complete information about vacancy postings. The necessary information about job postings, however, is unobservable and unverifiable. Therefore, firms have an incentive to falsely claim subsidy benefits. As a result, developed countries with relatively well-functioning labor markets may find the first-best policy impractical. Thus, my model is applicable to both developing and developed economies. Furthermore, from a public finance perspective, a subsidy on vacancy posting must be financed by the government, while a tariff would generate tax revenue for the government. Thus, the government may naturally wish to use an aggregate trade policy to solve an efficiency problem, since prices are common across both the informal sector and the formal sector, and unlike the subsidy, it can loosen the government's budget constraint.

[2]The informal sector in developing countries can be as large as 70% of all employment (Bosch & Esteban-Pretel, 2012; Djankov et al., 2002; Schneider, 2003). To make matters worse, trade liberalization increases the share of informal workers in certain countries, such as in the manufacturing sector in Colombia, where 47% informal workers in 1986 increased to 57% in 1996 (Goldberg & Pavcnik, 2003). Trade liberalization in Brazil also increases informal employment by 1 to 2.5% (Bosch, Goni-Pacchioni, & Maloney, 2012). In addition, a 10-percent reduction in the Brazilian import tariffs lowers the Brazilian industry-level average formal wage by 0.86% and expands the Brazilian informal sector by 1.51% (Paz, 2014).

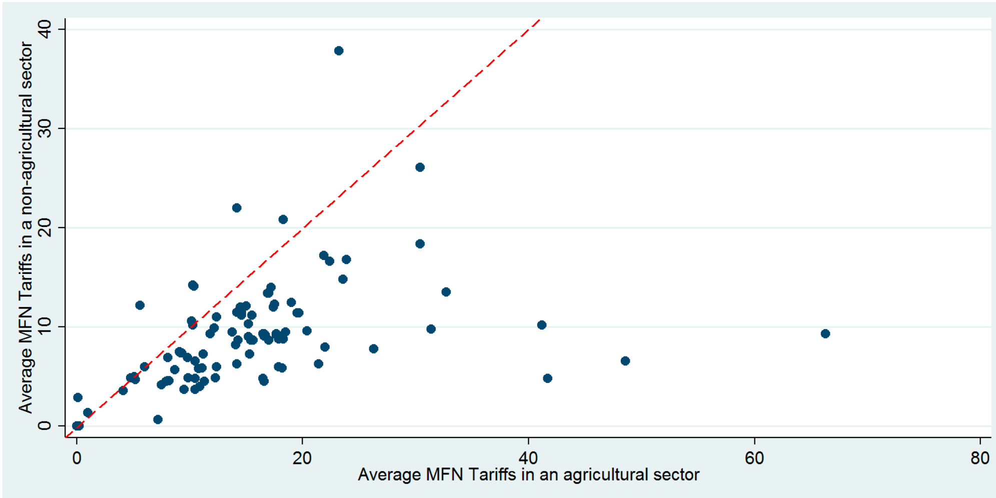

Figure 1: Average MFN applied tariffs of WTO members in 2011 (from World Tariff Profile 2012 by WTO). Each point represents a pair of each developing country's average import tariff in the agricultural sector and in the non-agricultural sector, and the red dashed line is a 45-degree line.

Perhaps concern about job losses is common to all countries, and the General Agreement on Tariffs and Trade (GATT) and the World Trade Organization (WTO) have recognized the effect of trade liberalization on labor markets as the GATT and WTO allow high tariffs and voluntary negotiation of reciprocal tariff cuts. The GATT and WTO might allow these high tariffs to ease employment concerns in labor-intensive industries such as agriculture. While the Agreement on Agriculture in the Uruguay Round discussed the reduction of subsidies and other forms of protection, the commitments that member countries are asked to make in the agricultural sector are, in general, still less restrictive than commitments in non-agricultural sectors. For example, the Agreement on Agriculture has no red (forbidden) box for domestic support — the agricultural sector is allowed to have more types of domestic supports. Although there have been attempts to lower import tariffs and trade barriers in the agricultural sector, on average this sector still has a higher applied tariff rate than other sectors.

Figure 1 shows average tariffs in an agricultural sector and average tariffs in a non-agricultural sector in non-industrialized countries. I include only developing countries because they are arguably considered small open economies and their labor markets are relatively underdeveloped, though there are obvious exceptions, e.g., China. In addition, because in reality the use of export subsidies has been restricted, Figure 1 only focuses on import tariffs. Normally countries export and import both labor-intensive goods and capital-intensive goods. Provided that agricultural goods are labor-intensive, if countries generally want to protect their domestic jobs, my model predicts that on average countries impose higher import tariffs on agricultural products. According to Figure 1, the average import tariff in the agricultural sectors is greater than that in non-agricultural sectors in most countries. Only 10 of 113 developing countries have their average import tariff on a non-agricultural sector higher than their average import tariff on the agricultural sector,[3] as shown by the points above the dashed line in Figure 1. On average, each country imposes roughly a 7-percent higher import tariff on the agricultural sector than on other sectors. Therefore, the data in Figure 1 supports model predictions. If one thinks in broad terms of non-industrialized countries as the most likely to be small open economies and of agricultural production as among the most labor-intensive activities in non-industrialized countries, then the data in Figure 1 is broadly consistent with the model predictions.

The rest of the paper is organized as follows: Section 2 discusses some important related literature. Section 3 introduces the model. I characterize the competitive equilibrium and the constrained-efficient equilibrium in Section 4. Comparative statics for impacts of price changes on factor prices, employment, and welfare are shown in Section 5. Section 6 derives an optimal trade policy. Section 7 extends the model from Section 3 to a two-country model and investigates the optimal trade policy for a large open economy. Section 8 illustrates some numerical examples. Section 9 discusses three variations of the main model: (i) a special case—a Ricardian model—in which labor is the only input of production, (ii) a dynamic model, and (iii) a case in which all policy instruments are available for a government. Section 10 offers a summary of the main results and directions for future work.

2 Related Literature

This paper bridges the gap between two literatures: the effect of trade liberalization on equilibrium unemployment and the motives behind trade policy. To the best of my knowledge, this paper is closest to Costinot (2009), which explores determinants of trade policy in a small open economy with search frictions in the labor market. Costinot (2009) extends the model in Pissarides (2000) and investigates how a trade tax in each sector responds to characteristics of the labor market in that sector, such as productivity of workers, the world price, and the job turnover rate. My model differs from Costinot (2009) in various ways. Despite identical workers and an aggregate matching technology in all sectors, in my model degrees of capital intensity and labor market inefficiency play an important role in determining trade policy. Trade taxes in Costinot (2009) exploit benefits from characteristics of sectoral labor markets to minimize distortions from trade taxes. In contrast, trade policy in my model attempts to resolve inefficiency in a labor market and improve the country's welfare.

In my another work, Suwanprasert (2016), I revisit Costinot (2009) by studying the motivation of trade policy in the same model but in a larger space of parameter values. In that paper, I show that Costinot (2009) assumes a parameter space that implicitly creates inefficiently high unemployment. Therefore, import tariffs that reduce unemployment can improve welfare. In the larger parameter space, unemployment can be inefficiently low and import subsidy is optimal. When the

[3] These countries are Bahamas (14.6%), Djibouti (7.8%), Comoros (6.6%), Brazil (3.9%), Argentina (3.7%), Brunei (2.8%), Maldives (2.5%), Uruguay (0.4%), Mauritius (0.4%), and Saudi Arabia (0.1%).

labor market is efficient, free trade is optimal, as concluded in this paper.

Brecher (1974a, 1974b) and Matschke (2006) take different approaches to address the same questions asked in this paper. Brecher (1974a) investigates unemployment caused by minimum wages in a Heckscher-Ohlin model and Brecher (1974b) studies an optimal trade policy in the same framework. This paper's main departure from these works is the source of labor market inefficiency. Brecher (1974a, 1974b) requires exogenous minimum wages to generate structural unemployment, whereas this paper begins with the microeconomic foundations of a labor market and allows for search frictions to naturally generate unemployment. Hence, my model is able to predict how the optimal trade policy responds to endogenous labor market inefficiency. Matschke (2006) adds minimum wages to the lobbying game in Grossman and Helpman (1994) and shows that unemployment may not result in an increase in trade protection. The key mechanism in Matschke (2006) is rent sharing between capital owners and workers. When neither capital owners nor workers are represented by a lobby, protection rents are lost and the equilibrium tariff is low. Furthermore, wages in Brecher (1974a, 1974b) and Matschke (2006) are exogenous due to minimum wages and are independent of trade policy, whereas my model endogenizes wages through Nash bargaining. As a result, the effect of trade policy in this paper not only comes from employment but also from equilibrium wages.

The equilibrium unemployment in international trade has not been widely investigated. Davidson, Martin, and Matusz (1988) introduce search frictions in one sector to describe how real returns react to output prices. In a follow-up study, Davidson, Martin, and Matusz (1999) then allow search frictions in markets for factor endowments. Their model can be considered a Heckscher-Ohlin model, in which both capital and labor markets are subject to search frictions. They show that different matching technologies can be a source of comparative advantage. Helpman and Itskhoki (2010) use a similar approach, but focus on labor as the only factor of production. They suggest that search frictions in a labor market can be the source of comparative advantage, assuming the existence of a numeraire sector that exogenously determines expected labor income. Although trade policy changes total employment, it does not affect labor income or welfare (Kwon, 2014). my model departs from Davidson, Martin, and Matusz (1988, 1999) and Helpman and Itskhoki (2010) in the sense that it incorporates labor market inefficiency and considers optimal trade policies that aim to correct this inefficiency.

Dutt et al. (2009) add search frictions to a Ricardian model and a Heckscher-Ohlin model. Hoon (1991) adds search frictions and efficiency wage, the wage that workers choose the efficient level of effort, to a Heckscher-Ohlin model. Their comparative statics shows that employment is increasing in the domestic price of the labor-intensive good. This paper goes further by considering the efficiency of the search equilibrium and characterizing the optimal trade policy in a simplified static version of Dutt et al. (2009). Felbermayr, Prat, and Schmerer (2011b) introduce labor market frictions into the Melitz model (Melitz, 2003): since labor is the only input of production, they conclude that unemployment always decreases after trade liberalization. In contrast, my model shows that an expanding capital-intensive export sector potentially raises the number of unemployed workers.

Dutt et al. (2009), Felbermayr, Prat, and Schmerer (2011a), and Hasan et al. (2012) show that trade openness reduces economy-wide unemployment. However, these studies define trade openness as a reduction in transportation costs instead of a price change as in my model, and do not classify sector-specific capital intensities. Thus, their results, which run counter to ours, arise because the reduction in transportation costs happens in both sectors and the effect of trade liberalization in labor-intensive sectors may dominate the effect of trade liberalization in capital-intensive sectors. Heid (2015) reports that in 13 Latin American and Caribbean countries, regional trade agreements have decreased the unemployment rate by 1.2% and decreased informal employment by 20.3%. Harrison, McLaren, and McMillan (2011) and Goldberg and Pavcnik (2007) study how trade liberalization affects income inequality. Other empirical works examine workers' mobility costs from sectoral reallocation, which may not reflect an impact on total unemployment (for instance, Artuc, Chaudhuri, & McLaren, 2010; Dix-Carneiro, 2014; Dix-Carneiro & Kovak, 2015). Belenkiy and Riker (2015) provide a useful review of the empirical and theoretical work on international trade and unemployment.

Another part of the literature studies the motives behind trade policies in small open economies. According to standard economic reasoning, unilateral free trade is the best policy for a small open economy, regardless of its trading partners' tariffs. In addition to the Grossman and Helpman (1994) paper which I discussed earlier, Maggi and Rodriguez-Clare (2007) link lobbying to the degree of capital mobility: when capital is less mobile, owners of capital in an import-competing sector suffer losses due to trade liberalization, because they cannot relocate their capital to an expanding export sector. Thus, capital owners lobby their government to grant them trade protection. Grossman and Helpman (1994), as well as Maggi and Rodriguez-Clare (2007) also provide another reason for the observed high protection associated with agricultural products, as shown in Figure 1: these agricultural sectors receive high protections because of lobbying. Grossman and Helpman (1994) and Maggi and Rodriguez-Clare (2007) differ from this work in the sense that their motive for trade policy arises from political economy motives while my mechanism arises naturally from labor market inefficiency. In Antras and Staiger (2012), a government may use trade policy to correct hold-up inefficiencies in bilateral bargaining between buyers and sellers during a process of offshoring intermediate inputs. Without trade policies, free trade yields an inefficiently low output level. My model, in contrast, contains the hold-up problem and the congestion externality of posting vacancies resulting in too low or too high of employment in free trade, and I show that the optimal trade policy aims to eliminate labor-market inefficiency.

3 Model

The model is a simplified static version of Dutt, Mitra, and Ranjan (2009). Generally speaking, it is an extended Heckscher-Ohlin model that includes a frictional Diamond-Mortensen-Pissarides labor market structure. It focuses on long-run adjustments where capital and workers can move freely between sectors, but hiring unemployed workers is costly.

Consider a small open economy in a static one-period model.[4] The economy is endowed with homogenous labor force L and capital K. There are two types of final goods in the world, called X and Y. The economy takes world prices as given and its trade policies do not alter world prices. Let p be the price of good X and normalize the price of good Y to unity; hence, p is the relative price of good X in terms of good Y. It is not necessary to define comparative advantage until Section 5 when I compare the world price and the autarky price. Without loss of generality, consider a trade policy in sector X that changes the domestic price from p to $(1+t)\,p$. When good X is an imported good, the trade policy is considered an import tariff (subsidy) if $t > 0$ $(t < 0)$. In contrast, when good X is an exported good, $t > 0$ $(t < 0)$ is an export subsidy (tax).[5]

3.1 Households

The preferences of a representative household are given by

$$U\left(X^d, Y^d\right) = \frac{\left(X^d\right)^{\alpha}\left(Y^d\right)^{1-\alpha}}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}},$$

where X^d and Y^d denote home-country aggregate demand of goods X and Y, respectively, and $0 < \alpha < 1$.

The budget constraint is

$$I\left(t\right) = \left(1+t\right) pX^d + Y^d,$$

where $I\left(t\right)$ is the net income of the country as a function of the trade policy t. The representative household takes net income as given. The determination of net income $I\left(t\right)$ is discussed later, after the production side is introduced. Note that this budget constraint is different from the actual budget constraint of the country due to a price distortion in the representative household's point of view.

3.2 Firms

As in a standard Heckscher-Ohlin model, production of X and Y use both capital K and labor L. Capital and employed workers can move freely between sectors, but finding workers is costly. Single-worker firms must produce a costly vacancy and then wait until the vacancy is randomly filled. Then firms and workers bargain bilaterally over the surplus of the successful match via Nash bargaining. After jobs are created, the firms rent capital at a market rate r, produce output, and sell it in the goods market. Hereafter, I use subscript $i \in \{X, Y\}$ to index a statement that holds true for any X and Y.

[4] Results in a case of a large open economy are in Section 7.

[5] Introducing a sectoral trade policy in both sectors, t_x and t_y, does not change my result because only the relative price matters to the equilibrium allocation. Thus I can interpret t as $1+t = \left(1+t_x\right) / \left(1+t_y\right)$. I also restrict the attention to $t \in [-1, t_{max}]$ such that unemployment drops to zero at $t = t_{max}$. Removing the upper bound, t_{max}, does not significantly change our result.

Vacancy Posting

To post an additional vacancy, firms must use a combination of x_V units of good X and y_V units of good Y as intermediate inputs to produce one new vacancy.[6] The production function for posting a vacancy is the same in both sectors. The input requirement for delivering one vacancy is

$$x_V^{\alpha} y_V^{1-\alpha} \geq f\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha},$$

where $f > 0$ is a parameter capturing production technology for posting a vacancy. Firms buy intermediate goods at domestic prices. The nominal cost of producing one vacancy is

$$\left(1+t\right) p x_V\left(p,t\right) + y_V\left(p,t\right).$$

Firms choose intermediate inputs $x_V^*\left(p,t\right)$ and $y_V^*\left(p,t\right)$ to minimize the nominal cost of producing one vacancy such that the input requirement is satisfied. The cost minimization problem is solved in Section 4.

Final Goods

After a vacancy is filled by a worker, a firm in sector i rents capital k_i for the worker to produce good i. For each employed worker, the production function for good i is

$$g_i\left(k_i\right) = k_i^{\phi_i},$$

where $0 < \phi_i < 1$ represents the capital intensity of good i. Without loss of generality, I assume that good X is labor-intensive, so that $\phi_y > \phi_x$. The total production is calculated from $X^s = k_x^{\phi_x} L_x = K_x^{\phi_x} L_x^{1-\phi_x}$ and $Y^s = k_y^{\phi_y} L_y = K_y^{\phi_y} L_y^{1-\phi_y}$, where L_i is total labor employed in sector i, and $K_i = k_i L_i$ is total capital employed in sector i. Because capital can freely move between sectors and is rented from a perfectly competitive market, its rental rate r is identical in both sectors and is equal to the value of the marginal product of capital, that is

$$r = \left(1+t\right)\phi_x p k_x^{\phi_x-1} = \phi_y k_y^{\phi_y-1}. \quad (1)$$

Therefore, after a vacancy is filled by a worker, the matched position generates a surplus

$$\begin{aligned} S_x &= \left(1+t\right) p k_x^{\phi_x} - r k_x = \left(1-\phi_x\right)\left(1+t\right) p k_x^{\phi_x}, \\ S_y &= k_y^{\phi_y} - r k_y = \left(1-\phi_y\right) k_y^{\phi_y}, \end{aligned}$$

[6] The vacancy cost can be interpreted as advertisement cost or time cost of human resources to search for and screen potential employees.

where S_i is a surplus of a filled position in sector i.

Matching and Wage Determination

Anticipating the surplus from a match, the worker and the firm bargain over wage w_i to determine the division of the surplus via Nash bargaining. The Nash bargaining solution gives a wage

$$w_i = \text{argmax}\left(S_i - w_i\right)^{\beta}\left(w_i\right)^{1-\beta}, \tag{2}$$

where $0 < \beta < 1$ is the bargaining power of the firm and $1-\beta$ is the bargaining power of the worker. I assume that β is common across the two sectors. Under Nash bargaining, the surplus is divided according to bargaining power. The worker's wage w_i and the firm's profit π_i for each matched position are

$$\begin{aligned} w_i &= (1-\beta) S_i, \\ \pi_i &= \beta S_i. \end{aligned}$$

Let V be the number of total economy-wide posted vacancies. I assume an exogenous matching technology which creates aggregate employment E from vacancy V and total population L:

$$E = M(V, L) = min\left\{V^{\lambda} L^{1-\lambda}, L\right\}, \tag{3}$$

where $0 \leq \lambda \leq 1$ is the (constant) elasticity of job creation with respect to vacancies. I restrict the parameter values such that the total employment is always less than the size of the workforce, $E < L$ to prevent negative unemployment. [7]

After the firm has paid a vacancy cost $(1+t)\,px_V + y_V$ to create a vacancy, the vacancy is randomly filled with the same probability E/V and the firm receives its share of surplus $\pi_i = \beta S_i$. Thus, the expected profit before firms post vacancies is

$$\mathbf{E}(\pi_i) = \frac{E}{V}\beta S_i - \left[(1+t)\,px_V + y_V\right],$$

where a bold $\mathbf{E}$ is used to denote expected value (as distinct from a non-bold E which denotes the country's aggregate employment).

Free entries and exits ensure that the equilibrium profit is driven down to zero:

$$\mathbf{E}(\pi_i) = 0. \tag{4}$$

[7] The parameter restriction is $\frac{\beta\Phi_2}{f}\left((1+t_{max})\,p\right)^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha} \leq 1$

Endowments Market

Sectoral allocations satisfy resource constraints in the labor market and the capital market:

$$L_x + L_y = E, \tag{5}$$

$$k_x L_x + k_y L_y = K. \tag{6}$$

The first constraint is the allocation of employed workers across sectors and the second constraint is how capital is allocated between two sectors. I restrict the parameter values to ensure that two sectors are active, i.e., $L_x > 0$ and $L_y > 0$.

4 Equilibrium

In this section, I solve for a competitive equilibrium of the economy described in Section 3 and solve for a constrained-efficient equilibrium from a social planner's problem. Then, I show a sufficient condition to ensure that the competitive equilibrium coincides with the constrained-efficient equilibrium in the social planner's problem.

4.1 Competitive Equilibrium

The definition of a competitive equilibrium is as follows.

> **Definition**: *A competitive equilibrium is the vector of factor prices, consumptions, final outputs, the allocation of endowments, intermediate inputs, and vacancy level* $\{w, r, X^d, Y^d, X^s, Y^s, k_x, k_y, L_x, L_y, x_V, y_V, V\}$, *given the world price ratio, the trade policy, and initial endowments* $\{p, t, K, L\}$, *that satisfies the following conditions: (i) the production cost of vacancy is minimized, (ii) firms maximize their profit, (iii) wage divides surplus according to Nash bargaining, (iv) rental price is given by a perfectly competitive market, (v) firms enter and exit freely, (vi) endowment markets are cleared, and (vii) the representative consumer maximizes his utility.*

To solve for a competitive equilibrium, I start with deriving the cost of producing a vacancy. The vacancy cost minimization problem is

$$\text{Firms} \quad \underset{x_V, y_V}{Min} \quad (1+t)\, p x_V + y_V,$$

$$\text{s.t.} \quad x_V^{\alpha} y_V^{1-\alpha} \geq f \alpha^{\alpha} (1-\alpha)^{1-\alpha}.$$

Demands for intermediate goods are

$$x_V^* = \alpha f \left((1+t)\, p\right)^{\alpha-1}, \tag{7}$$

$$y_V^* = (1-\alpha) f \left((1+t)\, p\right)^{\alpha}. \tag{8}$$

Therefore, the minimum cost of delivering one vacancy is

$$(1+t)\, p x_V^* (p,t) + y_V^* (p,t) = ((1+t)\, p)^{\alpha} f. \tag{9}$$

The parameter α captures how intermediate good x_v can be substituted by intermediate good y_v in the production of vacancy and thus exhibits how the vacancy cost responds to output prices and the trade policy. In the extreme case of $\alpha = 1$, an increase in the trade policy t will raise the cost of filling a vacancy by the same proportion as it increases the price of the labor-intensive good X, and therefore by a greater proportion than the price of producing the capital-intensive good Y. If the case of $0 < \alpha < 1$, the cost of filling a vacancy rises with the trade policy t but by less than the price of the labor-intensive good and by more than the price of the capital-intensive good. Finally, in the case of $\alpha = 0$, the cost of filling a vacancy does not change with the trade policy, as is true also of the price of the capital-intensive good.

These features will play a role in how a trade policy t affects unemployment in Section 5, and how the country's production possibility frontier is constructed. A change in the price of the labor-intensive good X does not only alter unemployment but also alter a combination of intermediate goods x_v and y_v used in the production of a vacancy.

Next, since firms can freely enter a market, the expected profit in all sectors is driven down to zero according to equation (4). Multiplying both sides of equation (4) by V results in

$$\beta S_i E = \left[(1+t)\, p x_V^* + y_V^*\right] V = ((1+t)\, p)^{\alpha} f V. \tag{10}$$

It implies $S_i = \frac{((1+t)p)^{\alpha} f V}{\beta E}$ where the right-hand side is common across both sectors. Therefore, $S_x = S_y = S$ in equilibrium, or

$$S = (1+t)\,(1-\phi_x)\, p k_x^{\phi_x} = \left(1-\phi_y\right) k_y^{\phi_y}. \tag{11}$$

Combined with the Nash bargaining solution, this results in $w_x = w_y = w$ and $\pi_x = \pi_y = \pi$; wages and profits are equalized across sectors. From now on, the sector-specific subscript i is dropped, unless its exclusion results in ambiguity.

Combining equations (1) and (11) leads to

$$k_x = \left(\frac{1-\phi_y}{1-\phi_x}\right)\left(\frac{\phi_x}{\phi_y}\right) k_y. \tag{12}$$

Equation (12) confirms that the capital-intensive (large ϕ) sector Y uses more capital per worker than the labor-intensive (small ϕ) sector X does, even once the indirect capital and labor used in posting job vacancies is also taken into account.

Capital in each sector can be solved from substituting equation (12) into equation (11):

$$k_x = \left(\frac{1-\phi_x}{1-\phi_y}\right)^{\frac{1-\phi_y}{\phi_y-\phi_x}} \left(\frac{\phi_x}{\phi_y}\right)^{\frac{\phi_y}{\phi_y-\phi_x}} ((1+t)\,p)^{\frac{1}{\phi_y-\phi_x}}\,, \tag{13}$$

$$k_y = \left(\frac{1-\phi_x}{1-\phi_y}\right)^{\frac{1-\phi_x}{\phi_y-\phi_x}} \left(\frac{\phi_x}{\phi_y}\right)^{\frac{\phi_x}{\phi_y-\phi_x}} ((1+t)\,p)^{\frac{1}{\phi_y-\phi_x}}\,. \tag{14}$$

A return on capital r, surplus of job S, and wage w can be solved by substituting equations (13) and (14) into equations (1) and (11):

$$r = \Phi_1\,((1+t)\,p)^{-\frac{1-\phi_y}{\phi_y-\phi_x}}\,, \tag{15}$$

$$S = \Phi_2\,((1+t)\,p)^{\frac{\phi_y}{\phi_y-\phi_x}}\,, \tag{16}$$

$$w = (1-\beta)\,\Phi_2\,((1+t)\,p)^{\frac{\phi_y}{\phi_y-\phi_x}}\,, \tag{17}$$

where $\Phi_1 = \left[\left(\frac{1-\phi_y}{1-\phi_x}\right)^{\left(1-\phi_y\right)(1-\phi_x)} \frac{\phi_y^{\phi_y(1-\phi_x)}}{\phi_x^{\phi_x\left(1-\phi_y\right)}}\right]^{\frac{1}{\phi_y-\phi_x}}$ and $\Phi_2 = \left[\left(\frac{\phi_x}{\phi_y}\right)^{\phi_x\phi_y} \frac{(1-\phi_x)^{\phi_y(1-\phi_x)}}{\left(1-\phi_y\right)^{\phi_x\left(1-\phi_y\right)}}\right]^{\frac{1}{\phi_y-\phi_x}}$ are constants that depend on ϕ_x and ϕ_y only.

Next, I find the equilibrium total vacancies V and total employment E from substituting equation (3) into equation (4):

$$V = \left(\frac{\beta S}{f}\right)^{\frac{1}{1-\lambda}} L.$$

Substituting S from equation (11) gives the equilibrium vacancy V and the equilibrium employment:

$$V = \left[\frac{\beta\Phi_2}{f}\,((1+t)\,p)^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{1}{1-\lambda}} L, \tag{18}$$

$$E = \left[\frac{\beta\Phi_2}{f}\,((1+t)\,p)^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L. \tag{19}$$

Given k_x and k_y from equations (13) and (14), we can solve for L_x and L_y that clear the resource markets according to equations (5) and (6):

$$\begin{aligned} L_x &= \frac{k_y E - K}{k_y - k_x}, \\ L_y &= \frac{K - k_x E}{k_y - k_x}. \end{aligned}$$

Note that, as in the standard Heckscher-Ohlin model, k_x and k_y are independent of initial endowments but L_x and L_y depend on the relative effective endowment ratio K/E.

The country's budget constraint is

$$(1+t)\,pX^s + Y^s - ((1+t)\,px_V + y_V)\,V + tp\left(X^d + X_f - X^s\right) = (1+t)\,pX^d + Y^d.$$

The left-hand side shows sources of income. The first two terms are producer income, given producers sell goods X^s and Y^s at domestic price $(1+t)\,p$ and 1, respectively. The next term is the vacancy cost that arises from posting V vacancies. The last term is tariff revenue if the country has excess demand in X (i.e., $X^d - X^s > 0$), or import subsidy cost if the country has excess supply in X (i.e., $X^d - X^s < 0$). The tariff revenue is redistributed back to the representative household in a lump-sum transfer. The right-hand side shows how income is spent on consumptions according to the demand functions X^d and Y^d.

Rewriting the budget constraint leads to

$$pX^c\,(p,t) + Y^c\,(p,t) = pX^d\,(p,t) + Y^d\,(p,t)\,, \tag{20}$$

where $X^c\,(p,t) \equiv X^s\,(p,t) - x_V\,(p,t)\,V\,(p,t)$ is a net output of good X and $Y^c\,(p,t) = Y^s\,(p,t) - y_V\,(p,t)\,V\,(p,t)$ is a net output of good Y. I emphasize the endogeneity of these variables by writing them as functions of the relative price p and the trade policy t. There are two important observations. First, the country's budget constraint is facing the free trade world price ratio. The trade policy causes a transfer between the government and consumers but does not change the total income. Second, because the country is small, the entire tax burden goes to domestic households.

The country's net income $I\,(t)$ is defined as the left hand side of equation (20):

$$\begin{aligned} I\,(p,t) &= pX^c\,(p,t) + Y^c\,(p,t) \\ &= \left[1 + \frac{t}{1+t}\left(\frac{1-\phi_y}{\phi_y - \phi_x}\right)\right] r\,(p,t)\,K + \left[1 - \frac{t}{1+t}\left(\frac{\phi_y}{\phi_y - \phi_x}\frac{1}{1-\beta} - \alpha\frac{\beta}{1-\beta}\right)\right] w\,(p,t)\,E\,(p,t)\,. \end{aligned} \tag{21}$$

The net income also can be written in terms of returns to factors of productions, capital income and labor income. Because the actual effective relative price is p, not $(1+t)\,p$, returns $r\,(t)$ and $w\,(t)$ inaccurately measure the returns on the endowment. Therefore, they are slightly discounted, as shown in equation (21). For convenience, I henceforth define a function $h\,(z) \equiv \frac{\phi_y}{\phi_y - \phi_x}\left(\frac{1}{1-z}\right) - \alpha\frac{z}{1-z}$.

The last part is to determine demand functions and aggregate welfare from a utility maximization problem.

The representative household maximizes his utility subject to his budget constraint:

$$
\begin{aligned}
\underset{X^d,Y^d}{Max} \qquad & U(X,Y) &= \frac{X^{\alpha}Y^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \\
\text{s.t.} \qquad & & I(p,t) = (1+t)\,pX^d + Y^d,
\end{aligned}
$$

where $I(p,t)$ is from equation (21). The demand for each good is

$$
X^d(p,t) = \frac{1}{(\alpha + (1-\alpha)(1+t))} \times \frac{\alpha I(p,t)}{p}, \tag{22}
$$

$$
Y^d(p,t) = \frac{(1+t)}{(\alpha + (1-\alpha)(1+t))} \times (1-\alpha)\, I(p,t)\,. \tag{23}
$$

Without the trade policy t, the demand functions are simplified to the standard demand functions from a Cobb-Douglas utility function. Substituting demand into the utility function yields a welfare function as a function of the relative price p and trade trade policy t:

$$
\begin{aligned}
U(p,t) &= U\left(X^d(p,t)\,, Y^d(p,t)\right) \\
&= I(p,t) \times P(p,t) \qquad (24)
\end{aligned}
$$

where $P(p,t) = \frac{(1+t)^{1-\alpha}}{(\alpha+(1-\alpha)(1+t))p^{\alpha}}$ is the inverted price level that captures the impact of the price distortion on the country's social welfare. The function $P(p,t)$ is strictly concave in t, and, for any given p, is maximized at $t = 0$. Tariffs always distort domestic prices perceived by the representative household and make consumption choice suboptimal.

4.2 Social-Planner's Problem

This section analyzes a benchmark constrained-efficient equilibrium by investigating how a benevolent social planner directly allocates endowments to maximize the country's welfare, when the planner is subject to the same search frictions.

I begin this section with the definition of a constrained-efficient equilibrium. It is an allocation of endowments in which a government, subjected to search frictions in a labor market, is allowed to directly assign the amount of resources used in each sector to maximize social welfare.

Definition: *A constrained-efficient equilibrium is the vector of consumptions, final outputs, allocations of endowments, intermediate inputs, and vacancy level,* $\{X^d, Y^d, X^s, Y^s, K_x, K_y, L_x, L_y, x_V, y_V, V\}$, *that maximizes social welfare, given the world price ratio and initial endowments* $\{p, K, L\}$, *and that satisfies the (i) production technologies, (ii) matching technology, and (iii) endowment constraints.*

The social planner's welfare maximization problem can be written formally as

$$\begin{aligned}
\text{Gov} \quad & \underset{X^d, Y^d, K_x, K_y, L_x, L_y, x_V, y_V, V}{Max} \quad U\left(X^d, Y^d\right) && = \frac{\left(X^d\right)^{\alpha}\left(Y^d\right)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \\
\text{s.t.} \quad & L_x + L_y && = V^{\lambda} L^{\lambda} \\
& K_x + K_y && = K \\
& x_V^{\alpha} y_V^{1-\alpha} && = \alpha^{\alpha}(1-\alpha)^{1-\alpha} f \\
& pX^d + Y^d && = pK_x^{\phi_x} L_x^{1-\phi_x} + K_y^{\phi_y} L_y^{1-\phi_y} - \left(px_V + y_V\right) V.
\end{aligned}$$

The Lagrangian equation associated with the welfare maximization problem is

$$\begin{aligned}
\mathscr{L} \quad = \quad & \frac{\left(X^d\right)^{\alpha}\left(Y^d\right)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} + \mu_1\left(V^{\lambda} L^{\lambda} - L_x - L_y\right) \\
& + \mu_2\left(K - K_x - K_y\right) + \mu_3\left(x_V^{\alpha} y_V^{1-\alpha} - \alpha^{\alpha}(1-\alpha)^{1-\alpha} f\right) \\
& + \mu_4\left(pK_x^{\phi_x} L_x^{1-\phi_x} + K_y^{\phi_y} L_y^{1-\phi_y} - \left(px_V + y_V\right) V - pX^d + Y^d\right).
\end{aligned}$$

The first-order conditions are

$$[V] \quad \mu_1 \lambda V^{\lambda-1} L^{1-\lambda} - \left(px_f + y_f\right) = 0 \tag{25}$$

$$\left[X^d\right] \quad \left(\frac{\alpha}{1-\alpha}\frac{Y^d}{X^d}\right)^{1-\alpha} - p\mu_4 = 0 \tag{26}$$

$$\left[Y^d\right] \quad \left(\frac{\alpha}{1-\alpha}\frac{Y^d}{X^d}\right)^{-\alpha} - \mu_4 = 0 \tag{27}$$

$$[L_x] \quad \left(1-\phi_x\right) pK_x^{\phi_x} L_x^{\phi_x} - \mu_1 = 0 \tag{28}$$

$$[L_y] \quad \left(1-\phi_y\right) K_y^{\phi_y} L_y^{\phi_y} - \mu_1 = 0 \tag{29}$$

$$[K_x] \quad \phi_x p_x K_x^{\phi_x-1} L_x^{1-\phi_x} - \mu_2 = 0 \tag{30}$$

$$[K_y] \quad \phi_y p_y K_y^{\phi_y-1} L_y^{1-\phi_y} - \mu_2 = 0 \tag{31}$$

$$[x_V] \quad -pV + \mu_3 \alpha x_V^{\alpha-1} y_V^{1-\alpha} = 0 \tag{32}$$

$$[y_V] \quad -V + \mu_3(1-\alpha) x_V^{\alpha} y_V^{-\alpha} = 0 \tag{33}$$

and constraints are

$$L_x + L_y = V^{\lambda} L^{\lambda} \tag{34}$$

$$K_x + K_y = K \tag{35}$$

$$x_V^{\alpha} y_V^{1-\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} f \tag{36}$$

$$pK_x^{\phi_x} L_x^{1-\phi_x} + K_y^{\phi_y} L_y^{1-\phi_y} - \left(px_V + y_V\right) V = pX^d + Y^d. \tag{37}$$

From equations (32), (33), and (36), the minimum cost of delivering one vacancy is

$$px_V^* + y_V^* = p^\alpha f.$$

Combining equations (28), (29), (30), and (31) and using $k_i = K_i/L_i$ gives

$$\begin{aligned} (1-\phi_x)\, pk_x^{\phi_x} &= \left(1-\phi_y\right) k_y^{\phi_y}, \\ \phi_x pk_x^{\phi_x - 1} &= \phi_y k_y^{\phi_y - 1}. \end{aligned}$$

These equations mean that the marginal product of labor and the marginal product of capital are equalized across sectors. This turns out to be the same expression as in equations (1) and (11) when $t = 0$. These two equation gives k_x and k_y that are identical to k_x and k_y from equations (13) and (14) in the competitive equilibrium. We can solve for the Lagrange multiplier μ_1 as

$$\mu_1 = \Phi_2 p_x^{\frac{\phi_y}{\phi_y - \phi_x}} = S|_{t=0}\,. \tag{38}$$

The Lagrange multiplier μ_1 represents a shadow price of having one more job, which is equal to S, the surplus of each job in the competitive equilibrium when $t = 0$.

Substituting $\mu_1 = S$ into equation 25 leads to

$$\lambda SE = p^\alpha f V. \tag{39}$$

The production set in the social planner's problem can be expressed by a well-defined concave set $\left\{(X^c, Y^c, V) \in \mathbb{R}^3\right\}$ that represents the amounts of net output of good X, net output of good Y, and vacancy V. The production set is called a *consumption possibility frontier (CPF)* instead of a traditional production possibility frontier (PPF) because it shows net outputs, which are available for consumptions, instead of the gross outputs, which include intermediate goods used in the production of vacancies.

An example of the CPF (given particular values of $\alpha, \lambda, f, \phi_x, \phi_y, K, L$) is illustrated in Figure 2. Note that the construction of the CPF is independent of the wage bargaining power β which only plays a role in a competitive equilibrium. Given the world price of the final goods (p for good X and 1 for good Y), the price of vacancy is p^α according to equation 9. A price plane is defined as a set $\left\{(X^c, Y^c, V) \in \mathbb{R}^3 \mid pX^c + Y^c - p^\alpha V = \bar{a}, \exists \bar{a} \in \mathbb{R}\right\}$ and the social planner chooses a production point on the CPF where the price plane is tangent to the CPF.

For a given vacancy V, a 2-D slice of the 3-D CPF is comparable to a standard PPF with fixed capital and labor. The only difference is that, as a production point moves along the 2-D slice of the 3-D CPF, the amount of net output X increases because of two reasons: (i) more endowments are reallocated to the production of good X, and (ii) the production of vacancies use more intermediate good y_v and less intermediate good x_v.

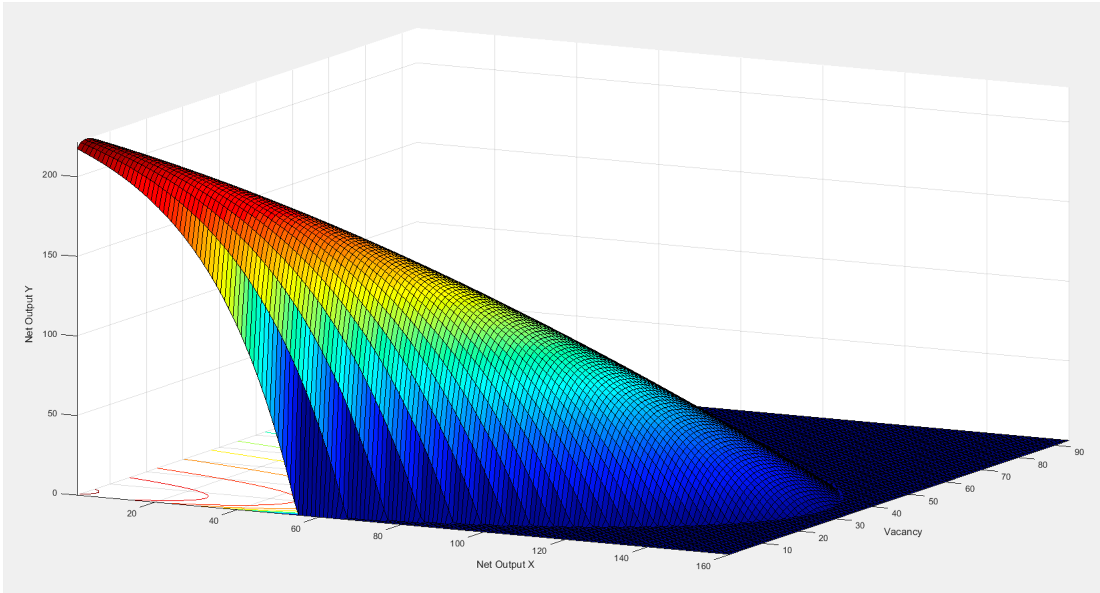

Figure 2: A 3-D consumption possibility frontier (CPF) showing a combination of net outputs X^c and Y^c for any vacancy V.

For example, in Figure 2, if the social planner wants to maximize the production of the capital-intensive good Y, it only posts around 10 vacancies to hire just enough workers to use capital. However, if the social planner wants to maximize the production of the labor-intensive good X, it has to post around 35 vacancies to hire more workers, since the production of the labor-intensive good requires more labor. Although posting more vacancies increases the *gross* production of good X, the additional output does not cover the input requirement for posting a vacancy and the *net* output declines.

4.3 Efficiency of Competitive Equilibrium

In this section, I characterize a condition under which the competitive equilibrium solved in Section 4.1 reaches the same welfare level as the constrained-efficient equilibrium.

The equilibrium conditions for the competitive equilibrium under free trade described in Section 4.1 are equivalent to the conditions for the constrained-efficient equilibrium in Section 4.2. The competitive equilibrium coincides with the constrained-efficient equilibrium when equation (10) of the competitive equilibrium is the same as equation (39) of the constrained-efficient equilibrium. The sufficient condition is summarized in Lemma 1.

Lemma 1. *A competitive equilibrium under free trade coincides with a constrained-efficient equilibrium if and only if $\beta = \lambda$.*

Proof. Straightforward from equations (10), (38), and (39). □

As a result of my model simplification, I can derive the explicit sufficient condition of labor market efficiency that cannot be shown by the original model in Dutt et al. (2009). Generally, in a frictional labor market there are two sources of labor market inefficiency. First, total vacancy can be too low because of the hold-up problem. With incomplete contracts, the upfront vacancy cost is not included in ex-post wage bargaining. As firms get only a fraction β of the surplus from a filled vacancy, they have a disincentive to post the efficient amount of vacancies, because the share of the surplus that firms receive may not be enough to cover the vacancy cost. Second, firms can post too many vacancies because they do not recognize the congestion externality on other firms. Firms post additional vacancies as long as it is profitable to do so. However, they do not consider the fact that posting another vacancy reduces other firms' expected profit through a lowered probability of any given vacancy being filled.

Constrained efficiency arises only when the two sources of inefficiency perfectly offset each other. Lemma 1 is consistent with the Hosios efficiency condition, which states the sufficient condition for labor market efficiency: the labor market is efficient when the bargaining power of firms is equal to the elasticity of the job-matching function with respect to vacancy level (Hosios, 1990). In this paper, I assume a constant elasticity of a matching function with respect to vacancy. Thus, the Hosios efficiency condition is reduced to $\beta = \lambda$. When $\beta < \lambda$, the hold-up problem dominates the congestion externality problem and the equilibrium employment is inefficiently low (unemployment is inefficiently high). When $\beta > \lambda$, the congestion externality problem dominates the hold-up problem and equilibrium employment is inefficiently high (unemployment is inefficiently low).

Definition. Labor market inefficiency is defined as $\lambda - \beta$.

I define labor market inefficiency as the difference between the elasticity of the job-matching function with respect to vacancy level and the bargaining power of firms. The labor market is efficient when $\lambda - \beta = 0$. Unemployment is inefficiently high when $\lambda - \beta > 0$, and it is inefficiently low when $\lambda - \beta < 0$.

In Figure 3, a 2-D CPF is constructed as a projection of the 3-D CPF in Figure 2 onto a $X^c \times Y^c$ plane by tracing each possible production point associated with one relative world price in a competitive equilibrium. In other words, for a given world price, vacancy V is created according to equation (18) when $t = 0$; the equilibrium net outputs X^c and Y^c associated with this vacancy V are recorded as one point on the 2-D CPF graph. Therefore, for any relative price, a point on a 2-D CPF where a price line is tangent to the 2-D CPF represents a competitive equilibrium associated with the price and exhibits the net outputs in that equilibrium immediately. Varying the bargaining power β does not affect the 3-D CPF but it alter the equilibrium vacancy V according to equation (18).

Figure 3 illustrates the 2-D CPF in three situations where the parameter λ is fixed, but the parameter β varies such that $\beta = \lambda$, $\beta < \lambda$, and $\beta > \lambda$. When $\beta = \lambda$, the competitive equilibrium corresponds with the social planner under free trade, and the social planner always picks the point where the price plane is tangent to the (fixed) 3-D CPF. When β is dropped below λ or raised

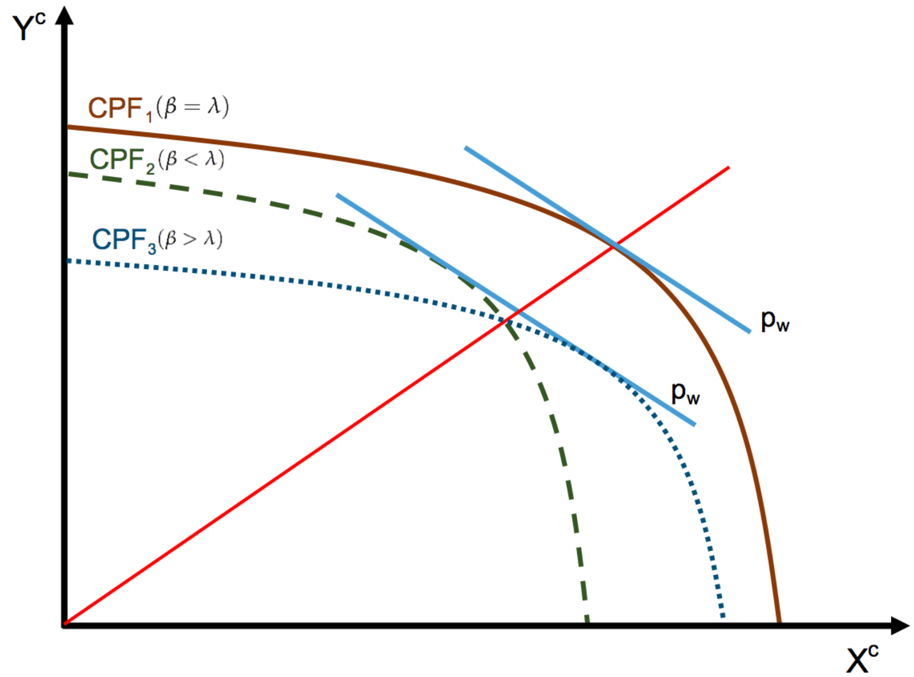

Figure 3: A combination of net outputs X^c and Y^c when vacancy V adjusts according to a relative price.

above λ, the competitive equilibrium will no longer correspond to the social planner (who would continue to pick the production point where the price plane is tangent to the (fixed) 3-D CPF), and the competitive equilibrium will pick a point where the 2-D projection is tangent to the price line conditional on a provision of vacancies that is either higher than the social planner would have chosen ($\beta > \lambda$) or lower than the social planner would have chosen ($\beta < \lambda$).

Suppose this country is facing a world price ratio p_w. CPF_1, the solid-brown curve represents the CPF in the case of an efficient labor market when $\beta = \lambda$. According to Lemma 1, when the labor market is efficient, the country's net income is maximized. The country produces at the point where the world price is tangent to the CPF_1 curve. The slope of the solid red line represents the relative net supply Y^c/X^c at the world price when the labor market is efficient.

In the case where $\beta < \lambda$, the bargaining power of firms is relatively low and the hold-up problem dominates the congestion externality, and in this case the labor market underproduces vacancies. This inefficiency leads to fewer available workers in the economy and employment is inefficiently low. Therefore, CPF_2, the green-dashed curve, lies inside the CPF_1 curve, because the country loses some potential workers who would be employed if the labor market was efficient. The economy would produce more outputs if it hired additional workers. At the same world price, p_w, the new production point, where the world price ratio is tangent to the CPF_2 curve, is to the left of the red-dashed line. This result shows the relative supply of output, Y^c/X^c, increases. The country produces more of good Y and fewer of good X.

In the case where $\beta > \lambda$, where the bargaining power is relatively high, CPF_3, the blue-dotted curve also lies inside the CPF_1 curve, but for a different reason than the previous situation. When $\beta > \lambda$, the labor market generates inefficiently high employment and results in overproduction of vacancies: firms post too many vacancies without realizing that the marginal social benefit of an additional vacancy in terms of job surplus is less than the posting cost. Therefore, despite having high employment and producing more output, a large amount of output is used to pay the production-of-vacancy cost, which decreases the country's net income. The country also produces more of good X and less of good Y, as the production point where the world price is tangent to the CPF_2 curve lies to the right of the solid red line.

5 Comparative Statics

In this section, I focus on the competitive equilibrium under free trade (where $t = 0$). Throughout this section, I slightly abuse notation by writing variables as a function of a relative price only, i.e., the trade policy t is dropped. I analyze the effect of a price change from the autarky price ratio p_A to the new world price ratio p_w.

5.1 Factor Prices and Allocation of Endowments

This subsection investigates how the competitive equilibrium adjusts when a country faces a price change from the market-clearing price in autarky to the new fixed world prices.

First I calculate real wages and real rental prices from equations (15) and (17):

$$\frac{r(p)}{p^{\alpha}} = \Phi_1 p^{-\frac{1-\phi_y}{\phi_y-\phi_x}-\alpha},$$
$$\frac{w(p)}{p^{\alpha}} = (1-\beta)\,\Phi_2 p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}.$$

According to equation (19), equilibrium employment is

$$E(p) = \left[\frac{\beta\Phi_2}{f} p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

We can easily see that the real return on capital is decreasing in p, but the real wage and employment are increasing in p.[8] Without loss of generality, I illustrate the main mechanism of the model by showing the case in which there is an increase in p. An increase in p improves the value of the labor-intensive product, raises marginal product of labor, and creates demand for labor in the labor-intensive sector. Firms intensively seek more workers: more vacancies in the labor-intensive sector are posted and more workers are employed. The opposite occurs in the capital-intensive sector: the value of capital intensive goods shrinks, marginal product of labor decreases, and so

[8] $\phi_y/(\phi_y-\phi_x) > 1 > \alpha$.

does demand for labor. Fewer jobs are available in the capital-intensive sector. Because the labor-intensive sector is where the demand for labor is largely created, the increase in labor demand in the labor-intensive sector dominates the decrease in labor demand in the capital-intensive sector. As a result, total employment increases. In addition, the decrease in the rental price reflects the fact that capital is less important in production.

This effect on employment is also exhibited in Dutt et al. (2009). Because this paper simplifies Dutt et al. (2009) by removing the time dimension, the elasticity of employment with respect to the relative price can be explicitly calculated and is summarized in Lemma 2.

Lemma 2. *The elasticity of employment with respect to the relative price p is*

$$\frac{dlog\left(E\left(p\right)\right)}{dlog\left(p\right)}=\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\left(\frac{\lambda}{1-\lambda}\right). \tag{40}$$

Proof. From equation (19).

□

The effect of a price change on employment comes through the job surplus that induces the job creation process. The first term, $\frac{\phi_y}{\phi_y-\phi_x}-\alpha$, represents the elasticity of job surplus with respect to the relative price. The effect is large when the capital intensity in the two sectors is similar. The term α comes from how the production-of-vacancy cost substitutes intermediate good X for intermediate good Y. The second term, $\frac{\lambda}{1-\lambda}$, represent the elasticity of employment with respect to the surplus. This effect is increasing in the elasticity of employment with respect to the vacancy.

Next, I focus on the allocation of endowments. To understand an expansion or contraction of production in each sector, I calculate the share of labor and capital used in each sector:

$$\begin{aligned}
L_x/E &= \frac{\left(1-\phi_x\right)\phi_y}{\phi_y-\phi_x}\left(1-K/k_yE\right),\\
L_y/E &= \frac{\left(1-\phi_y\right)\phi_x}{\phi_y-\phi_x}\left(K/k_xE-1\right),\\
K_x/K &= \frac{\left(1-\phi_y\right)\phi_x}{\phi_y-\phi_x}\left(k_yE/K-1\right),\\
K_y/K &= \frac{\left(1-\phi_x\right)\phi_y}{\phi_y-\phi_x}\left(1-k_xE/K\right).
\end{aligned}$$

Recall that k_iE is increasing in p. When p increases after trade liberalization, equations (15) and (17) show that capital becomes relatively cheaper than labor. Equations (13) and (14) confirm that firms in all sectors use more capital per worker. A price change affects L_x and L_y through two channels. First, as a standard price effect, firms use more capital per worker; k_x and k_y increase and causes L_x and L_y to be re-allocated to clear a capital market. Secondly and importantly, a

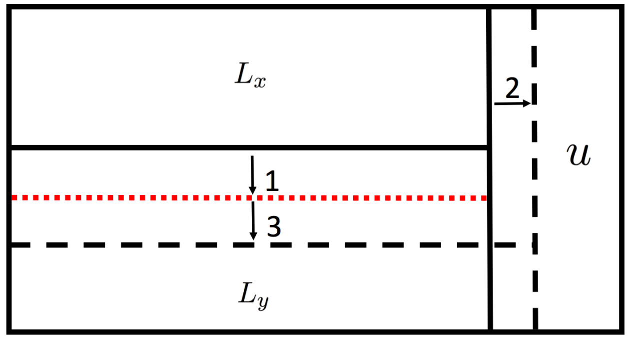

Figure 4: The adjustment in the labor market due to a price change.

price ratio changes an aggregate employment level E; this effect acts as a change in a relative endowment K/E and causes another adjustment, as in the Rybczynski theorem. An effect on K_i is quite straightforward because the only effect comes through k_x and k_y. As a result, both K_x and L_x increase and sector X — the labor-intensive sector — expands, while both K_y and L_y decrease and sector Y shrinks.

The adjustment in the labor market is illustrated in Figure 4. The box represents total population (L) in the economy, which is categorized initially into 3 groups by the solid black lines: workers in sector X (L_x), workers in sector $Y(L_y)$, and involuntarily unemployed workers (u). The left area, which combines L_x and L_y, represents the total employment E. Again, I continue to assume that a price ratio moves in favor of good X (p increases). An effect of the price change is decomposed into two parts. The first part is a standard adjustment due to a price change in a standard Heckscher-Ohlin model. Holding the amount of employed workers fixed, the price change relocates workers in sector Y to sector X. Arrow 1 captures this effect. Aggregate employment is unchanged (as the vertical solid black line does not move) but the reallocation shifts the horizontal solid black line down to the horizontal red short-dashed line. The second effect is a novel endowment effect. The labor-intensive sector searches intensively for workers, posts more vacancies and raises aggregate employment, despite the fact that the capital-intensive sector employs fewer workers. This endowment effect is captured by arrow 2. The total employment increases as the vertical solid black line shifts to the vertical black long-dashed line on the right. This effect causes another reallocation between two sectors; as in the Rybczynski theorem, when the total active labor force increases, the labor-intensive sector expands and the capital-intensive sector shifts down to the horizontal black long-dashed line, which shows that within the area of employed workers, more workers are in the labor-intensive sector.

I summarize the findings in Proposition 1:

Proposition 1. *When a relatively labor (capital)-abundant country moves from autarky to free trade:*

1. *Real wages increase (decrease) and real returns to capital decrease (increase)*
2. *Employment increases (decreases)*
3. *A labor-intensive sector expands (contracts) while a capital-intensive sector contracts (expands)*

Proof. See Appendix. □

This first result is consistent with the Stolper-Samuelson theorem, in which a relatively abundant factor gives higher real returns, while the other factor gives lower real returns. The novel result of this paper is the effect on employment. There is no fluctuation in employment in full-employment models. However, when I carefully model how a labor market actually works, I can analyze the motivation behind why firms hire workers. Intuitively, when the price of labor-intensive goods increases, firms want to use less capital and have more incentive to search for a worker and vice versa. The change in equilibrium employment also indirectly affects the expansion or contraction of a sector as if the total labor increases according to the Rybczynski theorem.

How does aggregate income respond to a price change?

The aggregate income can be written in terms of returns to factors according to equation (21):

$$I\left(p\right)=r\left(p\right)K+w\left(p\right)E\left(p\right).$$

The percentage change in aggregate income due to a one-percent change in price p can be written as

$$\frac{\triangle I}{I} = \left(\frac{rK}{rK+wE}\right)\frac{\triangle r}{r}+\left(\frac{wE}{rK+wE}\right)\left(\frac{\triangle w}{w}+\frac{\triangle E}{E}\right).$$

The total change in the aggregate income is decomposed into the change in capital income and the change in labor income. The main feature of this model is that the two effects in the change in labor income — the effect through the wages and the effect through employment — coexist. In the traditional Heckscher-Ohlin model, there is only the effect through the wages; the effect through employment is absent because of the assumption of full employment. In contrast, the effect through wages is absent in Brecher (1974a, 1974b) and Matschke (2006), in which the wage is exogenously bounded from below at the minimum wage. In Helpman and Itskhoki (2010), the two effects coexist but they move in the opposite direction; they completely offset each other, as the expected wage is fixed by an exogenous wage rate in a numeraire sector.

In the model, according to equations (15), (17), and (19), a percentage change of the aggregate income due to one percentage change of price p is

$$\frac{\triangle I}{I}=\left(\frac{rK}{rK+wE}\right)\underbrace{\left(-\frac{1-\phi_y}{\phi_y-\phi_x}\right)}_{\frac{\triangle r}{r}}+\left(\frac{wE}{rK+wE}\right)\left(\underbrace{\left(\frac{\phi_y}{\phi_y-\phi_x}\right)}_{\frac{\triangle w}{w}}+\underbrace{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\left(\frac{\lambda}{1-\lambda}\right)}_{\frac{\triangle E}{E}}\right).$$

In a traditional Heckscher-Ohlin model without unemployment, the returns to endowments w_T

and r_T adjust such that firms have zero profit. The conditions are $p = r_T k_x (w_T, r_T) + w_T l_x (w_T, r_T)$ and $1 = r_T k_y (w_T, r_T) + w_T l_y (w_T, r_T)$, where k_i (l_i) is a demand for capital (labor) to produce one unit of output. The equilibrium factor prices are $r_T = \Phi_1 p^{-\frac{1-\phi_y}{\phi_y - \phi_x}} = r$ and $w_T = \Phi_2 p^{\frac{\phi_y}{\phi_y - \phi_x}} = S = w/(1-\beta)$. The percentage of income with respect to one percentage change of price p is

$$\frac{\triangle I}{I} = \left(\frac{rK}{rK+wE}\right) \underbrace{\left(-\frac{1-\phi_y}{\phi_y - \phi_x}\right)}_{\frac{\triangle r}{r}} + \left(\frac{wE}{rK+wE}\right) \underbrace{\left(\frac{\phi_y}{\phi_y - \phi_x}\right)}_{\frac{\triangle w}{w}}.$$

The elasticities of real factor prices with respect to a price ratio in both a standard Heckscher-Ohlin model and my model are the same, but the labor income in this paper is more volatile because of the additional effect from employment. Therefore, when a capital-abundant country opens to trade, this model predicts that its labor income drops more than what a traditional Heckscher-Ohlin model predicts due to an additional decrease in the amount of labor employed.

I conclude the effect on the real factor income in Proposition 2:

Proposition 2. *The elasticities of real factor prices with respect to a price ratio in the Heckscher-Ohlin model with labor market frictions are identical to those predicted by the traditional Heckscher-Ohlin model. The employment effects reinforce the real income effects and make the labor income more volatile than in the standard Heckscher-Ohlin model.*

Proposition 2 also highlights that quantifying the distributional impacts of trade policy should take labor market frictions into account. Abstracting from the employment effects underpredicts the variation in labor income.

5.2 Welfare Gains From Trade

This subsection explores how introducing a frictional labor market into a Heckscher-Ohlin model shapes our understanding of welfare gains from trade. To begin with, I identify the autarky price ratio, the market-clearing price in a closed economy. Later I compare welfare at the autarky price in a closed economy and welfare at a new world price in an open economy under free trade.

In a closed economy, a domestic price ratio clears the output markets. According to Walras' law, I can identify the autarky price by using the market-clearing condition for good X. The demand for good X is from the representative household's consumption and the production of vacancy costs. The supply is solely from domestic production. Therefore, the value of excess demand of good X, denoted by $ED^X(p)$, is

$$p \times ED^X(p) \quad = \quad \alpha \left[pX^c(p) + Y^c(p)\right] - pX^c(p). \tag{41}$$

The autarky price p_A satisfies $ED^X(p_A) = 0$ and can be solved explicitly as

$$p_A = \left[\frac{1 - \left(\alpha\phi_x + (1-\alpha)\,\phi_y\right)}{\alpha\phi_x + (1-\alpha)\,\phi_y} \times \frac{\Phi_1}{\Phi_2} \times \left(\frac{f}{\beta\Phi_2}\right)^{\frac{\lambda}{1-\lambda}} \times \frac{K}{L} \right]^{\frac{1}{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}}}. \tag{42}$$

As in a traditional Heckscher-Ohlin model, the autarky price p_A is increasing in an endowment ratio K/L; the capital-labor ratio is a source of comparative advantage. When the country has access to international trade, if the country faces a world price $p_w > p_A$, it is relatively labor-abundant compared to the world and has a comparative advantage in good X. On the other hand, if it faces a world price $p_w < p_A$, it is relatively capital-abundant and has a comparative advantage in good Y.

According to equation (24), the welfare function in free trade given an arbitrary world price p_w is

$$U(p_w) = \frac{r(p_w)}{p_w^{\alpha}}K + \frac{w(p_w)}{p_w^{\alpha}}E(p_w). \tag{43}$$

The welfare function is strictly convex in the world price p_w and the welfare converges to infinity as the price converges to either zero or infinity.[9]

Next, I establish the sufficient statistic of welfare change due to price change.

Proposition 3. *The welfare change due to price change is*

$$dlog\left(U(p_w)\right) = \left[\gamma + \left(\frac{\lambda-\beta}{(1-\lambda)(1-\beta)}\right)\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{w(p_w)\,E(p_w)}{I(p_w)}\right] dlog\left(p_w\right),$$

where $\gamma = Export_x(p_W)/I(p_w)$ is the share of total output on the net export of good X.

The idea behind Proposition 3 is intuitive. In a simple case that the labor market is efficient, the result is simplified to $dlog(U(p_w)) = \gamma dlog(p_w)$. A country's welfare is higher when the world price of its exports increases; a country that exports good X ($\gamma > 0$) wants the price of good X to increases ($dlog(p_w) > 0$), and a country that exports good Y ($\gamma < 0$) wants the price of good Y to increases ($dlog(p_w) < 0$)

The welfare prediction includes how labor market inefficiency affects the magnitude of welfare changes. Corollary 1 emphasizes this novel result.

Corollary 1. *Given the same observed trade share, a labor-abundant (capital-abundant) country with inefficiently high (low) unemployment has substantially larger gains from trade than a labor-abundant (capital-abundant) country with efficient unemployment.*

An interesting result is that welfare gains from trade could still be quite large even for small trade share. Arkolakis et al. (2012) provide the condition summarizing the welfare gains from reduction of trade costs as

$$\widehat{Welfare} = \left(\widehat{1-|\gamma|}\right)^{1/\epsilon},$$

[9] $d^2U(p_w)/dp_w^2 > 0$, $lim_{p_w\to 0^+}U(p_w) = \infty$ and $lim_{p_w\to\infty}U(p_w) = \infty$.

where $1 - |\gamma|$ is the share of domestic expenditure, $\epsilon < 0$ is an elasticity of imports with respect to variable trade costs, and $\widehat{z} \equiv z_2/z_1$ is the change in variable z from the initial z_1 to the new z_2.

The main difference between this welfare prediction and the one in Arkolakis et al. (2012) is that the welfare formula in this paper includes the possibility of labor market inefficiency that can either reinforce or reduce the magnitude of welfare changes. Unlike Arkolakis et al. (2012), although trade share is small, the country can have large gains from trade if labor market inefficiency reinforces the welfare effect of price change. A labor-abundant country with inefficiently high unemployment and a capital-abundant country with inefficiently low unemployment may experience much bigger welfare gains from trade than the formula in Arkolakis et al. (2012) would indicate.

However, if labor market inefficiency weakens the welfare effect of price change, the country may have welfare losses from trade. To show a possibility of welfare loss from trade, I evaluate the elasticity of welfare change due to price change at the autarky price.

Corollary 2. *The elasticity of welfare change due to price change evaluated at $p_w = p_A$ is*

$$\left.\frac{dlog\left(U\left(p_w\right)\right)}{dlog\left(p_w\right)}\right|_{p_w=p_A} = \frac{\lambda-\beta}{\left(1-\lambda\right)\left(1-\beta\right)}\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{w\left(p_A\right)E\left(p_A\right)}{I\left(p_A\right)}.$$

In the standard literature, welfare gains from trade can be illustrated by the fact that for any world price ratio $p_w \neq p_A$, welfare under free trade is larger than welfare under autarky, $U\left(p_w\right) > U\left(p_A\right)$. In other words, the welfare function is minimized at the autarky price p_A. The above expression shows that the welfare function in this model reaches its global minimum at the autarky price ($dU\left(p_w\right)/dp_w|_{p_w=p_A} = 0$) only if $\lambda = \beta$, or when the labor market is efficient. The bottom line is that if $\beta \neq \lambda$, there exists a world price ratio p such that $U\left(p\right) < U\left(p_A\right)$. A small price deviation from the autarky price can hurt the welfare of the country if a labor market inefficiency exists. That is, a capital-abundant country with inefficiently high unemployment and labor-abundant country with inefficiently low unemployment may experience welfare loss from trade.

To see the mechanism, consider a country that has inefficiently low employment in autarky. A small reduction in price from the autarky price would cause job losses and put more pressure on the employment situation. An increase in return on capital is overwhelmingly dominated by a decrease in wage and employment. Initially, the country is worse off. If the price change is larger, the increase in capital income begins to dominate the loss in labor income and thus the country gains from trade. The opposite is true for a country beginning with inefficiently high employment.

I find that the country's welfare is minimized at the price p_{MinU} such that

$$p_{MinU} = \left[\frac{1-\lambda}{1-\beta}\times\frac{1-\left(\alpha\phi_x+\left(1-\alpha\right)\phi_y\right)}{\alpha\phi_x+\left(1-\alpha\right)\phi_y}\times\frac{\Phi_1}{\Phi_2}\times\left(\frac{f}{\beta\Phi_2}\right)^{\frac{\lambda}{1-\lambda}}\times\frac{K}{L}\right]^{\frac{1}{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}}}. \tag{44}$$

Proposition 4 formally concludes the finding.

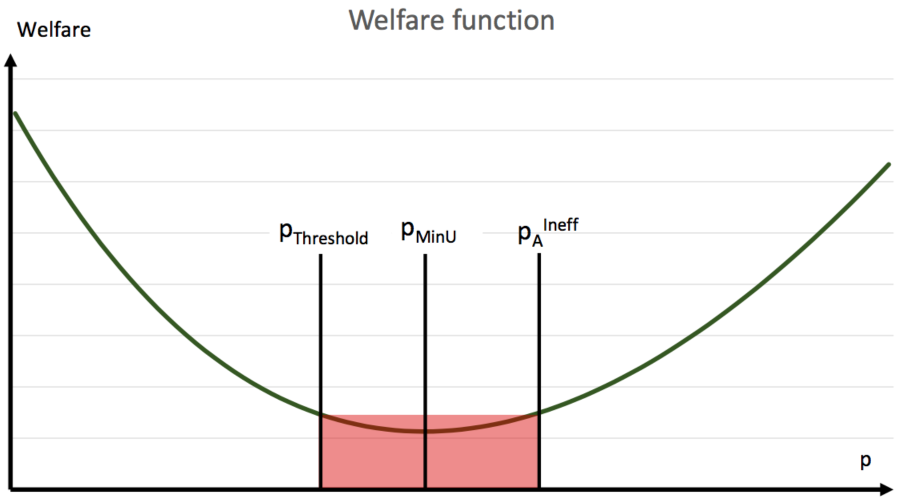

Figure 5: Welfare function.

Proposition 4. *A country's welfare is minimized at*

$$p_{MinU} = p_A \left(\frac{1-\lambda}{1-\beta} \right)^{\frac{1}{\left(\frac{\phi_y}{\phi_y - \phi_x} - \alpha \right) \frac{\lambda}{1-\lambda} + \frac{1}{\phi_y - \phi_x}}}.$$

Proof. From equations (42) and (44).

□

According to a traditional Heckscher-Ohlin model, a country always gains from trade whenever the world price is different from its autarky price. In other words, a country's welfare in a traditional Heckscher-Ohlin model is minimized at its autarky price p_A. This traditional prediction is true in this model when a labor market is constrained-efficient. A constrained-efficient labor market requires $\lambda = \beta$, which implies $p_{MinU} = p_A$. However, when a labor market is inefficient, this model provides a different prediction. One reason is that when a level of employment is inefficient, a price change may push the employment level away from the efficient level and hence worsen welfare.

For example, the welfare of a country that has inefficiently low employment ($\lambda > \beta$) is minimized at $p_{MinU} < p_A$. A country facing a new world price p_w, such that $p_{MinU} < p_w < p_A$, is worse off than it is in autarky. Despite how the new world price raises the return on capital, a reduction in the wage rate and job losses cut labor income such that the net income of the country shrinks and welfare decreases. In contrast, the welfare of a country that has inefficiently high employment ($\lambda < \beta$) is minimized at $p_{MinU} > p_A$. A new world price p_w, such that $p_{MinU} > p_w > p_A$, increases employment and enlarges the labor market inefficiency.

Figure 5 illustrates an example in which a country has inefficiently low employment ($\lambda > \beta$). The curve $U(p)$ represents the welfare of the country. In this case, the welfare function reaches its minimum at p_{MinU} and the autarky price p_A is on the right of the p_{MinU}, as predicted by Proposition 4. Starting at the autarky price, a country with inefficiently low employment prefers an increase in p, because exporting labor-intensive goods helps the country to create more jobs. A reduction in p is different, because exporting capital-intensive goods raises the unemployment in a country with high unemployment. Only when the world price is below $p_{threshold}$ does the welfare bounce back to be above the welfare in autarky because the increase in capital income is large enough to offset the loss of labor income. For any world price between $p_{threshold}$ and p_A, the country incurs welfare loss from trade. This is because a fluctuation in employment amplifies a fluctuation in labor income. A change in labor income is comparable to a first-order change, while a change in capital income is comparable to a second-order change. For a small change in the price, the first-order labor-income effect dominates the second-order capital-income effect. When the price change is large, the second-order capital-income effect dominates the other effect.

Corollary 3 concludes these remarks.

Corollary 3. *A capital-abundant country with inefficiently high unemployment and a labor-abundant country with inefficiently low unemployment may experience welfare loss from trade if the world price is not sufficiently different from the country's autarky price.*

Proof. From Proposition 3.

□

An interpretation of this corollary is that a country with a labor market inefficiency may not gain from trade if a price change is unfavorable because an increase in the return to one factor does not cover the loss from a fall in returns to the other factor. Corollary 3 sheds light on how a relatively capital-abundant country with inefficiently high unemployment may have welfare loss from trade, as shown graphically in Figure 5. Thus, the country would prefer autarky rather than free trade. This result suggests that a country may use trade policies to avoid a welfare loss from trade.

There are some interesting empirically relevant cases. First, according to Corollary 1, gains from trade of developing countries, who would seem to have inefficiently high unemployment and where international trade raises the price of their labor-intensive goods, are likely to be larger than Arkolakis et al. (2012) predict. Second, according to Corollary 3, developed countries are likely to have small gains from trade or losses from trade because they would also seem to have inefficiently high unemployment and the world prices are not sufficiently different from their autarky prices.

In addition, Corollary 3 simply implies that the new world price may not be large enough to induce sizable international trade and make a country better off. In other words, a country with an inefficient labor market that is trading a sufficiently small volume will lose from trade provided it is trading in the "bad direction". Thus, I evaluate the size of trade openness at the price p_{MinU} to measure the minimum required trade openness such that an additional price change moving away from the country's autarky price would be needed in order to locally improve the country's welfare.

At price p_{MinU}, the export as a share of GDP is

$$\frac{\text{Export}(p_{MinU})}{GDP\left(p_{MinU}\right)} = \left| (\lambda - \beta) \frac{\left(1 - \alpha\phi_x - (1-\alpha)\,\phi_y\right)\left(\alpha\phi_x + (1-\alpha)\,\phi_y\right)}{\left(\phi_y - \phi_x\right)\left[(1-\lambda) + (\lambda - \beta)\left(\alpha\phi_x + (1-\alpha)\,\phi_y\right)\right]} \right|.$$

Interestingly, the presence of labor market inefficiency requires that trade openness must be sufficiently large if the world price moves in an unfavored direction. A constrained-efficient labor market does not require a minimum size of export to generate gains from trade. The larger the labor market inefficiency, the larger the requirement for trade openness.

I also compare a relative return on factors at price p_{MinU} and the autarky price p_A,

$$\frac{w\left(p_{MinU}\right)E\left(p_{MinU}\right)}{r\left(p_{MinU}\right)K} = \left(\frac{1-\lambda}{1-\beta}\right) \times \frac{w\left(p_A\right)E\left(p_A\right)}{r\left(p_A\right)K}.$$

For example, a capital abundant country with inefficiently low employment begins with a relative return on factors, $w\left(p_A\right)E\left(p_A\right) / \left[r\left(p_A\right)K\right]$. As price p decreases, the country exports capital intensive goods. Labor income diminishes but capital income increases. The new welfare is less than the welfare in autarky. The welfare starts to recover as p drops below p_{MinU}, where the relative return on factor becomes $w\left(p_{MinU}\right)E\left(p_{MinU}\right)/r\left[\left(p_{MinU}\right)K\right]$.

6 Optimal Trade Policy

Having established how a price change causes an adjustment in employment and affects welfare, I now focus on the role of trade policy. More specifically, what is the optimal trade policy and under what conditions does a non-zero trade policy improve welfare? In this section, the government is restricted to use only a trade policy. Then I characterize the properties of the optimal trade policy, and find a sufficient condition for when a non-zero trade policy can improve a country's welfare.

6.1 Optimal Trade Policy - Small Open Economy

This subsection returns to a competitive equilibrium of a small open economy taking the world price as given. I consider the limited instrument case, where the only instrument available to a benevolent government is a trade policy t. Then I characterize the optimal trade policy.

The welfare maximization problem of the benevolent government is

$$
\begin{aligned}
\text{Gov } \underset{t}{Max}\ U(p,t) &= I(p,t) \times P(p,t) \\
&= \left[\left(1+\frac{t}{1+t}\left(\frac{1-\phi_y}{\phi_y-\phi_x}\right)\right) r(p,t)\,K + \left(1-\frac{t}{1+t}h(\beta)\right) w(p,t)\,E(p,t)\right] \\
&\quad \times \frac{(1+t)^{1-\alpha}}{(\alpha+(1-\alpha)(1+t))\,p^{\alpha}}.
\end{aligned}
$$

The government faces a trade-off between country's net income and a distorted price ratio. I establish the existence and uniqueness of the optimal trade policy in Lemma 3:

Lemma 3. *The optimal trade policy t^* in sector X exists and is unique. If t^* is an interior solution $t_{min} < t^* < t_{max}$, it satisfies $dU/dt = 0$.*

Proof. See Appendix.

□

Lemma 3 ensures the existence and the uniqueness of the optimal trade policy, and is helpful for the following analysis.

To find a necessary condition of the optimal trade policy, I calculate a derivative of the welfare function with respect to t,

$$
\begin{aligned}
\frac{dlogU(p,t)}{dt} = &-\frac{(1-\phi_x)(1-\phi_y)}{(\phi_y-\phi_x)^2}\frac{t}{(1+t)^2}\frac{r(p,t)\,K}{I(p,t)} - h(\lambda)(h(\beta)-1)\frac{t}{(1+t)^2}\frac{w(p,t)\,E(p,t)}{I(p,t)} \\
&-\frac{(1-\alpha)\,\alpha}{(\alpha+(1-\alpha)(1+t))}\frac{t}{(1+t)} \\
&+\frac{(\lambda-\beta)}{(1-\lambda)(1-\beta)(1+t)^2}\left(\frac{\alpha\phi_x+(1-\alpha)\,\phi_y}{\phi_y-\phi_x}\right)\frac{w(p,t)\,E(p,t)}{I(p,t)}.
\end{aligned}
$$

A small change in t affects welfare through three channels. First, it distorts production of final goods and causes an inefficient allocation of factors. The net returns to factors are smaller than they are under free trade. Second, the trade policy distorts the representative household's consumption choice. Lastly and importantly, the only benefit of a trade policy is to move the employment level closer to the efficient level of employment. A trade policy acts as a tax on the household with the proceeds used to subsidize job creation. An optimal trade policy balances the trade-offs; it adjusts employment while it sacrifices price distortions and production distortions. Without the effect on employment, the optimal policy would simply be $t^* = 0$.

To highlight the innovation of the model, I rewrite the first order condition as

$$
\frac{dlogU(p,t)}{dt} = \frac{(\lambda-\beta)}{(1-\lambda)(1-\beta)(1+t)^2}\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{w(p,t)\,E(p,t)}{I(p,t)} - \frac{t}{(1+t)^2}A_1(p,t),
$$

where $A_1(p,t) = \frac{(1-\phi_x)(1-\phi_y)}{(\phi_y-\phi_x)^2}\frac{r(p,t)K}{I(p,t)} + h(\lambda)\frac{w(p,t)E(p,t)}{I(p,t)} + \frac{(1-\alpha)\alpha(1+t)}{(\alpha+(1-\alpha)(1+t))} > 0$. This new equation simply divides the trade-offs into two parts. The first term is the gain from manipulating employment while the second term combines all standard social welfare loss due to a price distortion and a production distortion. One innovation of this paper is to introduce the first term that captures a government's incentive to intervene in an inefficient labor market.

Next, I find a sufficient condition where free trade is optimal by evaluating the first order condition at $t = 0$:

$$\left.\frac{dlogU(p,t)}{dt}\right|_{t=0} = \frac{(\lambda-\beta)}{(\phi_y-\phi_x)}\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{w(p,0)E(p,0)}{I(p,0)}. \tag{45}$$

Observe that the derivative is zero only when $\lambda = \beta$; the welfare function is maximized at $t = 0$ when a competitive equilibrium is constrained-efficient. The intuition is straightforward. When a labor market is constrained-efficient, the economy is at its second-best equilibrium and the government has no opportunity to improve its outcome. Therefore, free trade is optimal. When $\lambda \neq \beta$, the derivative is not zero, which means a trade policy can improve welfare in the competitive equilibrium.

The following proposition concludes the main finding:

Proposition 5. *Beginning at free trade, a trade policy improves a country's welfare if $\lambda \neq \beta$, and free trade is the optimal trade policy if $\lambda = \beta$.*

Proof. From equation (45).

□

Proposition 5 delivers two main results. First, it demonstrates a labor-market motive for a trade policy. Although the government does not target unemployment directly, employment potentially matters in decision-making as an inefficiency that needs to be corrected. The government wants to move equilibrium employment towards the efficient level of employment while sacrificing non-distorted prices. Second, a constrained-efficient labor market needs no protection. The government cannot improve social welfare beyond the best possible outcome under the presence of search frictions.

Figure 6 illustrates the intuition of an effect of a trade policy on welfare in the case of inefficiently low employment. Suppose that a government decides to use an export subsidy on its exports of labor-intensive goods. By raising the relative price from p to $(1+t)p$, the country raises the current inefficiently low employment toward the efficient level of employment. According to a 3-D CPF in Figure 2, the export subsidy causes a new path of vacancy V for every price p. The 2-D CPF, which is a projection of the 3-D CPF onto an $X^c \times Y^c$ plane, shifts out as the economy creates more vacancies at every world relative price. The slope of the budget line is the world relative price p but the consumption point and the production point are distorted and are chosen as if the relative price is $(1+t)p$. In Figure 6, this shift in the CPF is sufficiently large and leads to a higher

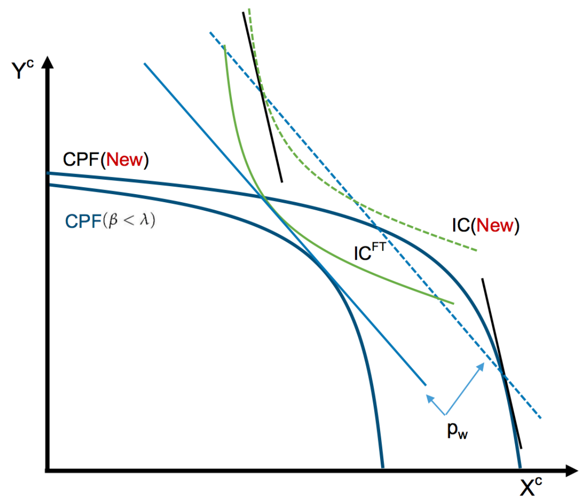

Figure 6: Price distortion causes a shift in CPF and thus the new welfare is higher than the initial welfare under free trade.

utility level as shown by a higher indifference curve. In this example, the trade policy can improve the country's welfare beyond the initial welfare under free trade.

As Figure 6 suggests, given the shifted-out CPF, a policy of free trade would be optimal, but the point is that under free trade the shifted-out CPF would not obtain and instead the original CPF would prevail. This suggests a potential commitment problem for a government that wants to intervene in this way, because once the vacancies are posted and the CPF has shifted out, the government would have an incentive to renege on its promised export subsidy. Therefore, this suggests a commitment role for a trade agreement perhaps, thought the commitment would be to higher prices of labor intensive goods for this kind of economy and the policies that deliver these higher prices.

The sign of the derivative depends on two terms: $\lambda - \beta$, which captures the magnitude of a labor-market inefficiency, and $\phi_y - \phi_x$ which describes the difference of capital intensity between two sectors when a trade policy is used in sector x.

Proposition 6. *In the Heckscher-Ohlin model, the optimal trade policy t^* in sector X has the same sign as the sign of $(\lambda - \beta) / (\phi_y - \phi_x)$.*

Proof. See Appendix. □

The sign of the optimal trade policy is independent of a country's comparative advantage. The idea of Proposition 6 is that if the sign of $(\lambda - \beta) / (\phi_y - \phi_x)$ is positive, then slightly increasing t locally raises welfare. Hence, the optimal trade policy is positive; [10] the trade policy raises the domestic price of labor-intensive goods $(\phi_y > \phi_x)$ when unemployment is inefficiently high $(\lambda > \beta)$ or raises the domestic price of capital-intensive goods $(\phi_y < \phi_x)$ when unemployment is

[10] The welfare function is strictly concave in t.

Sector where the trade policy is conducted	Total employment		
	Too low	Efficient	Too high
Labor intensive	Protect	-	Harm
Capital intensive	Harm	-	Protect

Table 1: Summary of the optimal trade policy.

inefficiently low $(\lambda < \beta)$. On the other hand, when the sign of $(\lambda - \beta) / (\phi_y - \phi_x)$ is negative, the optimal trade policy is also negative. There are two possibilities: to reduce the domestic price of labor-intensive goods $(\phi_y > \phi_x)$ when unemployment is inefficiently low $(\lambda < \beta)$ or to reduce the domestic price of capital-intensive goods $(\phi_y < \phi_x)$ when unemployment is inefficiently high $(\lambda > \beta)$. Table 1 summarizes the sign of the optimal trade policy in all cases.

I place more emphasis on the case of inefficiently high unemployment. A capital-abundant country uses tariffs on imported labor-intensive goods or an export tax on exporting capital-intensive goods. In contrast, a labor-abundant country raises the relative price of labor-intensive goods by subsidizing its exported labor-intensive goods and its imported capital-intensive goods. In other words, in the presence of inefficiently high unemployment, a capital-abundant country wants to decrease its trade openness but a labor-abundant country wants to expand its trade openness. The Lerner symmetry theorem applies here. An import tariff is equivalent to an export tax and an export subsidy is equivalent to an import subsidy — the main idea is to expand or contract trade openness. I conclude this model prediction in Corollary 4:

Corollary 4. *In the Heckscher-Ohlin model, a small open economy concerned about inefficiently high unemployment should use a trade policy to raise the domestic relative price of labor-intensive goods, regardless of its comparative advantage.*

Proof. From equation (19)□

□

In reality, countries export and import both labor-intensive goods and capital-intensive goods. Provided that countries want to avoid job losses and agriculture is labor-intensive, this model predicts that countries would use export subsidies and import tariffs on its agricultural products and the size of trade protections is decreasing in a capital-labor ratio. In other words, to protect domestic jobs, countries want to increase net exports of agricultural goods and reduce net exports of non-agricultural goods. These model predictions are consistent with data shown in Figure 1. On average, each country imposes its import tariff on an agricultural sector around 7 percent higher than it does in the other sectors.

One interesting policy application is that the government does not always protect its import-competing sector when unemployment is too high, because if the import-competing sector is capital-intensive, unemployment is elevated and worsens the situation. It is not only about protecting any domestic production but also about protecting the correct sector of production.

6.2 Impact of an Optimal Trade Policy on Employment

In this subsection, I show that the optimal trade policy adjusts employment to reduce labor market inefficiency; equilibrium employment moves toward efficient employment. To illustrate this, I compare employment levels in three cases: (1) employment in a competitive equilibrium under free trade E^{FT}, (2) employment in a competitive equilibrium when the government maximizes net labor income E^{LI}, and (3) efficient level of employment E^{Eff}.

According to equation (19), employment in a competitive equilibrium in free trade E^{FT} is

$$E^{FT} = \left[\frac{\beta\Phi_2}{f} p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

Because there is no explicit solution for the optimal trade policy t^* maximizing welfare, I consider an alternative trade policy t^{LI} that maximizes only the net labor income without considering its effect on the capital income or the price level. The ranking of employment that I focus on will be preserved when the welfare maximizing tariff rather than the labor-income maximizing tariff is considered. That is,

$$\begin{aligned} t^{LI} &= argmax \ \left(1 - \frac{t}{1+t} h(\beta)\right) w(t) E(t) \\ &= \frac{h(\lambda) - h(\beta)}{h(\lambda)(h(\beta) - 1)}. \end{aligned}$$

Recall that function $h(z)$ is strictly increasing in z, for all $z \in (0,1)$. Thus, $t^{LI} = 0$ only if $\lambda = \beta$. Without labor market inefficiency, the net labor income is maximized, and no intervention is needed. Employment level E^{LI} is calculated by substituting t^{LI} into equation (19):

$$E^{LI} = \left[\frac{\beta\Phi_2}{f}\left(\frac{h(\beta)}{h(\lambda)}\left(\frac{h(\lambda)-1}{h(\beta)-1}\right)p\right)^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

The last case, the efficient level of employment, is solved from equation (39) as

$$E^{Eff} = \left[\frac{\lambda\Phi_2}{f} p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

In the first situation where $\lambda > \beta$, the ranking is

$$E^{FT} < E^{LI} < E^{Eff}.$$

The inequality demonstrates that the competitive equilibrium generates inefficiently low employment and a government uses a trade policy to partially remove the inefficiency.

In contrast, when $\lambda < \beta$, the ranking becomes

$$E^{FT} > E^{LI} > E^{Eff}.$$

In this case, the competitive equilibrium generates inefficiently high employment and a government also uses a trade policy to reduce the inefficiency.

In these two cases, I show that the trade policy attempts to correct the labor market inefficiency by moving the employment toward the constrained-efficient level. However, its side effect is a distortion on wages. As a result, the after-tax employment does not fully reach the constrained-efficient employment.

The ranking is preserved when I consider the optimal trade policy t^* instead of the trade policy maximizing labor income t^{LI}. The intuition is the same, but the magnitude is smaller because the government considers the additional distortions on the return on capital and the price level.

7 Two-Country Model

In the previous model, the country is a small open economy that takes a world price as given. I relax the assumption of fixed prices by introducing another country. This allows the world price to be an equilibrium price that clears global markets of final goods. Therefore, a trade policy indeed affects the world price. In this section, I describe a two-country model and then investigate implications of labor market inefficiency on bilaterally optimal trade policy between two countries.

7.1 Model Description and Equilibrium World Price

This model extends the model in Section 3 by simply adding another country. There are two countries: country 1 and country 2. I use a subscript $j \in \{1, 2\}$ to denote a variable associated with country j. The two countries have identical preferences, identical technologies of productions, and identical structure in a labor market. As in a traditional Heckscher-Ohlin model, the two countries are different in terms of factor endowments K_j and L_j. In addition, the two countries may have a different bargaining power of firms, β_j, and a different elasticity of matching with respect to vacancy, λ_j.

In this paper, a comparative advantage is mainly due to endowment differences across countries. One variation of the model is to let two countries have the same amount of endowments and assume a heterogeneity in the labor market in two countries. This variation will give a similar prediction as would Helpman and Itskhoki (2010), that a degree of search frictions can be the source of a comparative advantage. In this model, this would mean a country with small search frictions has a comparative advantage in recruiting a worker and hence produces labor-intensive goods.

The market-clearing condition is similar to equation (41), but a difference is that the world price p_w adjusts such that the global excess demand is zero,

$$ED_1^X\left(p_w\left(t_1, t_2\right), t_1\right) + ED_2^X\left(p_w\left(t_1, t_2\right), t_2\right) = 0. \tag{46}$$

Trade policy t_j affects the global excess demand through the domestic goods market in country j only. The equilibrium world price is a function of two trade policies and is described as $p_w(t_1, t_2)$. This global market clearing condition in equation (46) implies that the equilibrium world price ratio is decreasing in both t_1 and t_2:[11]

$$\frac{dp_w(t_1,t_2)}{dt_1} < 0,$$
$$\frac{dp_w(t_1,t_2)}{dt_2} < 0.$$

A reason for this effect is that a trade policy t_j reduces domestic demand for good X and raises production of good X in country j. As a result, the total quantity of good X in the world market increases, while the total demand of good X in the world market decreases. Consequently, the equilibrium world price ratio has to fall.

To the best of my knowledge, the equilibrium world price in equation (46) does not have an explicit solution unless $\lambda_1 = \lambda_2$. In the case that $\lambda_1 = \lambda_2 = \lambda$, then the equilibrium world price is

$$p_w^{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}} = \frac{1-(\alpha\phi_x+(1-\alpha)\phi_y)}{\alpha\phi_x+(1-\alpha)\phi_y}\times\frac{\Phi_1}{\Phi_2} \times\left[\frac{\frac{(1+t_1)^{\frac{\phi_y-1}{\phi_y-\phi_x}}}{\alpha+(1-\alpha)(1+t_1)}K_1+\frac{(1+t_2)^{\frac{\phi_y-1}{\phi_y-\phi_x}}}{\alpha+(1-\alpha)(1+t_2)}K_2}{\frac{(1+t_1)^{h(\lambda)}}{\alpha+(1-\alpha)(1+t_1)}\beta_1^{\frac{\lambda}{1-\lambda}}L_1+\frac{(1+t_2)^{h(\lambda)}}{\alpha+(1-\alpha)(1+t_2)}\beta_2^{\frac{\lambda}{1-\lambda}}L_2}\right]. \tag{47}$$

The world price has a similar expression as the autarky price. It captures the total endowments in the world and trade policies. Under free trade, where $t_1 = t_2 = 0$, I have

$$p_w^{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}} = \frac{E_1}{E_1+E_2}\left(p_{A,1}\right)^{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}} +\frac{E_2}{E_1+E_2}\left(p_{A,2}\right)^{\left(\frac{\phi_y}{\phi_y-\phi_x}-\alpha\right)\frac{\lambda}{1-\lambda}+\frac{1}{\phi_y-\phi_x}}, \tag{48}$$

where E_j is the number of employed workers in country j evaluated at the world price p_w and $p_{A,j}$ is the autarky price in country j. The world price is simply a weighted average of two autarky prices. The weight is the ratio of the number of employed workers in the country to the total number of employed workers in the world.

One finding is that a world price is closer to the autarky price of a large country than it is to the autarky price of a small country. According to the analysis in Section 5.2, this finding suggests that a large country with an inefficient labor market is more likely to have welfare loss from trade. A reason for this is that the world price may fall into a price range that makes the country worse off,

[11] See Appendix for proof.

as in the example in Figure 5. In contrast, a small country with an inefficient labor market is more likely to escape this problem because the new world price is more likely to be sufficiently different from the country's autarky price. Although the new world price moves into an unfavorable direction, it changes dramatically and passes the unfavorable region. In the example in Figure 5, it is easy for a large country to face a world price $p_w \in (p_{treshold}, p_A)$, while for a small country it is more likely that the world price, if it falls, goes beyond $p_{threshold}$, i.e. $p_w < p_{threshold}$.

Using equation (48) and Proposition 4, I can conclude that in the case that $\lambda > \beta$, a condition that ensures loses from trade in the large-country model is equivalent to

$$1 - \frac{\lambda - \beta_1}{1 - \beta_1} \times \frac{E_1(p_w) + E_2(p_w)}{E_2(p_w)} < \left(\frac{p_{A,2}}{p_{A,1}}\right)^{\left(\frac{\phi y}{\phi y - \phi x} - \alpha\right)\frac{\lambda}{1-\lambda} + \frac{1}{\phi y - \phi x}} < 1.$$

This is a condition in which a small increase in the world price (the world price slightly moves toward country 1's autarky price) would improve the welfare of country 1. In contrast, a traditional Heckscher-Ohlin model predicts that a change in the world price towards a country's autarky price always makes that country worse off. The size of the price range is increasing in the size of labor market inefficiency and the relative size of country 1. This finding supports the above analysis.

In addition, in the case that $\lambda < \beta$, the condition is slightly different. A country is worse off if

$$1 < \left(\frac{p_{A,2}}{p_{A,1}}\right)^{\left(\frac{\phi y}{\phi y - \phi x} - \alpha\right)\frac{\lambda}{1-\lambda} + \frac{1}{\phi y - \phi x}} < 1 + \frac{\beta_1 - \lambda}{1 - \beta_1} \times \frac{E_1(p_w) + E_2(p_w)}{E_2(p_w)}.$$

If a country currently has inefficiently high employment, and its trading partner does not have a sufficiently large autarky price, then the new world price induces more employment and hurts the country's welfare.

7.2 Optimal Trade Policy - Large Open Economy

The optimal trade policy in Section 6 is for a small open economy when a country cannot manipulate the world price. In this two-country model, the analysis is more complicated and is unclear because of an unknown magnitude of terms-of-trade effect $dp_w(t_1, t_2)/dt_j$, which is increasing in the relative size of country j to the size of the world.

I begin by evaluating the derivative of the welfare function with respect to a trade policy t_1, when the trade policy can alter the world price.

$$\frac{dU_1(p_w(t_1,t_2),t_1)}{dt_1} = \underbrace{\frac{\partial U_1(p_w(t_1,t_2),t_1)}{\partial t_1}}_{\text{Labor-market inefficiency}} + \underbrace{\frac{\partial U_1(p_w(t_1,t_2),t_1)}{\partial p_w} \times \frac{\partial p_w(t_1,t_2)}{\partial t_1}}_{\text{Terms-of-trade manipulation}}.$$

A trade policy improves welfare for two channels: its direct effect to reduce labor market inefficiency and its indirect effect to manipulate the world price. The analysis in Section 6.1 can be considered a special case when $dp_w/dt_1 = 0$, for any t_1, which means that the country is sufficiently small and the world price is invariant to its trade policy. By allowing for the endogenous world price, the terms-of-trade manipulation becomes a government motivation to use a trade policy. The welfare of a country is improved if the world price of its export moves away from its autarky price. However, this paper shows that the welfare change also depends on labor market inefficiency. According to Section 6.1, an inefficient labor market may generate an undesirable employment situation and may cause a welfare loss that dominates welfare gains from changes in factor prices.

To see an optimal deviation from free trade, I evaluate the derivative of welfare with respect to t_1 at $t_1 = 0$:

$$\left.\frac{dU_1}{dt_1}\right|_{t_1=0} = \underbrace{\left.\frac{\partial U_1}{\partial t_1}\right|_{t_1=0}}_{+/-} + \underbrace{\left.\frac{\partial U_1}{\partial p_w}\right|_{t_1=0}}_{+/-} \times \underbrace{\left.\frac{\partial p_w}{\partial t_1}\right|_{t_1=0}}_{-}.$$

The lesson from Section 6.1 is $\partial U_1/\partial t_1$ can be either positive or negative, depending on the degree of labor market inefficiency $\lambda - \beta$. Normally, the second term $\partial U_1/\partial p_w$ is ambiguous because it depends on $p_w - p_{MinU}$, but, when it is evaluated at $t_1 = 0$, this term has the same sign as $\lambda - \beta$, according to the analysis in Section 5.2. The last term, $\partial p_w/\partial t_1$, is always negative as shown in Section 7.1. Evaluating at $t_1 = 0$, the sign of $\partial U_1/\partial t_1$ and the sign of $\partial U_1/\partial p_w$ are determined by $\lambda - \beta$. Combining all three terms, the sign of of dU_1/dt_1 is inconclusive.

Define t_L^* as a unilaterally optimal trade policy of a large country that satisfies

$$\frac{\partial U_1\left(p_w\left(t_L^*, t_2\right), t_L^*\right)}{\partial t_1} + \frac{\partial U_1\left(p_w\left(t_L^*, t_2\right), t_L^*\right)}{\partial p_w} \times \frac{\partial p_w\left(t_L^*, t_2\right), t_L^*}{\partial t_1} = 0,$$

and t_S^* as a unilaterally optimal trade policy of a small country solved in section 6 that satisfies

$$\frac{\partial U_1\left(p_w\left(t_S^*, t_2\right), t_S^*\right)}{\partial t_1} = 0.$$

In the case of an efficient labor market $\lambda = \beta$, I conclude that $t_L^* \neq t_S^* = 0$. While a small country cannot manipulate the world price and thus prefers free trade, a large country prefers to alter the terms of trade to benefit itself. The sign of t_L^* depends on the country's comparative advantage. A capital-abundant country has $\partial U_1/\partial p_w|_{t_1=0} < 0$ and prefers $t_L^* > 0$, which is an import tax. A labor-abundant country has $\partial U_1/\partial p_w|_{t_1=0} > 0$ and prefers $t_L^* < 0$, which is an export tax. The conclusion is consistent with the terms-of-trade incentive in standard literature.

One particular case of $\lambda \neq \beta$ is more interesting because a country has an additional incentive to relieve labor market inefficiency. Without loss of generality, I discuss only a case of $\lambda > \beta$; a case of $\lambda < \beta$ is simply the opposite of the following analysis. Due to inefficiently low employment when $\lambda > \beta$, the direct effect of t_1 on welfare evaluating at $t_1 = 0$ is positive, $\partial U_1/\partial t_1|_{t_1=0} > 0$. There are three possibilities. First, a capital-abundant country that faces $p_w < p_{MinU} < p_A$ prefers $t_L^* > 0$, because it benefits from the positive direct effect and a favorable terms-of-trade effect. Secondly, a capital-abundant country that faces $p_{MinU} < p_w < p_A$ has to balance trade-offs between the two effects. On one hand, it wants to correct labor market inefficiency (the direct effect). On the other hand, its intervention drives the price of labor-intensive goods down and puts more pressure on unemployment. The net result depends on the magnitudes of all terms, including the sensitivity of the world price with respect to the trade policy. Lastly, a labor-abundant country faces the same trade-off as the previous case: it wants to increase employment but by doing so it lowers the price of labor-intensive goods and causes more job losses.

In addition, a trade policy t_1 affects country 2 through a terms-of-trade externality only:

$$dU_2\left(p_w\left(t_1,t_2\right),t_2\right)=\underbrace{\frac{\partial U_2\left(p_w\left(t_1,t_2\right),t_2\right)}{\partial p_w}}_{+/-}\times\underbrace{\frac{\partial p_w\left(t_1,t_2\right)}{\partial t_1}}_{+/-}\times dt_1.$$

This model offers an interesting policy implication: under some circumstances, a unilaterally optimal trade policy may be a Pareto improvement; one country is better off while the other country is not worse off. I conclude this finding in Proposition 7:

Proposition 7. *A unilaterally optimal trade policy of a large open economy with labor market inefficiency may benefit the country's trading partners.*

Country 1 has an incentive to use its policy to reduce its labor market inefficiency. This action may affect the terms-of-trade and benefits country 2. There are two possibilities. First, a labor-abundant country with inefficiently high unemployment may use its unilaterally optimal trade policy, an export subsidy, to reduce its unemployment and improve its welfare. The country's trading partner, a capital-abundant country, benefits from the trade policy because of the better terms of trade. This situation is likely to happen because developing countries are generally labor-abundant. It implies that developed, capital-abundant countries are better off. Second, a capital-abundant country with inefficiently high employment may use an export subsidy on its capital-intensive exports. This intervention raises the price of good X and benefits the trading partner, a labor-abundant country, through the better terms of trade.

Corollary 5. *World free trade is not optimal if labor market inefficiency exists.*

Corollary 5 emphasizes that free trade may not be on the efficiency frontier of tariffs. Labor market inefficiency is bad and needs to be removed. Unless all countries can implement a first-best policy, a direct subsidy on vacancy postings, trade policy is more practical to reduce labor market inefficiency. As pointed out by Proposition 7, a unilaterally optimal trade policy can improve global welfare; it reduces the inefficiency in the domestic labor market and improves trading partners' welfare through better terms of trade.

8 Numerical Examples

This section begins with a review of the estimated values of two key parameters: the elasticity of the matching function with respect to vacancy λ and the wage bargaining power β. Then I numerically show predictions of this model as a function of the degree of labor market inefficiency.

Section 6.1 proposes that the new motive depends heavily on two parameters: λ and β. The first parameter in this model, λ, is based on an assumption of a constant-return-to-scale (CRS) Cobb-Douglas matching function, the ad-hoc specification that matches data well empirically but has no microeconomic foundation. Normally, a matching function gives a number of successful matches between unemployed workers and vacancies. In this model, it gives a number of matches between total population and vacancies — every worker starts with being unemployed. According to Patrongolo and Pissarides (2001), who provide a good survey of papers that structurally estimate matching functions in developed countries, the estimates of λ are between 0.19 and 0.88. In an earlier work, Pissarides (1986) uses quarterly data of U.K. male workers between 1967-1983 to estimate the restricted CRS Cobb-Douglas matching function and finds that $\lambda = 0.3$. Blanchard and Diamond (1990) use various specifications and λ is in a range of 0.54-0.71. Burda (1993) reports $\lambda = 0.44$ in the Czech Republic and $\lambda = 0.1$ in Slovenia. Burda and Wyplosz (1994) report $\lambda = 0.09$ in France, $\lambda = 0.27$ in Germany, $\lambda = 0.14$ in Spain and $\lambda = 0.22$ in UK. Yashir (2000) finds $\lambda = 0.87$ in Israel. Shimer (2005) finds that, in the U.S., $\lambda = 0.25$. These empirical works suggest a variation of λ across countries, but no final conclusion has been commonly used. Other estimates using U.S. data include 0.765 in Hall (2005), 0.539 in Nagypal (2009), 0.692 in Borowczyk-Martins, Jolivet, and Postel-Vinay (2013), and 0.66 in Sahin, Song, Topa,and Violante (2014).

The next parameter of interest is the wage bargaining power of firms (β). One simple inaccurate way to measure the bargaining power is to calculate a share of the firm's profit in total job surplus (firm's profit plus labor income). However, this method requires a lot of underlying assumptions and it does not work well when firms hire multiple workers. Hagedorn and Manovskii (2008) estimate that a one-percent increase in productivity raises real wages by 0.45%, or $\beta = 0.45$. Cahuc, Gianella, Goux, and Zylberberg (2002) estimate from French data from 1988-1992 and report β at around 0.8. Cahuc, Postel-Vinay, and Robin (2006) use French data from 1993-2000 and find that the bargaining power of firms against skilled workers is around 0.76. Flinn (2006) reports $\beta = 0.576$. These studies agree on an asymmetric bargaining power, $\beta > 0.5$.

Despite the empirical results, theoretical works simply assume $\lambda = 0.5 = \beta$ for several reasons. First, because there is no universally accepted estimated value of λ, it is natural to use $\lambda = 0.5$, the middle value, as a benchmark. In addition, it is reasonable to study a benchmark environment with an efficient labor market, i.e., $\beta = \lambda$. Also, assuming a symmetric Nash bargaining power is not arguably unacceptable. A common way to do this is to calibrate a coefficient of the matching function to be consistent with data. For example, Shimer (2009), Heid (2015) and Heid and Larch (2012) assume $\beta = 0.5$, and Shimer (2009), Heid (2015) and Satchi and Temple (2009) set $\lambda = 0.5$.

I begin with a numerical example of welfare as a function of a trade policy t. Again, I consider a trade policy t in a labor-intensive sector (sector X). In Figure 7, each curve represents a different degree of the labor market inefficiency: $\lambda = 0.5$ in the (black long-dashed black) U_1 curve, $\lambda = 0.65$ in the (blue short-dashed) U_2 curve, and $\lambda = 0.35$ in the (solid-orange) U_3 curve. Essentially, I vary the degree of labor market inefficiency $\lambda - \beta$ to -0.15, 0, and 0.15, while β is fixed at 0.5. For each

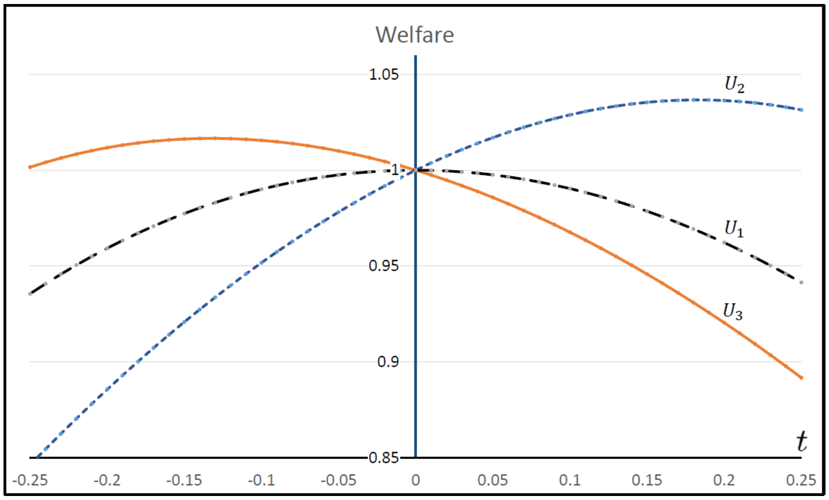

Figure 7: Welfare as a function of a trade policy. (Parameter values: $\beta = 0.5$, $\phi_x = 0.2$, $\phi_y = 0.8$, $p = 1$, $\alpha = 0.5$, $K = 150$, $L = 400$ and $f = 0.4$.)

situation, I calculate after-policy welfare normalized by the initial welfare under free trade; the welfare functions on the graph are equal to one at $t = 0$. Doing this allows us to see how many percentage points welfare can be improved by an intervention and what the optimal trade policy is.

In the benchmark case, U_1 represents a case of an efficient labor market where $\lambda = 0.5 = \beta$. As shown in Figure 7, the optimal trade policy is free trade as predicted by the model. In this situation, the labor market is working perfectly and a trade policy cannot improve the economy beyond perfection; a trade policy only causes a price distortion without generating any benefit. The second case where $\lambda = 0.65 > \beta$ represents a case of inefficiently high unemployment. The number of vacancies (and jobs) in free trade is inefficiently low. The optimal trade policy is to raise the domestic price of labor-intensive goods. As shown by the U_2 curve, welfare is maximized at $t = 0.18$ and the welfare can be improved by around 3.6%. In the last case, I set $\lambda = 0.35 < \beta$ to illustrate a case in which a labor market generates inefficiently low unemployment. The number of vacancies (and jobs) in free trade is inefficiently large, and the government has to distort domestic prices in favor of capital-intensive goods. As illustrated by the orange curve U_3 curve in Figure 7, the optimal trade policy is around -0.13 and welfare can be around 1.7% higher than it is in free trade.

Why is the optimal trade policy non-zero in the presence of an inefficient labor market? Figure 8 illustrates a decomposition of welfare in terms of aggregate income and a price index. I use an example where $\lambda = 0.65 > 0.5 = \beta$ in Figure 7. Again, all variables are normalized by their value in free trade. The welfare $U(t)$ is represented by the dashed blue curve, the aggregate income $I(t)$ is shown by the orange solid curve, and the inverted price level is shown by the green solid curve.

As can be seen in Figure 8, an inverted price level is always maximized at $t = 0$ where distortion does not exist. The aggregate income can be improved by a trade policy when the labor market inefficiency arises. In this example, employment is inefficiently low and a government can reduce

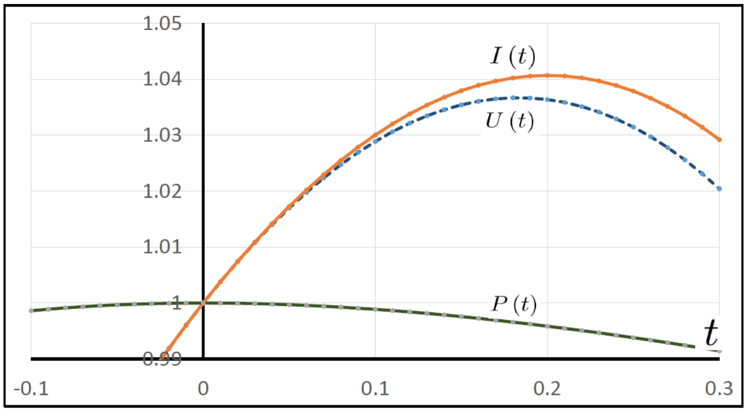

Figure 8: Decomposition of welfare in terms of aggregate income and a price level for a given trade policy t. (Parameter values: $\beta = 0.5$, $\lambda = 0.65$, $\phi_x = 0.2$, $\phi_y = 0.8$, $p = 1$, $\alpha = 0.5$, $K = 150$, $L = 400$ and $f = 0.4$.)

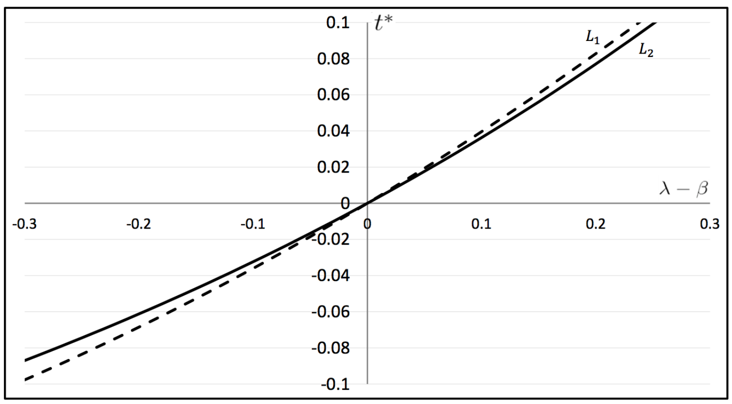

Figure 9: The optimal trade policy is increasing in $\lambda - \beta$. (Parameter values: $\beta = 0.5$, $\lambda = 0.65$ $\phi_x = 0.2$, $\phi_y = 0.8$, $p = 1$, $\alpha = 0.5$, and $f = 0.4$.)

labor market inefficiency. Raising the price of the labor-intensive good increases wages as well as employment. As a result, beginning at free trade, a small increase in trade policy improves income and welfare while levying a cost on a price distortion. The cost of price distortion is negligible when t is close to zero. In this example, the aggregate income is maximized at $t = 0.2$. However, to maximize welfare, the government has to consider a trade-off between labor market inefficiency and a distorted price level. Therefore, it would choose $t = 0.18$ to maximize welfare.

Figure 9 shows that an optimal trade policy is monotonically increasing in $\lambda - \beta$ and decreasing in capital-labor ratio. Two examples are illustrated: a relatively labor-abundant case ($K = 150$, $L = 600$) in the dashed line (L_1) and a relatively capital-abundant case ($K = 150$, $L = 100$) in the solid line (L_2).

When $\lambda < \beta$, employment is inefficiently high and the optimal policy is to tax production in the labor-intensive sector. In contrast, when $\lambda > \beta$, employment is inefficiently low and the optimal policy is to subsidize production in the labor-intensive sector. Free trade $t^* = 0$ is optimal when the labor market is already constrained-efficient, i.e., $\lambda = \beta$. The size of the optimal trade policy is always decreasing in capital-labor ratio. Because a trade policy causes a trade-off between capital income and labor income, a relatively capital-abundant country loses more capital-income when it corrects the labor market inefficiency. As a result, a relatively capital abundant country intervenes less than a relatively labor-abundant country does, or in other words, the optimal trade policy in a capital-abundant country is less sensitive to labor market inefficiency than it is in a labor-abundant country.

9 Discussion

In this section, I discuss three variations of the main model in Section 3: a special case, a simple Ricardian model, in which labor is the only factor of production; a dynamic model; and how a government can achieve constrained efficiency when its policy instruments are fully available.

9.1 Ricardian Model

This section shows a variation of the baseline model in the spirit of a Ricardian model in which labor is the only factor of production.

A model setup is almost identical to the baseline model in Section 3 except for production technology. Each employed worker in sector i can produce h_i units of output i. I can think of this technology as a special case where $\phi_x = \phi_y = 0$. As a result, capital is useless and the rental price is zero. I assume that the production technology of producing vacancy costs is still the same Cobb-Douglas function.

Equation (11), which states that job surplus in two sectors are equal, is slightly adjusted to

$$ph_x = h_y. \tag{49}$$

This is true only when

$$p = \frac{h_y}{h_x}.$$

The economy produces both types of goods only when its price ratio is equal to a constant slope of its production possibility frontier. The market-clearing price in autarky is pinned down by $p_A = h_y/h_x$.

As in a standard Ricardian model, in free trade, the country specializes in the production of goods where it has a comparative advantage. This is because the world price ratio is different from the autarky price ratio. Thus, equation (49) no longer holds and only one sector is active.

Without loss of generality, I assume that the country has a comparative advantage in good X. Therefore, the country produces only good X, exports good X, and imports good Y. I impose that an export subsidy and an import tariff are not too large, and that the pattern of trade is not reversed. I then consider two types of trade policies: (i) an export subsidy s, and (ii) an import tariff t.

Export subsidy s

In the first case, I study an optimal export subsidy s on an exportation of good X. Because the country produces only good X, the surplus from a job is $S = (1+s)\, ph_x$ and the zero-profit condition in equation (10) is changed to

$$\beta (1+s)\, ph_x E = ((1+s)\, p)^{\alpha} fV.$$

The equilibrium employment is

$$E(p,s) = \left[\frac{\beta h_x}{f} ((1+s)\, p)^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda}} L.$$

Similarly to equation (**??**), welfare function is now modified to

$$U(p,s) = \frac{(1+s)^{1-\alpha}}{(\alpha + (1-\alpha)(1+s))\, p^{\alpha}} \left[1 - \beta (\alpha + (1-\alpha)(1+s)) \right] ph_x \left[\frac{\beta h_x}{f} ((1+s)\, p)^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda}} L. \tag{50}$$

One feature of a traditional Ricardian model is that a trade policy does not distort a production point because the country always produces only one type of goods and the production possibility frontier is fixed. However, in this model, a trade policy distorts the production point through vacancy level V as the production possibility frontier shifts accordingly.

Next, following the analysis in Section 6.1, I start with finding a sufficient condition for a constrained efficiency, and then I investigate the optimal trade policy.

In the social planner's problem, a constrained efficiency arises when $pX^s - p^{\alpha} fV$ is maximized, or equivalently when $\lambda = \beta$. This sufficient condition is identical to what Lemma 1 concludes.

The derivative of the welfare function with respect to s is

$$\frac{dlogU(p,s)}{ds} = \frac{1-\alpha}{1+s} - \frac{1-\alpha}{\alpha + (1-\alpha)(1+s)} - \frac{\beta(1-\alpha)}{1 - \beta(\alpha + (1-\alpha)(1+s))} + \frac{\lambda}{1-\lambda} \frac{(1-\alpha)}{(1+s)}.$$

Evaluating the derivative of the welfare function with respect to s at $s = 0$ yields

$$\left. \frac{dlogU(p,s)}{ds} \right|_{s=0} = (\lambda - \beta) \frac{(1-\alpha)}{(1-\beta)(1-\lambda)}.$$

First, when a labor market is constrained-efficient, $\lambda = \beta$, there is no inefficiency that needs to be fixed. Therefore, free trade is optimal, as can be seen from $dlogU/ds|_{s=0} = 0$ when $\lambda = \beta$. Second, when employment is inefficiently low, $\lambda > \beta$, an export subsidy is optimal. Beginning at free trade, a slight increase in t improves the country's welfare as $dlogU/ds|_{s=0} > 0$ when $\lambda > \beta$. Lastly, when employment is inefficiently high, $\lambda < \beta$, an export tax is optimal.

This result is consistent with Proposition 6. However, it has a slightly different rule for the optimal export policy.

Proposition 8. *In the Ricardian model, the optimal export subsidy s^* has the same sign as the sign of $\lambda - \beta$.*

Proof. See Appendix.

□

When a country has a constrained-efficient labor market $\lambda = \beta$, it has no incentive to use an import tariff to adjust employment and, hence, it prefers free trade $s^* = 0$. When the country suffers from inefficiently high unemployment ($\lambda > \beta$), it has to create additional jobs by using an export subsidy ($s^* > 0$) to expand domestic production. The export subsidy makes the value of a filled job more profitable and convinces firms to add more vacancies. Thus, the unemployment problem is partially resolved. The opposite is also true. If the country has inefficiently low unemployment ($\lambda < \beta$), it uses an export tax ($s^* < 0$) to contract the production sector.

In this Ricardian version, the optimal trade policy satisfies

$$(1-\alpha)^2 \beta (1+s^*)^2 + \left[\alpha - \lambda + 2\alpha\beta - 2\alpha^2\beta\right](1+s^*) - \alpha + \alpha^2\beta = 0.$$

The explicit solution is

$$s^* = \frac{(\lambda-\beta) - (\beta+\alpha-2\alpha\beta) \pm \sqrt{(\lambda-\beta)^2 + 2(\lambda-\beta)(\beta-\alpha-2\alpha\beta+2\alpha^2\beta) + (\beta+\alpha-2\alpha\beta)^2}}{2(1-\alpha)^2\beta}.$$

In a case that $\lambda = \beta$, the solution is reduced to $s^* = 0$, as discussed previously.

Import Tariff t

In the second situation, I study the optimal import tariff on good Y. This tariff does not affect the price of good X but instead it changes the price of good Y from 1 to $1+t$. The surplus from a job is $S = ph_x$. Seemingly, the surplus itself is independent of the trade policy. However, the tariff affects firms through the vacancy cost.

The zero-profit condition in equation (10) is modified to

$$\beta p h_x E = (1+t)^{1-\alpha} p^{\alpha} f V.$$

The equilibrium employment is

$$E(p,t) = \left[\frac{\beta h_x}{f}\left(\frac{p}{1+t}\right)^{1-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

The new welfare function is

$$U(p,t) = \frac{(1+t)^{\alpha}}{(\alpha(1+t)+(1-\alpha))\,p^{\alpha}}\left[1-\beta\left(\alpha+\frac{1-\alpha}{1+t}\right)\right] p h_x \left[\frac{\beta h_x}{f}\left(\frac{p}{1+t}\right)^{1-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L. \quad (51)$$

The welfare function when the country uses an import tariff in equation (51) and the welfare function when the country uses an export subsidy in equation (50) are isomorphic, when I replace $(1+t) = 1/(1+s)$. This is consistent with the Lerner symmetry theorem, which states that an export subsidy $(s > 0)$ is equivalent to an import subsidy $(t < 0)$ and an export tax $(s < 0)$ is equivalent to an import tax $(t > 0)$.

Proposition 8 and the Lerner symmetry theorem lead to Proposition 9.

Proposition 9. *In the Ricardian model, the sign of the optimal import tariff t^* is opposite to the sign of $\lambda - \beta$.*

Proof. See Appendix. □

This idea is similar to the argument for the optimal export subsidy. The main difference is the source of profit. The optimal export subsidy raises the surplus of a filled position but the import subsidy reduces the nominal cost of vacancy cost through cheaper imported intermediate inputs. The two mechanisms reach the same conclusion: firms post more vacancies and employment increases.

Corollary 6 emphasizes results in Proposition 8 and Proposition 9.

Corollary 6. *In the Ricardian model, a small open economy that is concerned about inefficiently high unemployment should use either an export subsidy or an import subsidy.*

Proof. From Proposition 8 and Proposition 9. □

This result yields two interesting policy implications. First, to stimulate employment, a trade policy needs to encourage international trade in the form of an export subsidy or a reduction in an import tariff. The reason is that the country has to expand production activities by creating more demand on domestic good from abroad through an export subsidy or by reducing costs of intermediate inputs through an import subsidy or a reduction in an import tariff. Second, this result is opposite to the prediction in the Heckscher-Ohlin model because in that model, the government has to use the trade policy in the appropriate sector according to the summary in Table 1.

9.2 Dynamic Small Open Economy model

This subsection describes a dynamic model that extends the baseline model in Section 3 and re-investigates the rationale of trade policy in a small open economy.

9.2.1 Model Description

The main differences is the matching process. The matching function randomly creates new jobs by matching unemployed workers with vacancies posted. All variables in time period j are denoted by subscript j. Let N_j be the number of unemployed workers at time j. The matching function is now:

$$M\left(V_j, N_j\right) = min\left\{V_j^{\lambda} N_j^{1-\lambda}, N_j\right\}. \tag{52}$$

Existing jobs are destroyed at the rate $\delta > 0$.[12] Thus, unemployment evolves over time according to

$$N_j - N_{j-1} = -M\left(V_j, N_j\right) + \delta E_j.$$

A change in unemployment comes from two processes: some previously unemployed workers who have a job this period, and new unemployed workers who work in the previous period but lose their job.

Following Costinot (2009), I focus on a steady-state equilibrium in which all variables are unchanged over time. The main purpose of this approach is the tractability of the welfare effect of trade policy on a time path. Thus, henceforth subscript j will be dropped unless there is a risk of ambiguity.

In a steady state, unemployment remains constant over time, $N_j = N_{j-1}$. This implies

$$V^{\lambda} N^{1-\lambda} = \delta\left(L - N\right),$$

where the outflow of unemployed workers who get a job is equal to the inflow of newly unemployed workers.

9.2.2 Optimal Trade Policy

This subsection studies the rationale of trade policy in small open economy in a dynamic model.

As in Section 6.1, the government wants to maximize the country's welfare by using the trade policy t:

$$\begin{aligned} \text{Gov } \underset{t}{Max}\ U\left(p,t\right) &= I\left(p,t\right) \times P\left(p,t\right) \\ s.t. \quad & V^{\lambda} N^{1-\lambda} = \delta\left(L - N\right) \end{aligned}$$

[12]The static model is a case where $\delta = 1$. All workers lose their job at the end of each period. This results in $N_j = L$ for all $j \geq 0$.

The first-order condition is that

$$\frac{dlogU\left(p,t\right)}{dt}=\frac{\partial logI}{\partial t}+\frac{\partial logP}{\partial t}+\frac{\partial logI}{\partial V}\times\frac{\partial V}{\partial t}.$$

The government is facing three trade-offs. First, a trade policy distorts return to factors of production as firms are producing sub-optimally. Second, a trade policy distorts consumer prices. Lastly and importantly, a trade policy distorts vacancy postings in a competitive equilibrium.

Evaluating the first-order condition at $t=0$ shows that

$$\left.\frac{dlogU\left(p,t\right)}{dt}\right|_{t=0}=\underbrace{\frac{\partial logI}{\partial V}}_{(?)}\times\underbrace{\frac{\partial V}{\partial t}}_{(+)}.$$

Free trade is optimal only if $\partial logI/\partial V=0$ which is the case of labor market efficiency. When the labor market is inefficient, i.e., the vacancy level is not maximizing the net income, the country can use a trade policy to improve welfare. A country with inefficiently high unemployment ($\partial logI/\partial V>0$) can use a trade policy to raise the price of the labor-intensive goods and adjust unemployment toward the efficient level of unemployment. Also, a country with inefficiently low unemployment ($\partial logI/\partial V<0$) can use a trade policy to reduce the price of the labor-intensive goods.

This confirms that the rationale of a use of trade policy in a small economy concluded in Proposition 5 and Corollary 4 holds in the dynamic model. A country has an incentive to use a trade policy to reduce labor market inefficiency. Once the labor market is efficient, free trade is optimal.

9.3 Fully Available Policy Instruments

The main motive of trade protection in this paper is from labor market inefficiency. My previous analysis highlights the importance of trade policy for developing small open economies. It is reasonable to believe that a government should use an instrument that can eliminate the inefficiency without causing any distortion.

This section investigates an optimal intervention when a direct subsidy or tax on vacancy posting becomes available, which is likely possible in developed countries. The environment considered in this section is the same as the model described in Section 3. The only difference is that now a government is allowed to subsidize or tax vacancy postings at a price θ ($\theta>0$ if it is a subsidy and $\theta<0$ if it is a tax). A subsidy cost or a tax revenue is redistributed back to the representative consumer.

The direct labor market policy does not distort prices or factor prices. It only affects vacancy cost and after-tax labor income. The firms' profit in equation (4) is modified to

$$\mathbf{E}\left(\pi_i\right)=\frac{E}{V}\pi_i-\left(\left(\left(1+t\right)p\right)^{\alpha}f-\theta\right)=\frac{E}{V}\beta S_i-\left(\left(\left(1+t\right)p\right)^{\alpha}f-\theta\right)=0. \tag{53}$$

The competitive equilibrium is similar to the competitive equilibrium in Section 3. The only difference is that firms' expected profit includes the labor market policy θ.

The equilibrium total vacancies V and total employment E are slightly changed to capture a new effective vacancy cost as

$$\begin{aligned} V &= \left(\frac{\beta S}{\left((1+t)\,p\right)^{\alpha} f - \theta}\right)^{\frac{1}{1-\lambda}} L, \\ E &= \left(\frac{\beta S}{\left((1+t)\,p\right)^{\alpha} f - \theta}\right)^{\frac{\lambda}{1-\lambda}} L. \end{aligned}$$

The welfare maximization problem is

$$\text{Gov } \underset{t,\theta}{Max}\ U\left(p,t,\theta\right) = I\left(p,t,\theta\right) \times P\left(p,t\right).$$

The optimal policy is

$$\begin{aligned} \theta^* &= \frac{\lambda-\beta}{\lambda} p^{\alpha} f, \\ t^* &= 0. \end{aligned}$$

The government uses the labor market policy to adjust a vacancy level to reach the optimal vacancy level and, hence, the labor market inefficiency is completely eliminated. Therefore, the labor-market-inefficiency motive of a trade policy no longer exists and the optimal trade policy is free trade. This is consistent with Bhagwati (1971) that a first-best policy aims to eliminate the inefficiency at the source of the inefficiency and a trade policy is usually a second-best policy. However, in reality, the first-best policy is difficult to implement because it requires complete information about vacancy postings that are unobservable and unverifiable. Thus trade policy, which is a second-best policy, is more practical.

10 Conclusion

I introduce a frictional labor market into an otherwise standard Heckscher-Ohlin model of international trade. There are two inputs called capital and labor that can move freely across sectors. The two outputs are different in terms of input requirements; one is relatively labor-intensive and the other one is relatively capital-intensive. The main departure from the literature is that a labor market has Diamond-Mortensen-Pissarides search frictions where firms have to post costly vacancies before acquiring workers.

One positive finding is that a change in the relative price ratio can potentially affect the equilibrium economy-wide unemployment. When the price of labor-intensive (capital-intensive) goods increases, firms adjust their search intensity and equilibrium unemployment drops (rises).

I show that welfare gains from trade are affected by labor market inefficiency because the inefficiency alter the way the labor market responds to price changes. The effect of labor market inefficiency can reinforce or weaken welfare gains. As a result, conditional on having the same observed trade share, some countries might actually have larger welfare gains from trade and some countries might actually have welfare losses from trade.

Next, I investigate the rationale of the trade policy of a small open economy. One normative finding is that a trade policy can improve efficiency when the labor market is initially inefficient. The government can distort prices to motivate firms to employ more or fewer workers. This intervention also comes with a tradeoff of a price distortion from consumers' points of view. As a result, although the government can improve welfare, it cannot reach constrained efficiency.

The key lesson from this work is that raising employment is not about subsidizing an export sector or protecting an import-competing sector but how labor-intensive the sector is. The government needs to stimulate a relatively labor-intensive sector (for example, an agricultural sector).

I also compare my results with standard trade models and point out why these traditional models with full employment cannot deliver the same results. In addition, I look at a special case, the Ricardian model, where labor is the only factor of production. As in the standard literature, a country specializes in the production of goods where it has a comparative advantage. Hence, in order to promote employment, that country's government has to stimulate its only active sector, which is its export sector.

There are various possible extensions for future work. One possibility is to extend the model to a dynamic model and quantify welfare gain or a time path of the optimal trade policy. The model can be extended to *N* goods and *N* factors (for example, heterogeneous workers in terms of skills). Another possibility is to study its empirical counterpart.

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A Appendix

A.1 Figure

The estimate of tariffs in Figure.

	(1) Tariff Diff	(2) Tariff Diff	(3) Tariff Diff	(4) Agg MFN Tariff	(5) Non-Agg MFN Tariff
Capital per capita	1.014		1.058	0.0554	-0.958*
	(0.880)		(0.869)	(0.888)	(0.405)
GDP per capita	0.275	0.179		0.378	0.103
	(0.628)	(0.394)		(0.634)	(0.289)
Constant	-4.068	5.034	-1.395	12.14	16.21***
	(10.19)	(4.540)	(8.112)	(10.28)	(4.684)
Observations	71	122	71	71	71

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Data description:

- Tariffs are from World Trade profile 2012 by WTO.
- Real GDP and population are from Penn World Table by Aten, Heston, and Summers (2012). Penn World Table Version 7.1.
- Capital stocks are estimated by Berlemann and Wesselhoft (2014)

A.2 Proof of Proposition 1

Proposition 1: *When a relatively labor (capital)-abundant country moves from autarky to free trade,*

1. Real wages increase (decrease) and real returns to capital decrease (increase)

2. Employment increases (decreases)

3. A labor-intensive sector expands (contracts) while a capital-intensive sector contracts (expands)

Proof. The proof has three steps.

1. The effect on wages and rental prices are straightforward from equation (15) and (17). To find real returns, we divide nominal returns with prices:

$$\frac{r}{p^{\alpha}} = \Phi_1 p^{-\frac{1-\phi_y}{\phi_y-\phi_x}-\alpha},$$
$$\frac{w}{p^{\alpha}} = (1-\beta)\,\Phi_2 p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}.$$

2. The effect on total employment can be seen from

$$E = \left[\frac{\beta\Phi_2}{f} p^{\frac{\phi_y}{\phi_y-\phi_x}-\alpha}\right]^{\frac{\lambda}{1-\lambda}} L.$$

3. To claim that the production sector expands, we show that both capital and labor in that sector increase.

$$\begin{aligned}
L_x/E &= \frac{(1-\phi_x)\,\phi_y}{\phi_y-\phi_x}\left(1-K/k_yE\right) \\
L_y/E &= \frac{\left(1-\phi_y\right)\phi_x}{\phi_y-\phi_x}\left(K/k_xE-1\right) \\
K_x/K &= \frac{\left(1-\phi_y\right)\phi_x}{\phi_y-\phi_x}\left(k_yE/K-1\right) \\
K_y/K &= \frac{(1-\phi_x)\,\phi_y}{\phi_y-\phi_x}\left(1-k_xE/K\right)
\end{aligned}$$

Using equation (13) and (14), we have that K_i and L_i move in the same direction.
The effect of a change in relative prices on these variables are straightforward.

□

A.3 Proof of Lemma 3

Lemma 3: *The optimal trade policy t^* in sector X exists and is unique. If t^* is an interior solution $t_{min} < t^* < t_{max}$, it satisfies $dU/dt = 0$.*

Proof. The proof has three parts.

1. Begin with equation (**??**), we argue that $U(t)$ is continuous in t for all $t \in [t_{min}, t_{max}]$ where $t_{min} < 0 < t_{max}$. Because $[t_{min}, t_{max}]$ is a compact set, there exists t^* maximizing $U(t)$.

2. Uniqueness arises because $U(t)$ is strictly concave for all $t \in [t_{min}, t_{max}]$.

3. The condition is a necessary condition from the first order condition when the solution is interior.

□

A.4 Proof of Proposition 5

Proposition 5: *In the Heckscher-Ohlin model, the optimal trade policy t^* in sector X has the same sign as the sign of $(\lambda - \beta) / (\phi_y - \phi_x)$.*

Proof. Because $U(t)$ is strictly concave, $U'(t)$ is strictly decreasing in t for all $t \in [t_{min}, t_{max}]$. Then, we compare $U'(t^*)$ and $U'(0)$. When $(\lambda - \beta) / (\phi_y - \phi_x) > 0$, we have $U'(t^*) = 0 < U'(0)$ and conclude that $t^* > 0$. In contrast, when $(\lambda - \beta) / (\phi_y - \phi_x) < 0$, we have $U'(t^*) = 0 > U'(0)$ and conclude that $t^* < 0$. □

A.5 The World Price in Section 7

The world price when $t_1 \neq 0$, $t_2 \neq 0$ and $\lambda_1 = \lambda_2$ is

$$p_w^{\left(\frac{\phi_y}{\phi_y - \phi_x} - \alpha\right)\frac{\lambda}{1-\lambda} + \frac{1}{\phi_y - \phi_x}} = \frac{(1-u_1)L_1}{(1-u_1)L_1 + \left(\frac{1+t_2}{1+t_1}\right)^{h(\lambda)}\left(\frac{\alpha+(1-\alpha)(1+t_1)}{\alpha+(1-\alpha)(1+t_2)}\right)(1-u_2)L_2}\left(\frac{p_{A,1}}{1+t_1}\right)^{\left(\frac{\phi_y}{\phi_y - \phi_x} - \alpha\right)\frac{\lambda}{1-\lambda} + \frac{1}{\phi_y - \phi_x}}$$
$$+ \frac{(1-u_2)L_2}{\left(\frac{1+t_1}{1+t_2}\right)^{h(\lambda)}\left(\frac{\alpha+(1-\alpha)(1+t_2)}{\alpha+(1-\alpha)(1+t_1)}\right)(1-u_1)L_1 + (1-u_2)L_2}\left(\frac{p_{A,2}}{1+t_2}\right)^{\left(\frac{\phi_y}{\phi_y - \phi_x} - \alpha\right)\frac{\lambda}{1-\lambda} + \frac{1}{\phi_y - \phi_x}}.$$

A.6 Proof of Proposition 8

Proposition 8: *In the Ricardian model, the optimal export subsidy s^* has the same sign as the sign of $\lambda - \beta$.*

Proof. Because $U(s)$ is strictly concave, $U'(s)$ is strictly decreasing in s for all $s \in [s_{min}, s_{max}]$. Then, we compare $U'(s^*)$ and $U'(0)$. When $\lambda - \beta > 0$, we have $U'(s^*) = 0 < U'(0)$ and conclude that $s^* > 0$. In contrast, when $\lambda - \beta < 0$, we have $U'(s^*) = 0 > U'(0)$ and conclude that $s^* < 0$. □

A.7 Proof of Proposition 9

Proposition 9: *In the Ricardian model, the sign of the optimal import tariff t^* is opposite to the sign of $\lambda - \beta$.*

Proof. The proof applies Proposition 8 and the Lerner symmetry. When $\lambda - \beta > 0$, we conclude that $s^* > 0$ which implies $t^* = 1/(1+s) - 1 < 0$. In contrast, when $\lambda - \beta < 0$, we conclude that $s^* < 0$ which implies $t^* = 1/(1+s) - 1 > 0$. □