WORKING PAPER

FINANCIAL FRAGILITY AND OVER-THE-COUNTER MARKETS

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I propose a model to study whether trade frictions in an over-the-counter market for financial assets exacerbate or attenuate financial fragility. I model the financial sector as a large number of financial institutions, which I label banks. Each bank is a coalition of depositors and depositors are subject to privately observed liquidity shocks. The banks' problem is to maximize the welfare of depositors by implementing the efficient allocation of financial assets among them. I show that when banks use the *balanced team mechanism*, proposed by Athey and Segal (2013), there is always a truth-telling equilibrium which supports the constrained Pareto efficient allocation. When the frictions in the over-the-counter market are small, this equilibrium is unique. However, I provide numerical examples in which these frictions are severe and the economy has other equilibria. In one equilibrium depositors claim high liquidity needs, asset price falls, the trade volume collapses and, consequently, the equilibrium allocation is not constrained Pareto efficient. I label this equilibrium a bank-run equilibrium and I interpret the existence of bank-runs as a financial fragility. I propose two policies to eliminate bank-run equilibria. The first is a suspension scheme and the second is an opening of trade facilities similar to the ones established by the Federal Reserve Bank during the 2007-08 financial crisis. Both policies can eliminate bank runs when contingent on announcements of liquidity needs in a large number of banks.

KEYWORDS: Over-the-counter markets, banking, bank-runs, financial fragility, financial crisis, dynamic mechanism design, weak implementation.

1. INTRODUCTION

In the last decades, securitization has grown steadily for a wide range of financial assets. From household mortgages to long-term firm financing, it allowed financial institutions to transform illiquid investments into asset-backed-securities (ABS) which are traded over-the-counter. However, a panic during the 2007/2008 financial crisis hit particularly hard financial institutions deeply involved in the ABS market and coincided with a collapse of ABS trading volume and price. But

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what generates financial panics in an environment where the portfolio of financial institutions consists of assets traded in an over-the-counter market? Or, more generally, how is financial fragility affected by the tradability of financial assets?

The existing literature does not adequately address this issue. Jacklin (1984) investigates bank fragility in the context of a Diamond and Dybvig (1983) model with financial assets traded in a centralized market. Trading in centralized markets has the implication that panics are ruled out, as agents engage in perfect risk-sharing trough financial markets. However, the rise of securitization has substantially increased the relevance of over-the-counter markets and it is yet not understood how frictions created by an over-the-counter market might affect financial fragility.

I propose a new model of financial fragility where financial assets are traded in an over-the-counter market. The model builds on a discrete time version of Duffie et al. (2005) and Lagos and Rocheteau (2009), hereafter DGP and LR, with with three important modifications. Agents in my model belong to small coalitions in which they participate in risk-sharing arrangements. I call these coalitions banks and agents in my model I call depositors.[1] Depositors receive preference shocks over time, which generates stochastic liquidity needs, and these shocks are private information. With private information there is an issue of how to achieve truth-telling among depositors. I investigate the *balanced team mechanism*, proposed by Athey and Segal (2013), which always has a truth-telling equilibrium.[2] Lastly, I assume that each bank has random accesses to a centralized Walrasian market, which is a special case of DGP and LR.

I derive two main results from my model. First, I show that there always exists an equilibrium which supports the constrained Pareto efficient allocation in the economy. Second, and most important, I construct numerical examples of equilibria where depositors misrepresent their preference shocks and, consequently, the equilibrium allocation is not constrained Pareto efficient. In the inefficient equilibrium depositors announce a high demand for liquidity because they believe that the other depositors are doing the same—a form of self-fulfilling crisis which I call a bank run. During a bank run, the asset price falls and the trade volume collapses. I interpret the existence of runs as a fragility of the financial sector. Private information and agents belonging to a coalition are both essential extensions of DGP and LR in order for the economy to be fragile. If either assumption is dropped, the economy turns into a special case of LR which has a unique equilibrium.

[1]The interpretation of banks as a coalition of agents is not a novelty. For example, Bryant (1980) and Diamond and Dybvig (1983) model a bank as a coalition of agents engaging in risk-sharing arrangements trough a bank contract.

[2]The *balanced team mechanism* extends the mechanism proposed by Arrow (1979) and d'Aspremont and Gérard-Varet (1979), which is known as the AGV-Arrow mechanism, to dynamic environments. See Fudenberg and Tirole (1991) for a detailed description of the AGV-Arrow mechanism.

My findings imply that the benefits of reducing trade frictions in an over-the-counter market are twofold. The first is to be expected. Since agents face idiosyncratic preference shocks, there are gains from trading with each other. Thus, with less trading frictions, they explore the trading opportunities more efficiently. The second reason is that, when the over-the-counter market is efficient, the economy does not have bank-run equilibria. Therefore, if policy makers can eliminate, or substantially reduce, trade frictions, they can make the financial sector stable.

Of course, in many cases policy makers are unable to reduce trade frictions and, as an alternative, I investigate two policies to enhance financial stability in my model: a suspension scheme and the opening of a centralized exchange facility where banks can trade financial assets.[3] I consider an arrangement where either one of above policies is implemented once the aggregate distribution of announcements does not coincide with the true distribution of preference shocks, which is known since there is no aggregate uncertainty in the model. Depositors anticipate that, conditional on either policy being implemented, truth-telling is a best response because there is no risk-sharing among depositors in their own bank—only trade in a Walrasian market. And, as a result, they have no incentives to misrepresent their preference shocks. Therefore, there cannot exist an equilibrium where the distribution of announcements differs from the true distribution of types.

A large literature studies optimal bank mechanisms in the Diamond-Dybvig model.[4] This literature shows that for Diamond-Dybvig banks to be fragile under an optimal direct mechanism, banks need to face aggregate uncertainty and a sequential service constraint—payments must respect a first-come, first-served rule. Uncertainty is also a necessary condition for fragility in my model; however, the sequential service constraint is not. Several financial institutions considered part of the shadow bank sector finance its assets by issuing debt with specific due dates. Hence, sequential service does not seem a relevant constraint for those institutions and it is appropriate to have a model that explains financial fragility without imposing sequential service. A second finding of this literature is that an indirect mechanism can be used to prevent runs.[5] Although I consider this an interesting possibility, I do not address it in this paper.

The fact that some form of trade friction is an essential element of a fragile financial sector is

[3]This policy provides a rationale for a Fed intervention in the aftermath of the crisis, namely, the creation of the Term Asset-Backed Securities Loan Facility (TALF). The model suggests that the Fed should use such facility to eliminate the trade frictions by operating as a market-maker. Worth mentioning that the Fed also lent over $1 trillion dollars taking ABSs as collateral, which is inconsistent with the policy recommendation in my model.

[4]This literature includes, but is not limited to, Wallace (1988), Peck and Shell (2003), Green and Lin (2003), Andolfatto et al. (2007), Ennis and Keister (2009), Cavalcanti and Monteiro (2011) and Andolfatto et al. (2014).

[5]See Cavalcanti and Monteiro (2011) and Andolfatto et al. (2014) for a discussion of indirect mechanisms in the context of the Diamond-Dybvig model.

not new. Jacklin (1984) studies a version of the Diamond-Dybvig model where there is a Walrasian market to trade financial assets. In this case, the market can implement the efficient outcome and the economy is not fragile. In the limit case of my model where the market for financial assets is a centralized exchange, I obtain this same result. But if one considers only the limit case where there is a Walrasian market, as Jacklin (1984) shows, banks are redundant. In practice, we do observe banks and, therefore, it is a desirable property of the environment that banks are not redundant institutions. My model displays this property as long as some market friction exists, which is a realistic assumption when assets are traded in an over-the-counter fashion. One may ask why not to study only the other limit case where markets do not exist—as most of the Diamond-Dybvig literature does. I also study this case in the paper; however, if there is no financial market, one cannot study the implications of bank runs on asset prices and trade volume and how it relates with trade frictions.

Allen and Gale (2000) and Allen and Gale (2004), here after AG, study the implications of bank fragility in a setting were Diamond-Dybvig banks trade contingent claims in a static inter-bank market. There are two important differences between my work and AG. First, AG focus on understanding the implications of bank fragility in the market outcomes given different market structures—complete vs incomplete markets. But they don't ask the question of why banks are fragile. On the other hand, the main focus of my paper is exactly to understand why banks are fragile and how it relates with different market structures. The second difference is that AG study a static Walrasian market. While I study a dynamic over-the-counter market, which allows me to analyse how market outcomes evolve over time.

Lagos et al. (2011) study financial crises in the context of an over-the-counter market where dealers provide liquidity to the economy. The financial crisis is modelled as an exogenous aggregate shock that makes all agents have a low valuation of the underlying financial asset. There is evidence that some form of aggregate shock decreased the value of mortgage-backed securities during the 2007/08 crisis period. However, in practice, it is hard to differentiate whether such shock was exogeneous, as in Lagos et al. (2011), the result of bank runs, as in my model, or both.

Trejos and Wright (2014) generalize preferences in the DGP model to preferences that are separable but not quasi-linear.[6] They show that, for some parameters, there are multiple equilibria and the equilibrium dynamics can be the outcome of self-fulfilling prophecies—sunspots. The main reason for multiplicity in Trejos and Wright (2014) is that the asset can also be used as a means of

[6]Their work also integrates DGP and the monetary economy of Shi (1995) and Trejos and Wright (1995).

exchange. As a result, beliefs over whether people will accept the asset as payments in the future change the value of the asset in the present generating multiple equilibria. In my model assets do not have value as a means of exchange, instead, I explore agents long-term relationships as in a bank arrangement. I believe both approaches are complementary to each other.

The rest of the paper is organized as follows. Section 2 studies a simplified version of the model where there is only one bank and one period. I use this simple version to introduce the AGV-Arrow mechanism in a simple context and to provide intuition of why bank-run equilibria exists in the model. Section 3 extends this version with one bank to an infinite-horizon model. I use this one-bank infinite-horizon version to introduce the Athey and Segal (2013) balanced team mechanism mechanism and to discuss a different nature of bank runs that emerges in a dynamic setting. Section 4 introduces the complete model, where there are a large number of banks which interact in an over-the-counter market. Section 5 describes the balanced team mechanism for the complete version; characterizes the constrained Pareto efficient allocation; shows that this allocation is supported by an equilibrium; and also shows that, if over-the-counter frictions are small, the equilibrium that supports the constrained Pareto efficient allocation is the unique equilibrium. Section 6 provides a numerical example which resembles a financial crisis. Section 7 considers an extension where depositors have no commitment. Section 8 studies policies to enhance stability in the banking system. And section 9 discusses the results and possible extensions.

2. A SINGLE BANK ONE-PERIOD MODEL

In this section I study the simplest version of the model, where there is only one bank and one period. An advantage of this version is that the implementation result of the AGV-Arrow mechanism applies. Namely, the AGV-Arrow mechanism implements the efficient outcome in Bayesian equilibrium. However, I provide an example where the economy also has an inefficient equilibrium under the AGV-Arrow mechanism. I label this equilibrium a bank-run equilibrium.

Environment

The economy consists of a single coalition of agents, which I call a bank. In the bank there are $N \in \mathbb{N}$ ex-ante identical agents, called depositors, and there are two consumption goods: a *numéraire* good and *fruits*. Depositors have an aggregate endowment $\bar{M} > 0$ of the numéraire good and $\bar{A} > 0$ of a financial asset, where one unit of the asset bears one unit of fruit. Initially, both endowments are equally divided among depositors.

Depositors receive a preference type which is private information. The total utility of a depositor is given by $u(a;\theta)+m$, where θ is a preference type, $a \in \mathbb{R}_+$ is the consumption of fruits and $m \in \mathbb{R}_+$ is the consumption of the numéraire good. Trough out the paper I assume that the endowment $\bar{M}$ is large enough so the constraint $m \geq 0$ does not bind for any of the mechanisms I consider. Preference types are drawn from a known distribution π with finite support $\Theta \subset \mathbb{R}_+$, and are i.i.d. across depositors. For each type $\theta \in \Theta$, the function $u(\cdot;\theta)$ is twice continuous differentiable, strictly increasing, strictly concave, $u'(0;\theta) = \infty$ and $u'(\infty;\theta) = 0$.

Direct mechanisms and equilibrium

I focus on the class of direct mechanisms, where each depositor only announces his type, and I assume that depositors can commit with a direct mechanism.[7] The time of actions is the following. First, depositors observe their types and then they simultaneously make an announcement of it. Label $\boldsymbol{\theta} = (\theta^1, \dots, \theta^N) \in \boldsymbol{\Theta} := \Theta^N$ the announcement vector, which I assume is publicly observable. After announcements are made, the mechanism transfers numéraire goods and allocates financial assets—all contingent on the announcement vector.

Formally, a direct mechanism is a pair of asset and numéraire policies, $\mu = \{\boldsymbol{\chi}, \boldsymbol{\psi}\}$. An asset policy is a function $\boldsymbol{\chi} = (\chi^1, \dots, \chi^N) : \boldsymbol{\Theta} \to \mathbb{R}_+^N$ which assigns assets to each depositor contingent on the announcement vector. And a numéraire policy is a function $\boldsymbol{\psi} = (\psi^1, \dots, \psi^N) : \boldsymbol{\Theta} \to \mathbb{R}_+^N$ which assigns numéraire good to each depositor contingent on the announcement vector. An asset policy $\boldsymbol{\chi}$ is feasible if $\sum_n \chi^n(\boldsymbol{\theta}) = \bar{A}$ for every $\boldsymbol{\theta}$. That is, if the aggregate asset holdings is consistent with the total amount of assets in the economy. Analogously, a numéraire policy $\boldsymbol{\psi}$ is feasible if it satisfies $\sum_n \psi^n(\boldsymbol{\theta}) = \bar{M}$ for every $\boldsymbol{\theta}$. I label this the budget balanced condition. A direct mechanism is feasible if both its policies are feasible. Label $\mathscr{M}$ the set of feasible direct mechanisms. Figure 1 depicts the sequence of actions.

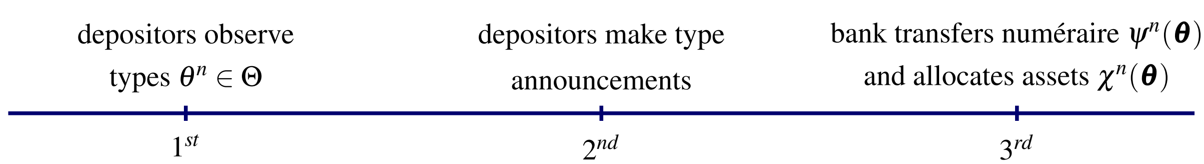

Figure 1: Sequence of actions

A direct mechanism $\mu = \{\boldsymbol{\chi}, \boldsymbol{\psi}\}$ should be interpreted as a trade mechanism. An announcement θ^n gives a depositor n a claim $\chi^n(\theta^n, \boldsymbol{\theta}^{-n})$ on the assets, where $\boldsymbol{\theta}^{-n}$ denotes the announcements from depositors $i \neq n$. The price the depositor pays/receives for this claim in terms of the

[7] In section 7 I discuss an extension of the model where there is no commitment.

numéraire good is $\frac{\bar{M}}{N} - \psi^n(\theta^n, \theta^{-n})$. Ultimately, the mechanism establishes trade quantities and prices contingent on the vector of announcements.

A feasible direct mechanism $\mu \in \mathscr{M}$ is associated with a Bayesian game for depositors. A depositor's pure strategy is an announcement contingent on his preference type, $\sigma(\theta) \in \Theta$. Label Σ the set of pure strategies, which is the same for every depositor. I restrict attention to equilibria in pure strategies, which is without loss of generality with respect to implementing the efficient outcome. The payoff of a depositor n, when he is of type θ^n and the announcement vector is $\hat{\boldsymbol{\theta}}$, is

$$(1) \quad v^n(\hat{\boldsymbol{\theta}}; \theta^n) = u\left(\chi^n(\hat{\boldsymbol{\theta}}); \theta^n\right) + \psi^n(\hat{\boldsymbol{\theta}}).$$

I label this game the depositors-game and I focus on its Bayesian equilibria.

The AVG-Arrow mechanism

The bank objective is to implement the efficient distribution of assets across depositors given any type realization. That is, the bank's problem is

$$(2) \quad \max\left\{\sum_n u(\chi^n; \theta^n);\ \sum_n \chi^n = \bar{A}\right\}$$

given any type vector $\boldsymbol{\theta}$. Label $\boldsymbol{\chi}^*(\boldsymbol{\theta})$ the asset policy associated with the solution of problem (2).

The bank designs a numéraire policy $\boldsymbol{\psi}$ in order to generate incentives for depositors to truthfully announce their preference type, so $\boldsymbol{\chi}^*(\boldsymbol{\theta})$ can be implemented. It is known that the VCG mechanism implements the efficient outcome in dominant strategies in this environment, but its transfer scheme is not budget balanced. As an alternative, I use the AGV-Arrow mechanism, which is a budget balanced mechanism.

Let the numéraire policy $\bar{\psi}^*$ be defined by

$$(3) \quad \bar{\psi}^{*n}(\boldsymbol{\theta}) = \frac{\bar{M}}{N} + \gamma^n(\theta^n) - \frac{1}{N-1}\sum_{i \neq n} \gamma^i(\theta^i),$$

where

$$(4) \quad \psi^{*n}(\boldsymbol{\theta}) = \sum_{i \neq n} u(\chi^{*i}(\boldsymbol{\theta}); \theta^i), \text{ and}$$

$$(5) \quad \gamma^n(\theta^n) = \mathbb{E}\left[\psi^{*n}(\boldsymbol{\theta})|\theta^n\right] - \mathbb{E}\left[\psi^{*n}(\boldsymbol{\theta})\right].$$

The AGV-Arrow mechanism is given by the pair $\mu^* = \{\boldsymbol{\chi}^*, \bar{\boldsymbol{\psi}}^*\}$. The term $\psi^{*n}(\boldsymbol{\theta})$ is the utility of depositors other than depositor n in state $\boldsymbol{\theta}$. In order to generate incentives for truth-telling, depositor n needs to internalize this term trough transfers. However, generically, we cannot make transfers associated with $\psi^{*n}(\boldsymbol{\theta})$ and balance the budget for every state $\boldsymbol{\theta}$. AVG-Arrow solve this problem by working with $\gamma^n(\theta^n)$, which is the change in the expected ψ^{*n} implied by depositor n's type. When the other depositors are announcing truthfully, the transfer γ^n is associated with the expected ψ^{*n} and, therefore, γ^n provides the same incentives for truth-telling as ψ^{*n}. For this reason, γ^n is labelled the incentive term of depositors n. Since $\gamma^n(\theta^n)$ depends only on θ^n, the budget can be balanced by making depositor n pays an equal share of depositors $i \neq n$ transfers without distorting incentives. Which leads to the transfer $\bar{\psi}^{*n}(\boldsymbol{\theta})$.

A strategy σ is a truth-telling strategy if $\sigma(\theta^n) = \theta^n$ for all $\theta^n \in \Theta$. Truth-telling is a Bayesian equilibrium of the game implied by the AVG-Arrow mechanism and the implied outcome achieves the maximum of problem (2). See Fudenberg and Tirole (1991) chapter 7 for a proof.

A bank-run example

The AVG-Arrow mechanism is an optimal mechanism in the sense that it has a Bayesian equilibrium associated with the efficient allocation of assets across depositors. Unfortunately, truth-telling is not necessarily the unique equilibrium. Consider the following illustrative example. There are $N = 3$ depositors and the total endowments are of $\bar{M} = 3.0$ and $\bar{A} = 3.0$. The utility function is a constant relative risk aversion $u(a;\theta) = \theta\frac{a^{1-\delta}-1}{1-\delta}$ with parameter $\delta = 6.0$. The type space is $\Theta = \{\theta_L, \theta_H\} = \{1.0, 1.5\}$. The probability of type θ_L is $\pi_L = 0.1$ and the probability of type θ_H is $\pi_H = 0.9$.

	θ_L,θ_L	θ_H,θ_L	θ_L,θ_H	θ_H,θ_H
θ_L	1.0000	1.0061	1.0061	1.0100
θ_H	0.9881	0.9903	0.9903	1.0000

Type θ_L

	θ_L,θ_L	θ_H,θ_L	θ_L,θ_H	θ_H,θ_H
θ_L	1.0000	0.9939	0.9939	0.9834
θ_H	0.9988	1.0007	1.0007	1.0000

Type θ_H

Figure 2: The depositors-game

Figure 2 depicts the depositors-game associated with the AVG-Arrow mechanism in this example. The first table contains the payoff of depositor n for each possible vector of announcements when his true type is θ_L. And the second table contains the payoff of depositor n for each possible vector of announcements when his true type is θ_H. The rows represent depositor n's announcement and the columns the possible combinations of the other depositors' announcements. Since the environment is symmetric, the AVG-Arrow mechanism is symmetric and, therefore, these two tables fully characterize the depositors-game.

The AVG-Arrow mechanism has a truth-telling equilibrium, but in this example it also has another equilibrium. Consider the strategy profile in which every depositor announces type θ_L independent of their true type. If a depositor is of type θ_L and deviate from the proposed equilibrium by announcing θ_H, his payoff is 0.9881 instead of 1.0000, which is not a profitable deviation. If a depositor is of type θ_H and deviate from the proposed equilibrium by announcing θ_H, his payoff is 0.9988 instead of 1.0000, which is also not a profitable deviation. Therefore, announcing type θ_L independent of your type is a Bayesian equilibrium of the game. I call this equilibrium a bank-run equilibrium because depositors announce a low demand for the bank assets anticipating that other depositors are going to do the same, which a form of self-fulfilling crisis.

The reason a bank-run equilibrium exists in this example is that, during a run, assets are relatively "cheap". The price a depositor pays for the asset is associated with the impact of his announcement on the expected utility of other depositors. When a depositor announces θ_H, the mechanism allocates more assets to him which causes a big reduction on the utility of the other depositors. Therefore, the price he pays when announcing θ_H is high. A depositor is willing to pay this high price under a truth-telling equilibrium because, if he misrepresent his type, he expects that the mechanism will allocate very little assets to him—in which case his marginal utility of holding more assets is high. Once a depositor believes everyone else will announce type θ_L, he believes the bank will allocate a reasonable amount of assets to him independently of his announcement. In this sense assets are "cheap". As a consequence, the depositor has a low marginal utility of holding more assets and he has no incentives to announce θ_H independently of his true type.

This form of bank run is in some sense opposite to the bank run presented in Diamond and Dybvig (1983). In the Diamond-Dybvig model depositors believe that resources are going to be scarce due to overpayments. Therefore, they run to the bank in order to get as much resources out of the bank as they can. In my model depositors believe that resources (financial assets) are going to be abundant due to a low demand for it. Therefore, they run to the bank in order to pay a lower price for those resources.

3. A SINGLE BANK INFINITE-HORIZON MODEL

An advantage of my model is that it can easily be extended to an infinite-horizon model. In a version with a single bank and infinite-horizon, the model is a special case of the model studied by Athey and Segal (2013) and I can use their results of implementation in perfect Bayesian equilibrium.

This extension is useful because it highlights a type of bank run which doesn't exist in static models. In dynamic settings, inefficient actions in the present can be supported as an equilibrium outcome by fear of even worse actions in the future. I label this form of inefficient equilibria a dynamic bank run. This is the type of bank runs I use to construct the financial crisis example in section 6.

Environment

The environment is an infinite repetition of the one discussed in the previous section. Time is discrete and goes from zero to infinite. All depositors discount the future at rate $\beta \in (0,1)$. Each preference type is drawn from a known distribution π^0, in date $t = 0$, and from $t = 1$ forwards it follows a Markov process with transition $Q : \Theta \times \Theta \to [0,1]$, which I assume has a unique ergodic distribution. As before, types are i.i.d. across depositors.

Direct mechanisms and equilibrium

In this setting, since types change over time, depositors announce their type at every date. Label $\boldsymbol{\theta}_t = (\theta_t^1, \dots, \theta_t^N) \in \boldsymbol{\Theta} := \Theta^N$ the date t announcement vector of depositors, and $\boldsymbol{\theta}^t = (\boldsymbol{\theta}_0, \dots, \boldsymbol{\theta}_t) \in \boldsymbol{\Theta}^t$ the history of announcement vectors from period zero up to period t.

A direct mechanism is a pair of asset and numéraire policies, $\mu = \{\boldsymbol{\chi}, \boldsymbol{\psi}\}$. An asset policy is a sequence $\boldsymbol{\chi} = \{\boldsymbol{\chi}_t\}_t$ of functions $\boldsymbol{\chi}_t = (\chi_t^1, \dots, \chi_t^N) : \boldsymbol{\Theta}^t \to \mathbb{R}_+^N$ which assigns assets to each depositor contingent on the history of announcement vectors $\boldsymbol{\theta}^t$. A numéraire policy is a sequence $\boldsymbol{\psi} = \{\boldsymbol{\psi}_t\}_t$ of functions $\boldsymbol{\psi}_t = (\psi_t^1, \dots, \psi_t^N) : \boldsymbol{\Theta}^t \to \mathbb{R}_+^N$ which assigns numéraire good to each depositor contingent on the history of announcement vectors $\boldsymbol{\theta}^t$. An asset policy $\boldsymbol{\chi}$ is feasible if $\sum_n \chi_t^n(\boldsymbol{\theta}^t) = \bar{A}$ for every date t and history $\boldsymbol{\theta}^t$. That is, if the aggregate consumption asset holdings is consistent with the total amount of assets in the economy. Analogously, a numéraire policy $\boldsymbol{\psi}$ is feasible if it satisfies the budget balanced condition, $\sum_n \psi_t^n(\boldsymbol{\theta}^t) = \bar{M}$, for every date t and history $\boldsymbol{\theta}^t$. A direct mechanism is feasible if both its policies are feasible. I label $\mathscr{M}$ the set of feasible direct mechanisms.

A depositor's pure strategy is a sequence of announcements contingent on his type and the history of announcements. Formally, a pure strategy for a depositor n is a sequence $\boldsymbol{\sigma} = \{\sigma_t\}_t$, where σ_t maps $(\boldsymbol{\theta}^{t-1}, \theta_t^n)$ into an announcement $\sigma_t(\boldsymbol{\theta}^{t-1}, \theta_t^n) \in \Theta$. Label Σ the set of pure strategies.

A feasible direct mechanism $\mu \in \mathscr{M}$ is associated with a dynamic Bayesian game for depositors. The strategy set, Σ, is the same for every depositor and, given a history of announcement vectors $\hat{\boldsymbol{\theta}}^t$, the date t payoff of a depositor n when he is of type θ_t^n is

$$(6) \qquad v^n(\hat{\boldsymbol{\theta}}^t;\theta_t^n) = u\left(\chi_t^n(\hat{\boldsymbol{\theta}}^t);\theta_t^n\right) + \psi_t^n(\hat{\boldsymbol{\theta}}^t).$$

As in the previous section, I label this game the depositors-game and consider its perfect Bayesian equilibria, from now on PBE.

The balanced team mechanism

The AVG-Arrow mechanism is designed for one-period environments, not dynamic ones. In dynamic environments types change over time and depositors are required to announce their type in every period. In this case, when a depositor misrepresent his type, his belief over the distribution of future types may differs from the bank's belief. Hence, it is possible that a depositor have incentive to lie in order to manipulate the bank's belief. Athey and Segal (2013) proposed an extension of the AGV-Arrow mechanism, which they called the *balanced team mechanism*, to deal with this case.

Let me start introducing the optimal asset policy. The bank objective is to implement the efficient allocation of assets across depositors in every period and every realization of types. That is, the bank's problem is

$$(7) \qquad \max\left\{\sum_n u(\chi^n;\theta_t^n);\ \sum_n \chi^n = \bar{A}\right\}.$$

for all date t and history $\boldsymbol{\theta}^t$. Label $\boldsymbol{\chi}^*$ the asset policy associated with the solution to (7).

The balanced team mechanism makes transfers so each depositor internalises the welfare of the other depositors in the bank, pretty much in the same way the AVG-Arrow mechanism does. The crucial difference is that in a dynamic setting each depositor needs to internalise not only the utility of others in the current period, but also the expect present value of their future utilities. In this way, depositors do not have incentives to manipulate the bank future beliefs. Formally, for all period t and history $\boldsymbol{\theta}^t$, the transfer to an depositor n in the balanced team mechanism is

$$(8) \qquad \bar{\boldsymbol{\psi}}_t^{*n}(\boldsymbol{\theta}^t) = \frac{\bar{M}}{N} + \gamma_t^n(\boldsymbol{\theta}^{t-1},\theta_t^n) - \frac{1}{N-1}\sum_{i\neq n}\gamma_t^i(\boldsymbol{\theta}^{t-1},\theta_t^i),$$

where

$$(9) \qquad \psi_t^{*n}(\boldsymbol{\theta}^t) = \sum_{i\neq n} u\left(\chi_t^{*i}(\boldsymbol{\theta}^t);\theta_t^i\right),$$

$$(10)\qquad \Psi_t^n(\boldsymbol{\theta}^t) = \mathbb{E}\left\{\sum_s \beta^{s-t}\psi_s^{*n}(\boldsymbol{\theta}^s) \;\Big|\; \boldsymbol{\theta}^t\right\}, \text{ and}$$

$$(11)\quad \gamma_t^n(\boldsymbol{\theta}^{t-1},\theta_t^n) = \mathbb{E}\left[\Psi_t^n(\boldsymbol{\theta}^t) \mid \boldsymbol{\theta}^{t-1},\theta_t^n\right] - \mathbb{E}\left[\Psi^n(\boldsymbol{\theta}_t) \mid \boldsymbol{\theta}^{t-1}\right].$$

The balance team mechanism is given by $\bar{\boldsymbol{\mu}}^* = \{\boldsymbol{\chi}^*, \bar{\boldsymbol{\psi}}^*\} \in \mathscr{M}$.

The environment here is a special case of Athey and Segal (2013). They show that truth-telling is a perfect Bayesian equilibrium of the balanced team mechanism, where a strategy $\boldsymbol{\sigma} = \{\sigma_t\}_t$ is a truth-telling strategy if $\sigma_t(\boldsymbol{\theta}^{t-1},\theta_t) = \theta_t$ for all t and $(\boldsymbol{\theta}^{t-1},\theta_t)$. Consequently, for each history of type realizations $\boldsymbol{\theta}^t$, the implied outcome achieves the maximum of problem (7).

A bank-run example

A dynamic environment generates an additional source of financial fragility. Once a depositor believes his announcement will trigger future "bad" behaviour among other depositors, he may have incentives to misrepresent his type even if it implies a welfare loss in the present. Consider the following illustrative example. There are $N = 3$ depositors and the total endowments are of $\bar{M} = 3.0$ and $\bar{A} = 3.0$. The utility function is a constant relative risk aversion $u(a;\theta) = \theta\frac{a^{1-\delta}-1}{1-\delta}$ with parameter $\delta = 6.0$. The type space is $\Theta = \{\theta_L, \theta_H\} = \{1.0, 1.5\}$. For simplicity, instead of consider a Markov process for types, I will look at the particular case where types are i.i.d. over time. The probability of type θ_L is $\pi_L = 0.7$ and the probability of type θ_H is $\pi_H = 0.3$. When types are i.i.d. over time, the balanced team mechanism is reduced to a repetition of an AVG-Arrow mechanism at every period, which simplifies the presentation of the depositors-game. This simplification is not essential in order to construct bank run examples.

	θ_L,θ_L	θ_H,θ_L	θ_L,θ_H	θ_H,θ_H
θ_L	1.0000	1.0020	1.0020	1.0016
θ_H	0.9871	0.9945	0.9945	1.0000

Type θ_L

	θ_L,θ_L	θ_H,θ_L	θ_L,θ_H	θ_H,θ_H
θ_L	1.0000	0.9898	0.9898	0.9761
θ_H	1.0071	1.0049	1.0049	1.0000

Type θ_H

Figure 3: The depositors-game

Figure 3 depicts the depositors-game associated with the balanced team mechanism for this example. As before, the first table represents the payoff of depositor n for each possible vector of announcements when his true type is θ_L. While the second table represents the payoff of depositor n for each possible vector of announcements when his true type is θ_H. The rows represent depositor n's announcement and the columns the possible combinations of the other depositors' announcements. While the second table is the payoff of depositor n for each possible vector of announcements when his true type is θ_H. Since types are i.i.d. over time, the stage game is the same in every date t and, therefore, it can be described with only two tables. If types followed a Markov process, the stage game would be contingent on the previous announcement vector.

This depositors-game has two interesting features. First, truth-telling at every period is a dominant strategy when β is close to zero. To see this note that, in the first table, when the depositor is type θ_L, his payoff of announcing θ_L is always higher than the payoff of announcing θ_H. And in the second table, when the depositor is type θ_H, his payoff of announcing θ_H is always higher than the payoff of announcing θ_L.

The second feature is that the dominant strategy implementation result does not hold when β is close to one. In this case, there are equilibria where depositors misreport their types in at least one period and the implied welfare is strictly lower than truth-telling. For example, consider the following strategy profile:

1. In date $t = 0$ announce θ_L independent of the true type, then go to 2.
2. If in date $t = 0$ all depositor have announced θ_L, then go to 4, otherwise, go to 3.
3. Announce θ_h until all depositors have announced θ_H for 5 consecutive periods, then go to 4.
4. Follow truth-telling.

Label $\boldsymbol{\sigma}_u$ the strategy profile described above.

CLAIM 1: *There exists a $\bar{\beta} \in (0,1)$ such that, for all $\beta \in [\bar{\beta}, 1)$, the strategy profile $\{\boldsymbol{\sigma}^n\}_n$, with $\boldsymbol{\sigma}^n = \boldsymbol{\sigma}_u$ for all depositor n, constitutes a perfect Bayesian equilibrium of the depositors-game depict in Figure 3. Furthermore, in the equilibrium path, all depositors announce θ_L in date zero.*[8]

The gains from trade within a bank occurs when depositors are from different types. In this case, the gains of trade are shared following the rules the balanced team mechanism in order to generate incentives for truth-telling. When all depositors misrepresent their types and announce θ_L, the gains from trade are not realized. This "inefficient" action can be supported as an equilibrium by the threaten of having depositors misrepresenting their type in the future. In other words, the one-period loss of announcing θ_L is lower than the five-periods loss of announcing θ_H. One may ask

[8]For this proof see Appendix A.

why to stay five periods announcing θ_H is a credible threat. It is credible because if some depositor deviate the punishment will be to stay at least one more period announcing θ_H. Of course, $\boldsymbol{\sigma}_u$ is only an equilibrium when depositors have high valuation of future utilities—β is close to one.

4. THE COMPLETE MODEL

In this section I introduce a version of the model where banks trade assets in an over-the-counter market. I emphasize one feature of over-the-counter markets, namely, trade happens with delay. When the trade delay is short, the inter-bank market resembles a centralized market like the NYSE, when the trade delay is long, it resembles an over-the-counter market like the ABS market.

Environment

There is a non-atomic unit measure of financial institutions which I call banks. Each bank is the same described in section 3, and preference shocks across depositors in different banks are independent. From period $t = 1$ forwards, a bank access a centralized Walrasian market with probability $\alpha \in (0,1]$. This assumption is equivalent to introducing a dealer, as as in DGP and LR, but giving all the bargaining power to the bank. When dealers have bargaining power, the Nash bargaining solution does not apply because depositors have private information. And, in order to keep the analysis simple, I do not consider this version of the model. Let $\iota_t \in \{0,1\}$ denotes whether a bank had access to the market, $\iota_t = 1$, or not, $\iota_t = 0$, in date t. I assume that ι_t is independent across banks and over time.

The price of assets in terms of numéraire good in the centralized is given by a stochastic process $\boldsymbol{p} = \{p_t\}$. For this section I assume that depositors observe a sunspot variable x_t, which is the same variable for all depositors in all banks. The sunspot random variables are i.i.d. over time with distribution F and finite support $\mathbb{S}$. Since there is a non-atomic measure of banks, and preference types and market accesses are independent across banks, the only source of aggregate uncertainty are the sunspots. Therefore, the price process is a deterministic function of sunspots. That is, a price is a function $p_t : \mathbb{S}^t \to \mathbb{R}_+$.

Direct mechanisms

Before formally define a direct mechanism, let me introduce some notation. I label $\boldsymbol{\theta}_t = (\theta_t^1, \ldots, \theta_t^N) \in \boldsymbol{\Theta} := \Theta^N$ the period t announcement vector, $\iota_t \in \{0,1\}$ whether the banks has access to the centralized market in period t or not, and p_t the period t price of assets in terms of numéraire good in the Walrasian market. The vector with the realization of these variables is denoted by

$h_t = (\boldsymbol{\theta}_t, \iota_t, p_t) \in H := \boldsymbol{\Theta} \times \{0,1\} \times \mathbb{R}_+$. As usual, I use superscript t to denote the history of variables from period zero up to period t.

A direct mechanism is a pair of asset and numéraire policies denoted by $\boldsymbol{\mu} = \{\boldsymbol{\chi}, \boldsymbol{\psi}\}$. An asset policy is a sequence $\boldsymbol{\chi} = \{\boldsymbol{\chi}_t\}_t$, where $\boldsymbol{\chi}_t = (\chi_t^1, \ldots, \chi_t^N) : H^t \to \mathbb{R}_+^N$ denotes the amount of assets allocated to depositor in the bank contingent on the history h^t. A numéraire policy is a sequence $\boldsymbol{\psi} = \{\boldsymbol{\psi}_t\}_t$, where $\boldsymbol{\psi}_t = (\psi_t^1, \ldots, \psi_t^N) : H^t \to \mathbb{R}^N$ denotes the transfer of numéraire good to each depositor contingent on the history h^t.

The difference between banks in this section and in the previous ones is that it can adjust its aggregate asset holdings when accessing the inter-bank market. In practice, this only changes the feasibility conditions of the direct mechanisms. That is, an asset policy $\boldsymbol{\chi} = \{\boldsymbol{\chi}_t\}$ is feasible if $\sum_n \chi_0^n = \bar{A}$ and $(1-\iota_t)\sum_n \left[\chi_t^n(h^t) - \chi_{t-1}^n(h^{t-1})\right] = 0$. The first condition is that the aggregate asset holding in a bank needs to be consistent with the initial distribution of assets among depositors. The second condition is that the bank can only adjust its aggregate asset holdings if it accesses the centralized market. Label Γ the set of all feasible asset policies. A numéraire policy $\boldsymbol{\psi} = \{\boldsymbol{\psi}_t\}$ is feasible if it satisfies the budget balanced condition, $\sum_n \psi_t^n(h^t) = \bar{M} + p_t \sum_n \left[\chi_{t-1}^n(h^{t-1}) - \chi_t^n(h^t)\right]$. That is, the aggregate numéraire transfers among depositors equals the aggregate transfer with the market. A direct mechanism is feasible if both policies are feasible. Label $\mathscr{M}$ the set of all feasible direct mechanisms.

A pure strategy is a sequence of announcements contingent on these variables. Formally, a depositor n strategy is a sequence $\boldsymbol{\sigma}^n = \{\sigma_t^n\}_t$ of measurable functions, where σ_t^n maps a vector $(h^{t-1}, \theta_t^n, x^t)$ into a announcement $\sigma_t^n(h^{t-1}, \theta_t^n, x^t) \in \Theta$. Label $\boldsymbol{\Sigma}$ the set of all pure strategies.

A feasible direct mechanism, $\boldsymbol{\mu} \in \mathscr{M}$, and a price process, $\boldsymbol{p}$, is associated with a stochastic game of incomplete information to depositors. Where the strategy set of each depositor n is $\boldsymbol{\Sigma}$ and his period utility in date t given a history h^t when he is of type θ_t^n is

$$v_t^n(h^t; \theta_t^n) = u\left(\chi_t^n(h^t); \theta_t^n\right) + \psi_t^n(h^t). \tag{12}$$

I label this game the depositors-game.

Equilibrium

I restrict attention to symmetric equilibria, meaning, every bank adopts the same mechanism and all depositors adopt the same strategy. Note that depositors from different banks can still make different announcements because they face different history of type realizations.

The only aggregate uncertainty in the economy comes from the public sunspot. Therefore, the price in a period t is a deterministic function of $x^t = (x_0, x_1, \dots, x_t)$. The market clearing condition requires that, for every period t and public sunspot realization x^t, the demand of assets equal the total offer of assets in the economy.

Formally, the transition probability of types, Q, the probability of accessing the centralized market, α, the distribution of sunspot, the price process, $\boldsymbol{p}$, a feasible mechanism, $\boldsymbol{\mu} \in \mathscr{M}$, and a strategy profile $\{\boldsymbol{\sigma}^n\}_n \in \boldsymbol{\Sigma}^n$, generate a sequence of measures $\boldsymbol{\eta} = \{\eta_t\}_t$ over the space of histories H^t. These measures are defined in the usual way. The aggregate demand for assets in the centralized market in period t is

$$D_t(x^t) := \int \sum_n \chi_t^n(h^t) d\eta_t(h^t|x^t). \tag{13}$$

Note that the aggregate demand for assets is a function of the sunspot history x^t. Markets clear at a period t if $D_t(x^t)$ equals $\bar{A}$ almost surely.

DEFINITION 1: Given a feasible direct mechanism $\boldsymbol{\mu} \in \mathscr{M}$, a symmetric equilibrium is a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ such that: (i) the strategy profile $\{\boldsymbol{\sigma}^n\}_n$, with $\boldsymbol{\sigma}^n = \boldsymbol{\sigma}$ for all n, is a perfect Bayesian equilibrium of the depositors-game associated with $\boldsymbol{\mu}$ and $\boldsymbol{p}$; and (ii) markets clear at every period.

5. EFFICIENCY RESULTS

In this section I show that there exists a price process and a direct mechanism which supports the constrained Pareto efficient allocation as an equilibrium outcome. Where constrained means constrained by the market access (ι equal to zero or one).

The balanced team mechanism

Let me start characterizing the optimal asset policy. Given a price process $\boldsymbol{p}$, the expected aggregate utility of a bank implied by an asset policy $\boldsymbol{\chi}$ is

$$W_{\boldsymbol{p}}(\boldsymbol{\chi}) = \mathbb{E}\left\{\sum_t \beta^t \left[\sum_n u(\chi_t^n, \theta^n) + \bar{M} + p_t \sum_n (\chi_{t-1}^n - \chi_t^n)\right]\right\}. \tag{14}$$

Label $\boldsymbol{\chi}_{\boldsymbol{p}}^*$ the asset policy that maximizes $W_{\boldsymbol{p}}$ among all feasible asset policies. The policy $\boldsymbol{\chi}_{\boldsymbol{p}}^*$ is only defined for a given price process $\boldsymbol{p}$, but trough out the text I will omit the argument $\boldsymbol{p}$ to keep the notation short whenever it is convenient. Note that $\boldsymbol{\chi}_{\boldsymbol{p}}^*$ may not exist for a particular price process. However, if $\boldsymbol{\chi}_{\boldsymbol{p}}^*$ exists it is unique since $u(\cdot, \theta)$ is strictly concave for all θ.

Before I proceed, let me introduce some notation. Label $U(A, \boldsymbol{\theta})$ the maximum aggregate period utility of a bank with total assets equal to A and vector type $\boldsymbol{\theta}$. That is,

$$(15)\quad U(A;\boldsymbol{\theta}) = \max\left\{\sum_n u(a^n;\theta^n);\ \sum_n a^n = A\right\}.$$

The function $U(\cdot;\boldsymbol{\theta})$ inherit all the properties of $u(\cdot;\theta)$. Namely, it is twice continuous differentiable, strictly increasing, strictly concave, $U'(0;\boldsymbol{\theta}) = \infty$ and $U'(\infty;\boldsymbol{\theta}) = 0$. Label $\{t_k\}_{k=1}^{\infty}$ the random sequence of periods in which a bank access the centralized market, and $d_k = t_{k+1} - t_k$ the time length between the accesses. Note that d_k follow a geometric distribution with parameter α. That is, the probability of the next access to the market be in d periods is $(1-\alpha)^{d-1}\alpha$.

PROPOSITION 1: *Consider a feasible asset policy, $\boldsymbol{\chi}$, and a sequence $\mathbf{A} = \{A_t\}_t$ satisfying $A_t(h^t) = \sum_n \chi_t^n(h^t)$ almost surely. A sufficient condition for $\boldsymbol{\chi}$ to solve problem (14) is that for all t and h^t: (i) the policy $\chi_t(h^t)$ solves problem (15) for $\boldsymbol{\theta} = \boldsymbol{\theta}_t$ and $A = A_t(h^t)$ almost surely; (ii) if the bank has its k-th access to the centralized market in period $t = t_k$, then $A_t(h^t)$ satisfies*

$$(16)\quad p_t - \mathbb{E}\left\{\beta^{d_k} p_{t+d_k} \mid h^t\right\} = \mathbb{E}\left\{\sum_{d=0}^{d_k-1} \beta^d U'(A_t;\boldsymbol{\theta}_{t+d}) \,\Big|\, h^t\right\}$$

almost surely; and (iii) the transversality condition, $\lim_{K\to\infty}\mathbb{E}\{\beta^{t_K} p_{t_K} A_{t_K}\} = 0$, holds.[9]

Proposition 1 provides sufficient conditions for a feasible asset policy to be optimal, which are given by the first order conditions of the bank problem. I use this conditions later when I show that there exists an equilibrium which supports the constrained Pareto efficient allocation.

The numéraire policy in the balanced team mechanism is constructed so a depositor internalizes the aggregate welfare of the bank. Label $\psi_t^{*n}(h^t)$ the difference between depositor n utility and the aggregate utility of the bank in date t given history h^t, net from the cost of depositor n's own assets $p_t\left(\chi_{t-1}^n - \chi_t^n\right)$. That is,

$$(17)\quad \psi_t^{*n}(h^t) = \sum_{i\neq n}\left[u\left(\chi^*(h^t);\theta_t^i\right) + p_t\left(\chi_{t-1}^i(h^{t-1}) - \chi_t^i(h^t)\right)\right].$$

Label $\Psi_t^n(h^t)$ the period t expected present value of the sequence $\{\psi_s^{*n}\}_{s=t}^{\infty}$ given h^t. That is,

$$(18)\quad \Psi_t^n(h^t) = \mathbb{E}\left\{\sum_{s=t}^{\infty}\beta^{s-t}\psi_s^{*n}(h^s) \,\Big|\, h^t\right\}.$$

[9]See Appendix B for all the proofs in this section.

The agent n incentive term of announcing θ_t^n in period t is given by

$$(19)\quad \gamma_t^n(h^{t-1}, p_t, \theta_t^n) = \mathbb{E}\left[\Psi_t^n(h^t) \mid h^{t-1}, p_t, \theta_t^n\right] - \mathbb{E}\left[\Psi^n(\boldsymbol{\theta}_t) \mid h^{t-1}, p_t\right].$$

For all period t and history h^t, the transfer to a depositors n in the balanced team mechanism is

$$(20)\quad \bar{\boldsymbol{\psi}}_t^{*n}(h^t) = \frac{\bar{M}}{N} + p_t\left(\chi_{t-1}^n(h^{t-1}) - \chi_t^n(h^t)\right) + \gamma_t^n(h^{t-1}, p_t, \theta_t^n) - \frac{1}{N-1}\sum_{i \neq n} \gamma_t^i(h^{t-1}, p_t, \theta_t^i).$$

The balance team mechanism is given by $\bar{\boldsymbol{\mu}}_{\boldsymbol{p}}^* = \{\boldsymbol{\chi}_{\boldsymbol{p}}^*, \bar{\boldsymbol{\psi}}_{\boldsymbol{p}}^*\} \in \mathscr{M}$. As in the case with only one bank, the depositors-game implied by the balance team mechanism has a perfect Bayesian equilibrium in truth-telling strategies. See Athey and Segal (2013) for a proof.

The constrained Pareto efficient allocation

In this class of models the numéraire good allows depositors to transfer utility with a linear technology. Therefore, an outcome is constrained Pareto efficient if, and only if, it maximizes depositors aggregate welfare. In this subsection I characterize this allocation. Note that constrained here refers to the constraint on market accesses, not the incentive compatibility constraint.

An allocation is a sequence $\boldsymbol{a} = \{\boldsymbol{a}_t\}_t$ of measurable functions $\boldsymbol{a}_t = (a_t^1, \ldots, a_t^N) : \boldsymbol{\Theta}^t \times \{0,1\}^t \to [0,B]^N$, where $B > 0$ is an upper bound on depositors asset holdings. I impose two restrictions on the allocation: there is an upper bound on asset holdings and the allocation is symmetric. Both restrictions are without loss of generality with respect to maximize aggregate welfare. The upper bound B can be taken large enough so it does not bind.[10] And the strictly concavity of $u(\cdot;\theta)$ implies that the welfare maximizing allocation is symmetric.

A symmetric allocation $\boldsymbol{a}$ is feasible if for all $(\boldsymbol{\theta}^t, \iota^t)$ it satisfies

$$(21)\quad \sum_n a_t^n(\boldsymbol{\theta}^t, \iota^{t-1}, 0) = \sum_n a_{t-1}^n(\boldsymbol{\theta}^{t-1}, \iota^{t-1}),$$

$$(22)\quad \sum_{\boldsymbol{\theta}^t}\sum_{\iota^t} \mathbb{P}(\boldsymbol{\theta}^t, \iota^t) \sum_n a_t^n(\boldsymbol{\theta}^t, \iota^t) = \bar{A}, \text{ and}$$

$$(23)\quad \sum_n a_0^n = \bar{A}.$$

[10] A necessary condition for an allocation $\boldsymbol{a}$ to maximize welfare is that, $\forall(\boldsymbol{\theta}^t, \iota^t)$, $(\boldsymbol{\theta}'^t, \iota'^t)$ such that $\iota_t = \iota_t' = 1$ and $\boldsymbol{\theta}_t = \boldsymbol{\theta}_t'$, we have $a_t(\boldsymbol{\theta}^t, \iota^t) = a_t(\boldsymbol{\theta}'^t, \iota'^t)$. This result comes from the fact that $u(\cdot;\theta)$ is strictly concave and that the stochastic type process is Markov. In addition, $u(\infty;\theta) = 0$ for each $\theta \in \Theta$ and the measure of banks with type vector $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ is bounded away from zero at any period t. Combined with the fact that assets exist in finite supply $\bar{A}$, these features imply that there is a positive real number, B, such that for all feasible $\boldsymbol{a}$, if $a_t(\boldsymbol{\theta}^t, \iota^t) > B$ for some $(\boldsymbol{\theta}^t, \iota^t)$, then there exists a Pareto superior and feasible outcome $\boldsymbol{a}'$ that is bounded above by B.

The first equation means that, in case the bank does not access the market, its total amount of assets is the same of the previous period. The second equation means that the total amount of assets held by depositors equals the total amount of assets existent in the economy. The last equation is the requirement that, since the initial distribution of assets is uniform, every bank should hold exactly $\bar{A}$ assets at period zero. Let $\mathscr{F}$ denotes the set of all feasible allocations $\boldsymbol{a}$.

The aggregate welfare of the economy implied by an allocation $\boldsymbol{a} \in \mathscr{F}$ is

$$(24) \quad W(\boldsymbol{a}) = \mathbb{E}\left\{\sum_t \beta^t \sum_n u(a_t^n; \theta_t^n)\right\}.$$

An allocation is constrained Pareto efficient if it achieves the maximum aggregate welfare among all feasible allocations. It is easy to show that such allocation exists and it is unique. Label this allocation $\boldsymbol{a}^*$. The following proposition characterizes $\boldsymbol{a}^*$.

PROPOSITION 2: *Consider a feasible allocation, $\boldsymbol{a} \in \mathscr{F}$, and a sequence $\boldsymbol{A} = \{A_t\}_t$ defined by $A_t(\boldsymbol{\theta}^t, \iota^t) = \sum_n a_t^n(\boldsymbol{\theta}^t, \iota^t)$. A necessary and sufficient condition for $\boldsymbol{a}$ to maximizes (24) is that for all $(\boldsymbol{\theta}^t, \iota^t)$: (i) the allocation rule $\boldsymbol{a}_t(\boldsymbol{\theta}^t, \iota^t)$ solves (15) for $\boldsymbol{\theta} = \boldsymbol{\theta}_t$ and $A = A_t(\boldsymbol{\theta}^t, \iota^t)$; and (ii) there exists a sequence of Lagrange multipliers, $\boldsymbol{\lambda} = \{\lambda_t\}_t$, such that, if $\iota_t = 1$, then $A_t(\boldsymbol{\theta}^t, \iota^t)$ satisfies*

$$(25) \quad \lambda_t = \mathbb{E}\left\{\sum_{d=0}^{d_k - 1} \beta^d U'(A_t; \boldsymbol{\theta}_{t+d}) \,\middle|\, \boldsymbol{\theta}^t, \iota^t\right\}.$$

Proposition 2 provides sufficient conditions for a feasible asset allocation to be constrained Pareto efficient, which are given by the first order conditions implied by the maximization of (24) constrained by equations (21)-(23).

The constrained Pareto efficient equilibrium

The balanced team mechanism weakly implements the optimal outcome within a bank for a given price. In addition, as long as depositors follow truth-telling strategies, it also sustain the constrained optimal outcome as an equilibrium outcome.

PROPOSITION 3: *There exists a price process $\boldsymbol{p}$, a strategy $\boldsymbol{\sigma}$ and a feasible mechanism $\bar{\boldsymbol{\mu}}^* = \{\boldsymbol{\chi}^*, \bar{\boldsymbol{\psi}}^*\} \in \mathscr{M}$ such that: (i) $\bar{\boldsymbol{\mu}}^* = \bar{\boldsymbol{\mu}}^*_{\boldsymbol{p}}$ is the balanced team mechanism associated with the price process $\boldsymbol{p}$; (ii) $\boldsymbol{\sigma}$ is the truth-telling strategy; (iii) $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium associated with the direct mechanism $\bar{\boldsymbol{\mu}}^*$; and (iv) the implied asset allocation is constrained Pareto efficient.*

The proof of proposition 3 follow two steps. First, I use the Lagrange multipliers of proposition

2 in order to build a price sequence such that the optimal asset policy is associated with the constrained Pareto efficient allocation. This is done by equalizing the price in the first order conditions of the bank, equation (16), to the Lagrange multiplier in equation (25). The second step is to generate incentives for truth-telling. I use the balanced team mechanism which always has a truth-telling equilibrium.

A uniqueness result when the inter-bank market is efficient

From previous sections, we know that banks are fragile when they are isolated—$\alpha = 0$. Is the inter-bank market able to eliminate bank fragility? I am able to provide a positive answer to this question when the over-the-counter market frictions are small.

PROPOSITION 4: *There exist $\bar{\alpha} \in (0,1)$ such that, for all $\alpha \in [\bar{\alpha}, 1]$, the constrained Pareto efficient allocation is the unique equilibrium outcome associated with the balanced team mechanism.*

When α is close to one, there is very little risk-sharing among a bank depositors because the bank adjusts its portfolio immediately after a depositor announcement. As a result, the balanced team mechanism only replicates the allocation of assets that each depositors would choose if they were by themselves accessing the market. And, because this is the Walrasian demand, depositors cannot get better by misrepresent their types.

6. A FINANCIAL CRISIS EXAMPLE

I have shown that the model with a single bank generates bank-run equilibria, and that when a large numbers of banks trade assets in a efficient inter-bank market such equilibria disappears. But what happens when the inter-bank market features severe over-the-counter trade frictions?

In this section I provide a numerical example of an equilibrium which resembles a financial turmoil in an economy where the over-the-counter market frictions are severe. Consider the following parametrization. There are $N = 3$ depositors and the total endowments are $\bar{M} = 3.0$ and $\bar{A} = 3.0$. The utility function is a constant relative risk aversion $u(a;\theta) = \theta \frac{a^{1-\delta}-1}{1-\delta}$ with parameter $\delta = 6.0$. The type space is $\Theta = \{\theta_L, \theta_H\} = \{1.0, 1.5\}$. The distribution of types over time is driven by a Markov process Q, where $Q(\theta_L, \theta_L) = 0.95$ and $Q(\theta_H, \theta_H) = 0.2$, and the economy starts in the steady state distribution of Q. Depositors discount the future at rate $\beta = 0.98$. The probability of accessing the market is $\alpha = 0.5$. And assume that there exists $\bar{x} \in \mathbb{S}$ such that $F(\bar{x}) = 10^{-8}$, where F is the distribution of sunspots.

Let me construct the strategy profile that will be supported in equilibrium. Label t_u the first time period in which the sunspot, x_t, is in the interval $[0,\bar{x}]$. Consider the following strategy:

1. Announce truthfully in all periods $t < t_u$.
2. In period t_u announce θ_L.
3. In period $t > t_u$
   - if in period $t-1$ all depositors have announced θ_L, then keep announcing θ_L.
   - otherwise, announce θ_H until all depositors have announced θ_H for 2 consecutive periods. Then announce θ_L.

Label this strategy $\boldsymbol{\sigma}_u$. To summarize, depositors announce truthfully until a sunspot hits the economy, which happens with probability 10^{-8}. The period of this shock is labelled t_u and I interpret it as the period the crisis starts. From t_u forwards, depositors "run" against the bank by announcing θ_L independent of their types. Having depositors announcing θ_L independent of their types can be supported as an equilibrium outcome by the fear of going to a cycle of having every depositors announcing θ_H. That is, this is a dynamic bank-run equilibrium.

I numerically verify that there exists an equilibrium price process, $\boldsymbol{p}$, such that $\{\boldsymbol{\sigma}_u, \boldsymbol{p}\}$ constitutes an equilibrium associated with the balanced team mechanism. This equilibrium display interesting features. The price is constant until period t_u, when the crisis occurs, and then it drops by 9.94 percent. The trade volume has a more interesting dynamic, as depicted in Figure 4. At the moment of the crisis the trade volume increases by 168 percent. This increase is due to a "fire-sale" effect: every bank try to reduce their asset holdings and, as a result, price drops and the trade volume increases. As time pass by, assets are reallocated and the trade volume converges to zero, which I interpret as a collapse of the over-the-counter asset market.

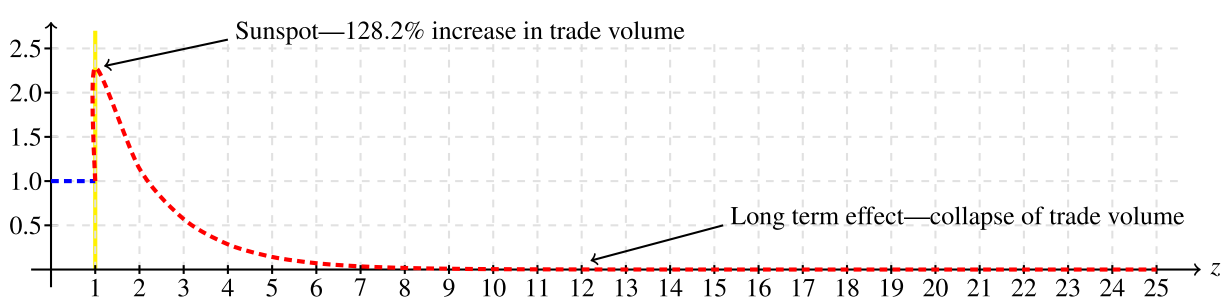

Figure 4: Trade volume

7. THE NON-COMMITMENT CASE

A weakness of the results in the previous sections is that depositors need to be able to commit with a long term relationship, which is a strong assumption. For instance, in a mutual fund there is nothing preventing a depositor from selling his respective share and permanently leave the fund. In this section, I consider the non-commitment version of the model, where depositors cannot commit with future actions. I show that the balanced team mechanism implements the constrained Pareto optimal allocation if β is high. That is, there exists $\bar{\beta} \in (0,1)$ such that for all $\beta \in [\bar{\beta},1)$ the constrained Pareto efficient allocation is an equilibrium outcome. I also show that there exists at least one inefficient equilibrium, which is autarky.[11]

The sequence of actions is the following. Depositors start the period with whatever assets they finish in the period before. Then they announce their types. After announcements are made, the mechanism will suggest how much assets each depositor should transfer to each other, how much assets to trade with the market in case there is access, and how much of the numéraire good to transfer. The key difference here is that the mechanism will only suggest the transfers, whether they are made or not depends on depositors' decision. That is, after observe the suggested transfers, each depositor have the option to deviate from what was suggested (transferring whatever amounts he so desires) and go to autarky, or to sticky with the transfers suggested by the mechanism. I assume that, once a depositor goes to autarky, he cannot get back into the bank. This is without loss of generality with respect to implement the best bank outcome since, in order to generate incentives, we always want to give a depositor the worse punishment for a deviation. The probability of accessing the market once in autarky is the same probability of the bank accessing the market.

The balanced team mechanism need to be extended to account for the possibility of some depositor deviating from the mechanism and going to autarky. The modification I propose is simple. The mechanism follow the balanced team mechanism until a depositor deviate. After a deviation the mechanism recommend autarky to all remaining depositors. Label this mechanism $\bar{\boldsymbol{\mu}}^*$ and call it the modified balanced team mechanism.

PROPOSITION 5: *There exist $\bar{\beta}$ such that, for all $\beta \in [\bar{\beta},1)$, the constrained Pareto efficient allocation is an equilibrium outcome associated with the modified balanced team mechanism $\bar{\boldsymbol{\mu}}^*$.*

The intuition for this result is that going to autarky prevents depositors from engaging in risk sharing. Even though there may be some gains from doing this in the short run, due to some

[11] See Appendix C for a formal discussion.

particular shocks, in the long run it definitely represents a cost. If depositors are patient enough, the long run cost exceeds any short term benefit from a deviation.

Proposition 5 shows that truth-telling is an equilibrium when β is high. On the other hand, autarky is always an equilibrium associated with any feasible mechanism and any β. Given any feasible mechanism, depositors are indifferent between deviating from the mechanism or not when all other depositors are going to autarky. As a result, autarky is always an equilibrium.

8. OPTIMAL POLICY

The model I developed here has a fragile economy in the sense that different equilibria arise and some are associated with low welfare.[12] In this section I consider again the commitment version of the model which I extend to study two alternative policies to reduce financial fragility.

Suspension

Assume that banks can monitor the distribution of announcement vectors in the whole economy. Label $\tilde{\Pi}_t$ the distribution of announcement vectors in period t and $\tilde{\Pi}^t$ the history of $\tilde{\Pi}_t$ realizations. Note that I use capital pi to denote distribution over vector types and non-capital pi to denote distribution over types.

The relevant state for a bank, h_t, needs to be extended in order to account for $\tilde{\Pi}_t$. That is, let $h_t = (\boldsymbol{\theta}_t, \iota_t, p_t, \omega_t, \bar{p}_t)$. The asset policy, the transfer policy, the depositors strategies and the equilibrium definition are extended to capture the extension in the space of histories.

I consider a very simple modification of the balanced team mechanism. Let t_u denotes the first period in which $\tilde{\Pi}_t \neq \Pi_t$, where Π_t is the true distribution of bank type vectors. The modified mechanism follows exactly the balanced team mechanism until $t = t_u$. For $t \geq t_u$, the modified mechanism maximize each depositor utility separately, which is a reversion to DGP and LR where depositors are isolated. That is, for $t \geq t_u$ the bank maximizes

$$(26) \quad \mathbb{E}\left\{\sum_t \beta^{t-t_u}\left[u(\chi_t^n, \theta^n) + \frac{\bar{M}}{N} + p_t(\chi_{t-1}^n - \chi_t^n)\right]\right\}$$

for each depositor n. And the numéraire consumption is $\bar{\psi}^{n*} = \frac{\bar{M}}{N} + p_t(\chi_{t-1}^n - \chi_t^n)$. Label $\bar{\boldsymbol{\mu}}_{mod}^*$ the modified balanced team mechanism. I refer to this policy as a suspension because from t_u forward

[12]One may suggest that the use of a different mechanism, instead of the balanced team mechanism, can eliminate the inefficient equilibria. Unfortunately, the recent developments in implementation theory do not provide such mechanism. Unique, or full, implementation is only guaranteed under a particular set of assumptions and as a limit result as the discount rate, β, goes to one. See, for example, Renou and Tomala (2013).

the bank suspend all risk-sharing among depositors.

PROPOSITION 6: *Consider a price process, $\boldsymbol{p}$, and a associated modified balanced team mechanism, $\bar{\boldsymbol{\mu}}^*_{mod}$. If a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium for the mechanism $\bar{\boldsymbol{\mu}}^*_{mod}$ then $\boldsymbol{\sigma}$ generates a distribution of type vectors that equals the true distribution Π_t at every date t.*[13]

The reason proposition 6 does not guarantee that the equilibrium is truth-telling is because suspension only occurs when the bank knows depositors are misrepresenting. If depositors are playing an strategy that generates the true distribution of announcements, but is not truth-telling, the bank cannot differentiate this from truth-telling. Consequently, there is not threat of suspension. Fortunately, generically this cannot happen, where generically here means on a full measure set of the space of initial distribution of types, π^0, Markov processes, Q, and sunspot distributions F.

PROPOSITION 7: *Generically, given a price process $\boldsymbol{p}$ and associated balanced team mechanism, $\bar{\boldsymbol{\mu}}^*_{\boldsymbol{p}}$, if a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium for the mechanism $\bar{\boldsymbol{\mu}}^*_{mod}$ then $\boldsymbol{\sigma}$ is a truth-telling strategy.*

Suspension prevents bank runs from a different reason that it does in Diamond and Dybvig (1983). In Diamond-Dybvig agents don't run against the bank during a suspension regime because they know there will be enough resources for later depositors. While in my setting suspension works because it kills risk-sharing and, therefore, any possibility for strategic complementarity.

Trading facilities

Assume that there exists a benevolent policy-maker with access to a commitment technology and a trading facility. The policy-maker can produce or consume the numéraire good with a linear technology as other depositors,[14] but he derives no utility from holding assets. A trading facility is a place that can be open or closed. When the facility is closed, the physical environment is that same discussed before and no one can trade with the policy-maker. When the facility is open, every bank has access to it and they can trade with the policy-maker. There is an operational cost $c \geq 0$ in terms of the numéraire good for the policy-maker to open the facility.

The sequence of actions within a period are the following. First, depositors make their announcements in the bank. Second, each bank reports the announcement vectors to the policy-maker. Without loss of generality, I assume that banks cannot misreport depositors announcements. This assumption is without loss of generality because, under the policy I consider, banks will have

[13]See Appendix D for the proofs in this section.
[14]This assumption can be replaced by the assumption that the market-maker can collect taxes.

no incentives to misreport. For the last, the policy-maker decides whether to pay the operational cost and trade occurs.

A policy-maker decision is family $\boldsymbol{f} = \{f_t\}_t$, where $f_t(\tilde{\Pi}_t) = \big(\omega_t(\tilde{\Pi}_t), \bar{p}_t(\tilde{\Pi}_t)\big)$. $\omega_t(\tilde{\Pi}_t) \in \{0,1\}$ denotes whether the policy-maker paid the operational cost $c \geq 0$ in order to trade with the banks, and $\bar{p}_t(\tilde{\Pi}_t) \in \mathbb{R}_+$ is the price the policy-maker is willing to buy and sell assets. Which means the policy-maker acts as a market-maker. Let W^M denotes the maximum welfare attained if the policy-maker operates at every period. Formally, we have that

(27) $W_M = \sum_t \beta^t \sum_\theta \pi_t(\theta) u(a_t^M(\theta)),$

where $\{\pi_t\}$ is the true distribution of types and $\{\boldsymbol{a}_t^M(\theta)\}_t$ satisfies

(28) $\sum_\theta \pi_t(\theta) a_t^M(\theta) = \bar{A}$, and

(29) $u'\big(a_t^M(\theta);\theta\big) = u'\big(a_t^M(\tilde{\theta});\tilde{\theta}\big)$ for all $\theta, \tilde{\theta} \in \Theta$.

Since there is a non-atomic measure of depositors and shock are *i.i.d.*, there is no uncertainty over π_t. That is, $\{\pi_t\}_t$ is a known deterministic sequence.

When the operational cost is low enough such that $W_M - \frac{c}{1-\beta} \geq W(\boldsymbol{a}^*)$, the optimal policy is to pay the operational cost at every period and set the price $\bar{p}_t$ to satisfy the difference equation

(30) $u'\big(a_t^M(\theta);\theta\big) = \bar{p}_t - \beta \bar{p}_{t+1}$ for all period t

for all θ. Label this decision policy $\boldsymbol{f}^M$. For this decision policy there will be a unique equilibrium associated with the balanced team mechanism.

PROPOSITION 8: *Given $\boldsymbol{f}^M$, for any price process, $\boldsymbol{p}$, and associated balanced team mechanism, $\bar{\boldsymbol{\mu}}^*$, the implied depositors-game features a unique equilibrium which is truth-telling.*

Proposition 8 implies that in this setting we can always guarantee stability and a welfare of $W_M - \frac{c}{1-\beta}$. On the other hand, if the operational cost c is so high that $W_M - \frac{c}{1-\beta} < W(\boldsymbol{a}^*)$, using $\boldsymbol{f}^M$ leads to a welfare loss. Is it possible to achieve stability under this circumstances without sacrificing welfare?

The answer to the above question is yes. The simple threat of implementing $\boldsymbol{f}^M$ is enough to guarantee that the equilibrium will be unique. Consider the following policy-maker policy decision. Let t_u denotes the first period in which $\tilde{\Pi}_t \neq \Pi_t$, where Π_t is the true distribution of bank type vectors. The policy-maker decision is $\omega_t = 0$ for all $t < t_u$, and, the policy $\boldsymbol{f}^M$ from t_u forward. Label this decision policy $\boldsymbol{f}^S$.

PROPOSITION 9: *Given $\boldsymbol{f}^S$, for any price process, $\boldsymbol{p}$, and associated balanced team mechanism, $\bar{\boldsymbol{\mu}}^*_{\boldsymbol{p}}$, if a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium for the mechanism $\bar{\boldsymbol{\mu}}^*_{\boldsymbol{p}}$ then $\boldsymbol{\sigma}$ generates a distribution of type vectors that equals the true distribution Π_t at every period t with probability one.*

The reason proposition 9 does not guarantee that the equilibrium is truth-telling is the same of 6. If depositors are playing a strategy that generates the true distribution of announcements, the policy-maker cannot differentiate this from truth-telling, so there is not threat of intervention. But again this cannot happen in general.

PROPOSITION 10: *Generically, given $\boldsymbol{f}^S$, a price process, $\boldsymbol{p}$, and associated balanced team mechanism, $\bar{\boldsymbol{\mu}}^*$, if a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium associated with the mechanism $\bar{\boldsymbol{\mu}}^*$ then $\boldsymbol{\sigma}$ is a truth-telling strategy.*

As before, generically here means on a full measure set of the space of initial distribution of types, π^0, Markov processes, Q, and sunspot distributions F.

9. DISCUSSION AND FUTURE EXTENSIONS

My model helps us to identify one of the causes for the 2007/08 financial crisis. In recent years the market for asset-backed securities (ABSs) has expanded extremely fast. Annual issuance of ABS went from $10 billions in 1986 to $893 billions in 2006, as reported by Agarwal et al. (2010). And a growing shadow bank sector has purchased most of these assets, which are usually traded in an over-the-counter fashion. Moreover, ABSs are complex financial instruments, which make it hard to trade since only very specialized traders are able to evaluate those assets. As a result, in 2007 the financial sector featured a large number of financial institutions operating a market with severe over-the-counter market frictions—all important elements for a fragile bank sector, as suggested by my model.

The most common prescription for enhancing financial stability is to regulate the contracts offered by financial institutions. For example, recently, the Securities and Exchange Commission (SEC) announced a set of proposals to enhance financial stability, which includes a recommendation for the MMF board of directors to impose fees and gate payments in times of heavy redemption activity.[15] And Cochrane (2014) calls for an narrow bank sector funded 100% by equity.

There are two downsides of directly regulating bank contracts. First, each type of financial institution serves a different type of depositor and, therefore, requires a different bank contract. As a result, the regulation needs to be specific to the type of institution. That is, we need one particular regulation for commercial banks, one for mutual funds, one for structured investment vehicles, etc. Which results in a complex regulatory system doomed to feature loopholes and regulatory arbitrage possibilities. The second downside is that, even if we are willing to write complex regulations to every type of financial institution, it is not clear what regulations we should impose. Even a glimpse through the Diamond-Dybvig literature shows that the optimal contract depends on several details of the environment. When you consider models other than Diamond-Dybvig, the possible regulations grow exponentially. Besides, more often than not, regulations have a welfare cost. In which case, the optimal regulation also depends on how the policy-maker evaluates welfare.

In this paper I suggest a different approach to enhance financial stability. Instead of focusing on the particular contract financial intermediaries are offering, I focus on the market where the underlying assets are traded. I show that, if we reduce trade frictions, we enhance financial stability with no need to regulate individual banks. Of course, much research still needs to be done to understand how robust this result is to alternative specifications. However, I believe it offers a much more promising path to financial stability than an overly complex regulatory framework.

There are possible extensions that I do not explore in this paper. To start with, over-the-counter trading relates to two frictions; trade delay due to search for trade partners, and bargaining due to bilateral trade. But in this paper I only explore the trade delay friction. A natural extension is to explicitly model the bargaining process. This extension is challenging because involves bargaining under private information.

Each bank in my model has a fixed group of depositors. In the real world, depositors change their bank with some frequency and banks are always searching for new depositors. An extension that allows for these possibilities would help to understand bank formation and the effect of financial crisis on the size distribution of banks. The challenge of this extension is computational, since the state of a bank grows exponentially with the number of depositors.

My model completely abstracts from the real side of the economy. An interesting extension would be to build a channel through which bank runs have real effects. One way of establishing this channel could be to have a real side of the economy producing financial assets from loans to firms

[15]See SEC (2013).

and consumers—a form of securitization trough which assets are created. In this case, a drop in asset prices would reduce incentives for lending.

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APPENDIX A: INEFFICIENCY RESULTS

Proof of claim 1

PROOF: I will use the one deviation principle to show the result. First, assume a depositor deviates in a period $t = 0$ by announcing θ_H. If the depositor is of type θ_L, the total gain of deviation is

$$-0.0129-\beta\frac{1-\beta^5}{1-\beta}0.0023.$$

The first term is the gain of the deviation. And the second is the expected gain of deviation implied by staying 5 consecutive periods announcing θ_H. Since both are negative, this deviation is not profitable. Now, assume the depositor is of type θ_H. In this case his gain of deviation is

$$0.0071-\beta\frac{1-\beta^5}{1-\beta}0.0023<0 \iff$$
$$0.0071<\beta\frac{1-\beta^5}{1-\beta}0.0023\leq\lim_{\beta\nearrow 1}\beta\frac{1-\beta^5}{1-\beta}0.0023=0.0117.$$

Thus, there exist $\bar{\beta}_0$ such that, if $\beta \geq \bar{\beta}_0$, the gain of deviation is strictly negative.

For the last, let us check a deviation in periods $t > 0$. If the depositors are supposed to announce truthfully forever, then a deviation is welfare decreasing since truth-telling is a strictly dominant strategy of the stage game. Assume someone deviate in period $t = 0$ and depositors are suppose to announce θ_H for five consecutive periods—the punishment stage described in bullet three. If the depositor is of type θ_L, the total gain of deviation is

$$0.0016-\beta^{5-s}0.0023,$$

where s is the number of consecutive periods all depositors have announced θ_H before. The gain from deviation is bounded above by

$$0.0016-\beta^5 0.0023<0 \Leftrightarrow 0.0016<\beta^5 0.0023\leq\lim_{\beta\nearrow 1}\beta^5 0.0023=0.0023.$$

Thus, there exist $\bar{\beta}_1$ such that, if $\beta \geq \bar{\beta}_1$, the gain of deviation is strictly negative. If the depositor is of type θ_H, the total gain of deviation is

$$-0.0239-\beta^{5-s}0.0023$$

which is always negative.

Label $\bar{\beta}$ the maximum between $\bar{\beta}_0$ and $\bar{\beta}_1$. Then, we can conclude that, if $\beta \in [\bar{\beta}, 1)$, the the gain of any deviation is strict negative and the proposed strategy profile constitutes a perfect Bayesian equilibrium of the depositors-game. □

APPENDIX B: EFFICIENCY RESULTS

Poof of proposition 1

PROOF: Consider the problem

$$\sup\left\{\mathbb{E}\textstyle\sum_t \beta^t \left[U(A_t, \boldsymbol{\theta}_t) + p_t(A_{t-1} - A_t)\right];\, \boldsymbol{A} = \{A_t\}_t \in \mathscr{A}\right\}, \tag{31}$$

where $\mathscr{A}$ denotes the set of random sequences, $\boldsymbol{A} = \{A_t(h^t)\}_t$, satisfying, for all $h^t \in H^t$, $A_0 = \bar{A}$ and $(1 - \iota_t)[A_{t-1}(h^{t-1}) - A_t(h^t)] = 0$.

Let me start showing that conditions (ii) and (iii) are sufficient for $\boldsymbol{A} \in \mathscr{A}$ to solve problem (31). Consider a sequence $\boldsymbol{A} \in \mathscr{A}$ satisfying (ii) and (iii) and another arbitrary sequence $\tilde{\boldsymbol{A}} \in \mathscr{A}$. The difference in the objective function of problem (31) implied by $\boldsymbol{A}$ and $\tilde{\boldsymbol{A}}$ is

$$\begin{aligned}
D = \lim_{K\to\infty} \mathbb{E}\Big\{&\textstyle\sum_{k=1}^{K-1} \beta^{t_k}\big\{\sum_{d=0}^{d_k-1}\beta^d\left[U(A_{t_k};\boldsymbol{\theta}_{t_k+d}) - U(\tilde{A}_{t_k};\boldsymbol{\theta}_{t_k+d})\right] \\
&- \left[p_{t_k} - \beta^{d_k} p_{t_k+d_k}\right]\left[A_{t_k} - \tilde{A}_{t_k}\right]\big\} - \beta^{t_K} p_{t_K}\left[A_{t_K} - \tilde{A}_{t_K}\right]\Big\} \\
\geq \lim_{K\to\infty} \mathbb{E}\Big\{&\textstyle\sum_{k=1}^{K-1} \beta^{t_k}\big\{\sum_{d=0}^{d_k-1}\beta^d U'(A_{t_k};\boldsymbol{\theta}_{t_k+d})\left[A_{t_k} - \tilde{A}_{t_k}\right] \\
&- \left[p_{t_k} - \beta^{d_k} p_{t_k+d_k}\right]\left[A_{t_k} - \tilde{A}_{t_k}\right]\big\} - \beta^{t_K} p_{t_K}\left[A_{t_K} - \tilde{A}_{t_K}\right]\Big\} \\
= \lim_{K\to\infty} \mathbb{E}\Big\{&\textstyle\sum_{k=1}^{K-1} \beta^{t_k}\mathbb{E}\Big\{\sum_{d=0}^{d_k-1}\beta^d U'(A_{t_k};\boldsymbol{\theta}_{t_k+d}) - \left[p_{t_k} - \beta^{d_k} p_{t_k+d_k}\right]\Big| h^{t_k}\Big\}\left[A_{t_k} - \tilde{A}_{t_k}\right] \\
&- \beta^{t_K} p_{t_K}\left[A_{t_K} - \tilde{A}_{t_K}\right]\Big\}.
\end{aligned}$$

Condition (ii) implies that

$$\mathbb{E}\Big\{\textstyle\sum_{d=0}^{d_k-1}\beta^d U'(A_{t_k};\boldsymbol{\theta}_{t_k+d}) - \left[p_{t_k} - \beta^{d_k} p_{t_k+d_k}\right]\Big| h^{t_k}\Big\}$$

equals zero almost surely. Hence,

$$D \geq -\lim_{K\to\infty} \mathbb{E}\left\{\beta^{t_K} p_{t_K}\left[A_{t_K} - \tilde{A}_{t_K}\right]\right\} \geq -\lim_{K\to\infty} \mathbb{E}\left\{\beta^{t_K} p_{t_K} A_{t_K}\right\}.$$

The last term of the above equation equals zero due to condition (*iii*). Therefore, we can conclude that conditions (*ii*) and (*iii*) are sufficient for a sequence $\boldsymbol{A} \in \mathscr{A}$ to solve problem (31). And we have by construction that if $\boldsymbol{A}$ solves problem (31) then a policy $\boldsymbol{\chi}$ satisfying condition (*i*) solves problem (14). □

Existence and uniqueness of a constrained Pareto efficient allocation

LEMMA 1: *There exists a unique constrained Pareto efficient allocation.*

PROOF: Note that W is a continuous map and $\mathscr{F}$ is a compact space when equipped with the norm $||\cdot||_\beta$, which defined as

$$||\boldsymbol{a}||_\beta = \sum_t \beta^t \sup\{|a_t(\boldsymbol{\theta}^t, \iota^t)|;\ \boldsymbol{\theta}^t, \iota^t \in \boldsymbol{\Theta}^t \times \{0,1\}^t\}.$$

Therefore, by the Weierstrass theorem, there exists an allocation $\boldsymbol{a}$ that maximizes W among all outcomes in $\mathscr{F}$. The uniqueness result comes from the strict concavity of $u(\cdot;\theta)$, for each $\theta \in \Theta$, and the convexity of the set $\mathscr{F}$. □

Proof of proposition 2

PROOF: Consider the problem

$$\text{(32)} \quad \max \left\{ \mathbb{E} \sum_t \beta^t U(A_t; \vec{\theta}_t);\ \boldsymbol{A} \in \mathscr{A} \right\},$$

where $\boldsymbol{A} = \{A_t\}_t$ is a sequence of maps $A_t : \boldsymbol{\Theta}^t \times \{0,1\}^t \to [0, NB]$ and $\mathscr{A}$ is the set of sequences $\boldsymbol{A} = \{A_t\}_t$ satisfying

$$A_t(\boldsymbol{\theta}^t, \iota^{t-1}, 0) = A_{t-1}(\boldsymbol{\theta}^{t-1}, \iota^{t-1}),$$
$$\sum_{\boldsymbol{\theta}^t} \sum_{\iota^t} \mathbb{P}(\boldsymbol{\theta}^t, \iota^t) A_t(\boldsymbol{\theta}^t, \iota^t) = \bar{A}, \text{ and}$$
$$A_0(\boldsymbol{\theta}_0) = \bar{A}.$$

It is easy to see that an outcome $\boldsymbol{a}$ is constrained optimal if, and only if, there exists $\boldsymbol{A} = \{A_t\}_t \in \mathscr{A}$ such that: $A_t(\boldsymbol{\theta}^t, \iota^t) = \sum_n a_t^n(\boldsymbol{\theta}^t, \iota^t)$, $\boldsymbol{a}_t(\boldsymbol{\theta}^t, \iota^t)$ solves (15) for $A = A_t(\boldsymbol{\theta}^t, \iota^t)$, and $\boldsymbol{A}$ achieves the maximum of problem (32). Hence, I just need to show that the existence of the Lagrange multipliers satisfying equation (25) is necessary and sufficient for an $\boldsymbol{A} \in \mathscr{A}$ to solve problem (32). This result can be derived from theorem 1, section 8.3, and theorem 1, section 8.4, in Luenberger (1969). □

Proof of proposition 3

PROOF: Let $\boldsymbol{\lambda} = \{\lambda_t\}_t$ denotes the Lagrange multipliers in proposition (2). And consider the deterministic price sequence $\boldsymbol{p} = \{p_t\}_t$ defined by

(33) $p_t = \lambda_t - \alpha\beta\sum_{d=0}^{\infty}\beta^d\lambda_{t+1+d}$

for all t. Let $\bar{\boldsymbol{\mu}}_{\boldsymbol{p}}^* = \{\boldsymbol{\chi}_{\boldsymbol{p}}^*, \bar{\boldsymbol{\psi}}_{\boldsymbol{p}}^*\} \in \mathscr{M}$ be the balanced team mechanism associated with the price sequence $\boldsymbol{p}$, and let $\boldsymbol{\sigma}$ be the truth-telling strategy. From Athey and Segal (2013), $\boldsymbol{\sigma}$ is a perfect Bayesian equilibrium of the depositors-game implied by $\bar{\boldsymbol{\mu}}_{\boldsymbol{p}}^*$ and $\boldsymbol{p}$. From equation (33), it is easy to see that the sequence $\boldsymbol{p}$ satisfies

(34) $\lambda_t = p_t - \sum_{d=1}^{\infty}\beta^d(1-\alpha)^{d-1}\alpha p_{t+d}$.

Equation (34), combined with propositions (1) and (2), implies that the optimal asset policy satisfies $\chi_{pt}^{*n}(\boldsymbol{\theta}^t, \iota^t, p^t) = a^{*n}(\boldsymbol{\theta}^t, \iota^t)$. Since $\boldsymbol{a}^*$ is feasible, the market clearing condition must be satisfied. And for the last, the implied allocation coincides with the constrained Pareto efficient allocation $\boldsymbol{a}^*$.
□

Proof of proposition 4

PROOF: For $\alpha = 1$, the optimal mechanism reflects the Walrasian demand associated with the depositors type and the market cost of the implied demand of assets. As a result, the balanced team mechanism only replicates the allocation of assets that each depositor would choose if they were by themselves accessing the market. And, because this is the Walrasian demand associated with a strictly continuous utility function, depositors get strictly worse off by misrepresent their types. Since payoffs in the depositors-game are continuous in α, depositors also get strictly worse off by misrepresent their type when α is in a neighbourhood of 1. □

APPENDIX C: THE NON-COMMITMENT CASE

Let me extend the notation introduced before to account for the depositors decision to go to autarky. Let $\tilde{\boldsymbol{\Theta}} := \Theta \cup \{aut\}$ where $\theta^n \in \Theta$ denotes that depositor n is not in autarky and have announced type θ^n and $\theta^n = aut$ denotes that depositor n is in autarky. In a given period t, let $\boldsymbol{\theta}_t = (\theta_t^1, \ldots, \theta_t^N) \in \tilde{\boldsymbol{\Theta}} := \tilde{\Theta}^N$ denotes the vector. I denote the period t realized variables $h_t = (\boldsymbol{\theta}_t, \iota_t, p_t) \in H := \tilde{\boldsymbol{\Theta}} \times \{0,1\} \times \mathbb{R}_+$, and the history of realizations $h^t = (\boldsymbol{\theta}^t, \iota^t, p^t) \in H^t$.

An asset policy is a sequence $\boldsymbol{\chi} = \{\boldsymbol{\chi}_t\}_t$, where $\boldsymbol{\chi}_t = (\chi_t^1, \dots, \chi_t^N) : H^t \to \mathbb{R}_+^N \cup \{aut\}$. If $\boldsymbol{\chi}_t \in \mathbb{R}_+^N$ then it denotes how much assets to allocate for each depositor in the bank. If $\boldsymbol{\chi}_t \in \{aut\}$ then it denotes a suggestion for depositors to go to autarky. An asset policy $\boldsymbol{\chi} = \{\boldsymbol{\chi}_t\}$ is feasible if $\sum_n \chi_0^n = N\bar{A}$; for all $h^t \in H^t$, either $\iota_t = 1$ or $\sum_n \mathbb{1}_{\theta_t^n \neq aut} \chi_t^n(h^t) = \sum_n \mathbb{1}_{\theta_t^n \neq aut} \chi_{t-1}^n(h^{t-1})$; and $\theta_t^n = aut$ implies $\chi_t^n(h^t) = 0$. The first condition is the same as before. The second condition is that the bank can only adjust its aggregate asset holdings once it accesses the centralized market, but it can only make with depositors that are not in autarky. The last condition is that the suggested transfer for people in autarky is always zero. Let Γ denotes the set of all feasible asset policies.

Note that a asset policy can be decentralized in many different ways. How much assets depositor 1 transfer to depositor 2 or depositor 3 can be set in different ways so it implies the same aggregate transfer. For simplicity, I impose this arrangement to be proportional. For instance, suppose depositors 1 and 3 are the depositors to receive net transfers of assets. And suppose that depositor 1 is going to receive 60% of the net transfer of assets while depositor 3 is going to receive 40%. That is,

$$\frac{\chi_t^1(h^t) - \chi_{t-1}^1(h^{t-1})}{\sum_{n=1,3} \chi_t^n(h^t) - \chi_{t-1}^n(h^{t-1})} = 0.6 \quad \text{and} \quad \frac{\chi_t^3(h^t) - \chi_{t-1}^3(h^{t-1})}{\sum_{n=1,3} \chi_t^n(h^t) - \chi_{t-1}^n(h^{t-1})} = 0.4.$$

Then, each depositor $n \neq 1,3$ will transfer 60% to depositor 1 and 40% to depositor 2 of the net amount he is suppose to transfer, $\chi_{t-1}^n(h^{t-1}) - \chi_t^n(h^t)$.

A numéraire policy is a sequence $\boldsymbol{\psi} = \{\boldsymbol{\psi}_t\}_t$, where $\boldsymbol{\psi}_t = (\psi_t^1, \dots, \psi_t^N) : H^t \to \mathbb{R}^N \cup \{aut\}$. If $\boldsymbol{\psi}_t \in \mathbb{R}_+^N$ then it denotes how much numéraire good to transfer to each depositor in the bank. If $\boldsymbol{\psi}_t \in \{aut\}$ then it denotes a suggestion for depositors to go to autarky. A transfer policy $\boldsymbol{\psi} = \{\boldsymbol{\psi}_t\}$ is feasible if $\sum_n \mathbb{1}_{\theta_t^n \neq aut} \psi_t^n(h^t) = p_t \sum_n \mathbb{1}_{\theta_t^n \neq aut} \left[\chi_t^n(h^t) - \chi_{t-1}^n(h^{t-1})\right]$ for all $h^t \in H^t$; and $\theta_t^n = aut$ implies $\psi_t^n(h^t) = 0$. That is, the transfers between depositors that have not gone to autarky are budget balanced and a depositor in autarky does not receive any numéraire transfer.

A pure strategy for an depositor n is a sequence $\boldsymbol{s}^n = \{\boldsymbol{\sigma}^n, \boldsymbol{a}^n, \boldsymbol{y}^n\} = \{\sigma_t^n, \boldsymbol{a}_t^n, \boldsymbol{y}_t^n\}_t$ of measurable functions where, σ_t^n maps $(h^{t-1}, \theta_t^n, x^t)$ into Θ and $(\boldsymbol{a}_t^n, \boldsymbol{y}_t^n)$ maps (h^t, θ_t^n, x^t) into $\mathbb{R}_+^{N-1} \times \mathbb{R}^{N-1}$. σ_t^n is the announcement decision, as before. $(\boldsymbol{a}_t^n, \boldsymbol{y}_t^n)$ is the asset and numéraire transfer to each other $N-1$ depositors. The implied depositors-game is analogous to the commitment case.

As before, the only aggregate uncertainty in the economy comes from the public sunspots. Therefore, the price in a period t is a deterministic function of the public signal vector x^{pt}. The market clearing condition requires that, for a given realization of the sequence x^{pt}, the implied demand of assets equal the total offer of assets at a period t.

Let $\boldsymbol{\eta} = \{\eta_t\}_t$ be the sequence of measures over the space H^t which is generated by Q, α, F, $\boldsymbol{p}$, $\boldsymbol{\mu} \in \mathscr{M}$, and $\{\boldsymbol{s}^n\}_n$. The aggregate demand for assets in the centralized market in period t is

(35) $D_t(x^{pt}) := \int \sum_n \sum_{i \neq n} a^n_{ti}(h^t) d\eta_t(h^t | x^{pt}).$

Markets clear at a period t if $D_t(x^{pt})$ equals $\bar{A}$ for all x^{pt} in the support $\mathbb{S}^{pt}$.

DEFINITION 2: Given a feasible mechanism $\boldsymbol{\mu} \in \mathscr{M}$, a symmetric equilibrium is a pair $\{\boldsymbol{s}, \boldsymbol{p}\}$ such that: (i) $\boldsymbol{s}^n = \boldsymbol{s}$ for all depositors form a perfect Bayesian equilibrium of the depositors-game implied by $\boldsymbol{\mu}$; and (ii) markets clear at every period.

The balanced team mechanism need to be extended to account for the possibility of some depositor deviate and go to autarky. The modification I propose is simple. The mechanism follow the balanced team mechanism until an depositor deviate. After a deviation the mechanism recommend autarky to all remaining depositors.

PROPOSITION 11: *There exist $\bar{\beta}$ such that, for all $\beta \in [\bar{\beta}, 1)$, there exists a price process $\boldsymbol{p}$, a strategy $\boldsymbol{s} = \{\boldsymbol{\sigma}, \boldsymbol{a}, \boldsymbol{y}\}$ and a feasible mechanism $\bar{\boldsymbol{\mu}}^* = \{\boldsymbol{\chi}^*, \bar{\boldsymbol{\psi}}^*\} \in \mathscr{M}$ such that: (i) $\bar{\boldsymbol{\mu}}^* = \bar{\boldsymbol{\mu}}^*_{\boldsymbol{p}}$ is the modified balanced team mechanism associated with the price $\boldsymbol{p}$; (ii) $\{\boldsymbol{s}, \boldsymbol{p}\}$ is a symmetric equilibrium associated with the mechanism $\bar{\boldsymbol{\mu}}^*$; (iii) $\boldsymbol{\sigma}$ is the truth-telling strategy ; (iii) $\boldsymbol{a}$ and $\boldsymbol{y}$ follow the mechanism recommendation; and (iv) the implied asset allocation is constrained Pareto efficient.*

PROOF: The proof of this proposition follows exactly the same steps of the proof of proposition 15. The only difference being that we cannot apply Athey and Segal (2013) to show that follow the mechanism transfer recommendation is an equilibrium since it requires commitment. Athey and Segal (2013) have an extension of their result for the environment without commitment which guarantees that cooperation with the mechanism is an equilibrium for β high, this result is given by proposition 4 in their paper. The reason we cannot directly apply proposition 4 in Athey and Segal (2013) is because the payoffs of the depositors-game, and also the optimal mechanism, varies with β. On the other hand, if we exam equation (34) and (25) we see that the equilibrium price and Lagrange multipliers, normalized by $(1-\beta)$, converge as β goes to 1. Therefore, we can apply the proposition 4 in Athey and Segal (2013) to show that cooperation is an equilibrium of this limit game. And, since autarky makes the depositor strictly worse off than cooperation, the result will also hold for β high. □

PROPOSITION 12: *Autarky is an equilibrium of the depositors-game implied by any price process, $\boldsymbol{p}$, and feasible mechanism, $\bar{\boldsymbol{\mu}}$.*

PROOF: If all depositors in the bank go to autarky, a depositor payoff of staying in the bank is bounded above by the autarky payoff. Therefore, going to autarky must be a best response. □

APPENDIX D: OPTIMAL POLICY

Proof of proposition 6

PROOF: Suppose by the way of contradiction that a pair $\{\boldsymbol{\sigma}, \boldsymbol{p}\}$ is an equilibrium and $\boldsymbol{\sigma}$ generates a sequence $\tilde{\Pi}_t \neq \Pi_t$ along the equilibrium path. Let t_u denotes the the first period in which $\tilde{\Pi}_t \neq \Pi_t$. Note that from date t_u forward depositors know that the bank will maximize his utility individually, hence, truth-telling is a dominant strategy. Thefore, $\boldsymbol{\sigma}$ need to be consistent with truth-telling for $t = t_u$ forwards, which contradicts the fact that $\tilde{\Pi}_t = \Pi_t$. □

Proof of proposition 7

PROOF: At any period t, the distribution of announcements needs to be generated by the distribution of previous types and sunspots realizations. That is, the announcement vector in period t is a function $g_t(\boldsymbol{\theta}^t, x^t)$. If g_t generates the same distribution of announcement vector Π_t, then it needs to satisfy, for all $\boldsymbol{\theta}_t \in \boldsymbol{\Theta}$,

$$\begin{aligned}(36)\quad \mathbb{P}[g_t(\boldsymbol{\theta}^t, x^t) = \boldsymbol{\theta}_t] &= \textstyle\sum_{\tilde{\boldsymbol{\theta}}^t, \tilde{x}^t \in g_t^{-1}(\boldsymbol{\theta}_t)} \mathbb{P}(\tilde{x}^t)\Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s) \\ &= \Pi_t(\boldsymbol{\theta}_t) = \textstyle\sum_{\tilde{\boldsymbol{\theta}}^t \in \boldsymbol{\Theta}^t} \mathbb{1}_{\tilde{\boldsymbol{\theta}}_t = \boldsymbol{\theta}^t} \Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s).\end{aligned}$$

Note that the above equation always holds if $g_t^{-1}(\boldsymbol{\theta}_t) = \{\tilde{\boldsymbol{\theta}}^t, \tilde{x}^t \in \boldsymbol{\Theta}^t \times \mathbb{S}^t;\ \tilde{\boldsymbol{\theta}}_t = \boldsymbol{\theta}_t\}$, that is, under truth-telling. Now suppose this equation holds for some function g_t such that $g_t^{-1}(\boldsymbol{\theta}_t) \neq \{\tilde{\boldsymbol{\theta}}^t, \tilde{x}^t \in \boldsymbol{\Theta}^t \times \mathbb{S}^t;\ \tilde{\boldsymbol{\theta}}_t = \boldsymbol{\theta}_t\}$. Consider a initial distribution $\hat{\Pi}_0 \neq \Pi_0$ such that $||\hat{\Pi}_0 - \Pi_0|| < \varepsilon$ and a transition $\hat{Q}_c$ such that $||\hat{Q}_c - Q_c|| < \varepsilon$. Since $\boldsymbol{\Theta}^t \times \mathbb{S}^t$ is finite, any change in g_t implies a discontinuous jump on the left hand side of equation (36). As a result, equation (36) cannot be satisfied for a different g_t if we take ε to be small enough. Note that

$$\textstyle\sum_{\tilde{\boldsymbol{\theta}}^t, \tilde{x}^t \in g_t^{-1}(\boldsymbol{\theta}_t)} \mathbb{P}(\tilde{x}^t)\Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s)$$

and

$$\sum_{\tilde{\boldsymbol{\theta}}^t \in \boldsymbol{\Theta}^t} \mathbb{1}_{\tilde{\boldsymbol{\theta}}_t = \boldsymbol{\theta}^t} \Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s)$$

are two different weighted sums of the terms $\Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s)$. Therefore, when the weights are fixed, any perturbation in the terms $\Pi_0(\tilde{\boldsymbol{\theta}}_0) \bigtimes_{s=1}^{t} Q_c(\tilde{\boldsymbol{\theta}}_{s-1}, \tilde{\boldsymbol{\theta}}_s)$ will, generically, breaks the equality. Which concludes the proof. □

Proof of proposition 8

PROOF: Since depositors access the market-maker at every period they don't cause externalities to each other. Therefore, the balanced team mechanism allocates the optimal demand associated with the Walrasian demand given the price sequence $\{\bar{p}_t\}_t$. Since this demand is optimal, the depositor cannot get any better by misrepresenting his type. □

Proof of proposition 9

PROOF: Analogous to proposition 6. □

Proof of proposition 10

PROOF: Analogous to proposition 7. □