A Model of Competitive Saving Over the Life-cycle, and its Implications for the Saving Rate Puzzle in China*

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Abstract

This paper develops a life-cycle model with multi-dimensional matching in a frictionless marriage market where a single man's rank is determined by his age and wealth relative to others. Using stable matching patterns, I analyze how people's marital age and saving behavior jointly respond to marriage market imbalances. In particular, with wealth as a status good for mating competition, an otherwise standard age profile of saving rates is downward-sloping in the pre-marital cohorts under the marriage squeeze. This result is consistent with recent empirical findings from China. In addition, both the age gap between spouses and the aggregate saving rate rise when the sex ratio increases. Neglecting the response in marital ages will lead to an overstatement of the effect of the sex ratio imbalance on the increase of the aggregate saving rate.

Keywords: marriage market, saving rate, multi-dimensional matching.

JEL classification: J11, J12, D10, D91, E21, O53.

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1. Introduction

The permanent income hypothesis predicts that people should save less when their income is growing. However, China's saving rates in recent years exhibit the opposite pattern on both macroeconomic and microeconomic levels. On the one hand, China's urban household saving rate rose by 10 percent of disposable income from 1990 to 2007 against a background of rapid income growth and a constantly low real interest rate. On the other hand, researchers such as Chamon and Prasad (2010) and Song and Yang (2010) document that the age profile of the savings rate exhibits a downward-sloping pattern in early ages with younger households having higher saving rates than middle-aged cohorts even when the lifetime earning profile follows the common hump-shaped path that peaks in middle age. These unusual patterns are denoted as the *saving rate puzzle* in China.

Several recent papers such as Wei and Zhang (2011) and Du and Wei (2013) propose a competitive saving motive as an explanation for the rapid rise of the aggregate saving rate. Single men accumulate savings in a competitive manner in order to improve their relative position in the marriage market, and this competition gets fiercer with an increase in the sex ratio (the number of men per women) in the pre-marital cohort. In Wei and Zhang (2011), the authors estimate that such a saving motive accounts for about half of the observed increase in the aggregate household saving rate in recent years.

Although the theory about the competitive saving motive is insightful, it leaves many important questions unanswered. For example, why is the age profile of saving rates downward-sloping in the pre-marital cohorts? Would such a competitive saving motive be mitigated by other responses of single people such as a change in the age gap between spouses - a well documented phenomenon under the marriage squeeze (for example, see Anderson, 2007; Abramitzky et al., 2011)? The goal of this paper is to develop a unified model to clarify these questions. In particular, I will show to what extent the current saving rate puzzle can be explained by a competitive saving motive when single men have the option to postpone their marriage age in addition to increasing their savings.

In this paper I construct an overlapping generations model with two-dimensional matching on the marriage market, in which men are differentiated by wealth and age. Starting from age 1, single men allocate their liquid wealth between consumption and saving with the understanding that when they reach their marital ages, men who accumulate more wealth are strictly preferred[1]. Since their labor income is subject to independently and identically distributed transitory shocks at each age, wealth inequality increases with age which reflects the cumulative differences in the effect of luck on savings. With an unbalanced sex ratio, men from the top of the saving distribution can be matched at their earliest marriageable age, while men from the bottom of the saving distribution who fail to be matched stay single and rejoin the marriage market in the next period. For those men who postpone marriage, although increased age in the next period serves as a disadvantage for them to get a better match, they may compensate for this by accumulating more wealth since they now have additional periods to save and could possibly experience good labor income shocks in those periods. In other words, a young woman may find men with different {saving, age} combinations equally attractive since although marrying an older man provides less periods of companionship, she can enjoy more pre-marital saving brought into the family by him. I find that a stable matching always exists in this model but it may fail to be pure due to the trade-off between wealth and age. Some general properties of the matching patterns are derived, and under common functional forms, the stable matching can be characterized.

I show that when the sex ratio in the marriage market is highly unbalanced as is the case in China, the age profile of saving rates of the pre-marital cohorts is downward-sloping even with an increasing age-income profile due to the different level of uncertainties about marriage prospects single men are subjected to when they approach marriageable age. For a newly born man, no matter what his current income is, his marriage prospect is highly uncertain because he will be subject to other income shocks between his current age and marriageable age, and his position in the marriage market is determined by his wealth at the marriageable age only. As a result, all age 1 men choose to save most part of their income when the chance of improving their marriage prospects is still high. However, as a man approaches the marriageable age, uncertainties in labor income are realized gradually and his position in the marriage market is in large part determined by the wealth he has

[1]In reality, accumulating more saving may not be the only way to improve one's rank. Single men can also choose to invest in their human capital. Since these two channels are complementary, adding human capital into this model framework may dampen the reaction of saving rate to the sex ratio but should not change the direction of reaction. To limit the dimension of the matching game and keep the model tractable, human capital accumulation is dropped from the model and left for future research.

accumulated. Consequently, there is little chance for him to change his position in the marriage market now. This leads to a reduction in returns to saving and a decrease in the saving rate at this age.

A natural question is how the saving behaviors of the women respond to men's saving behaviors when they can share their husbands' wealth after marriage. I assume women's relative position in the marriage market does not depend on their wealth, and thus young women accumulate much less saving than men. After we aggregate the savings of men and women in each age cohort, the overall age profile of saving rates of single agents is still downward-sloping.

To check whether the model can explain the observed saving rate puzzle, I employ quantitative calibrations and find the following results: (1) The saving rate profile of people younger than 36 exhibits similar patterns to those seen in the data. (2) If the age of marriage is exogenous, as the sex ratio rises from 1 to 1.18, the aggregate household saving rate would rise by 6.50% of GDP. However, if single men can postpone their marriage age, pressure on the saving rate is tempered and the aggregate saving rate rises by 3.35% of GDP instead.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents key facts on household saving in China. Section 4 presents the two-period benchmark model to show how a competitive saving motive arises when wealth is a status good. Section 5 presents a marriage model with an endogenous age gap between spouses and shows how saving and marital age jointly react to marriage market imbalances. Section 6 extends the benchmark model to a multi-period framework. Section 7 calibrates the model using data from the Chinese economy and derives numerical results as an explanation of the saving rate puzzle in China. Concluding remarks are offered in Section 8.

2. Related Literature

This paper is related to several strands of literature. First, there is literature on pre-marital investment. To the best of my knowledge, this is the first paper to propose and solve pre-marital investment and stable matching patterns in a multi-dimensional matching framework. Previous literature on pre-marital investment focuses on exactly one characteristic on which the matching process is based. Various studies have thus investigated the resulting matching patterns and welfare implications (Peters and Siow, 2002; Iyigun and Walsh, 2007; Cole et al., 2001; Chiappori et al., 2009). Although the basic mathematical structure underlying matching models does not require a one-dimensional characteristic, the multi-dimensional framework often raises difficult problems in applications and thus are rarely analyzed in the literature[2]. Chiappori et al. (2012) developed a two-dimensional matching model using a transferable utility framework where individuals are characterized by a continuous trait (socioeconomic status) and a discrete trait (whether a smoker). Stable matching patterns are characterized in closed form in their paper. However, all the characteristics are exogenous in their model and pre-marital investment is not considered.

A second strand of literature is on the marriage squeeze and the age of marriage. The marriage squeeze refers to an imbalance between the numbers of marriageable men and women in the marriage market, potentially due to war, unbalanced sex ratio at birth or shocks to population growth rate since men on average marry younger women. The key potential margin of adjustment to such imbalances is via the age gap at marriage. If the age gap increases in response to excess men in the marriage market, such an imbalance can be reduced, if not eliminated. Existing empirical studies find that the marriage age exhibits considerable flexibility to accommodate the marriage squeeze problem (Bergstrom and Lam, 1989b; Brandt et al., 2008; Abramitzky et al., 2011). For example, Brandt et al. (2008) study the marriage market consequence of the Chinese 1959-1961 famine, and find that the marriage rates of the famine born cohort were not significantly changed. This result is noteworthy given this famine reduced the cohort size by 75%. On the other hand, applied theoretical studies of the age gap between spouses usually either assume age is the only trait that differentiates people, or assume that other socioeconomic characteristics are independent of age (Bergstrom and Lam, 1989a; Anderson, 2007; Bhaskar, 2013). Although such treatment offers a simple and elegant way to study the age gap problems, it comes at the cost of realism. People's socioeconomic status such as income, wealth, profession, education and physical conditions do change with age, and usually these characteristics affect people's choice of marriage age.

[2] See Chiappori et al. (2010) for a survey and analysis of the mathematical tool of multi-dimensional matching problems.

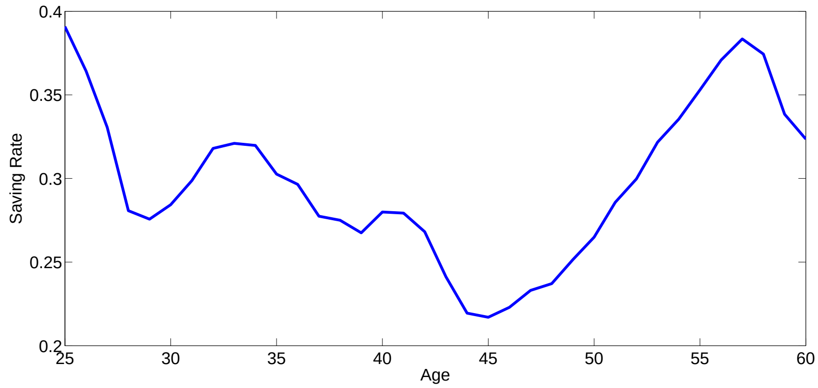
This paper is also related to the status good literature, such as Cole et al. (1992), Hopkins and Kornienko (2004) and Bhaskar and Hopkins (2013). One common insight of these papers is that if individuals care about their status defined as their rank in the distribution of spending on one "positional good", consumers' choices will be strategic and depend on the consumption choices of others. In such a situation, spending on the "positional good" generally shows an over-spending pattern. While this literature focuses on welfare analysis of an exogenously changed income distribution, my primary goal is to understand the effect of marriage market imbalance on an individual's choice of saving when each man's ranking in the marriage market is determined by his wealth relative to others.

Finally, there is a sizable literature offering explanations of the saving rate puzzle in China from different perspectives[3]. Among these papers, in Du and Wei (2013) a life-cycle matching model which also highlights the competitive saving motive is developed. They assume single men are endowed with the same income and save the same amount. In equilibrium, everyone's ranking in the marriage market is determined by a random draw of pizazz. When their model is calibrated to the Chinese economy, they find that the aggregate saving rate will increase by 6% of GDP as the sex ratio rises from 1 to 1.15, which is similar to the result in this paper. What makes this paper different from theirs is that the marriage market here is dynamic in which the age gap between spouses can adapt in response to marriage market imbalances. As illustrated in the calibration, allowing for an endogenous change of the age gap significantly tempers the pressure on the aggregate saving rate from an unbalanced sex ratio. In addition, this paper offers an explanation for the downward-sloping age profile of saving rates in China.

The contributions of this paper are both theoretical and empirical. On the theoretical aspect, to the best of my knowledge this is the first paper which solves the stable matching patterns in a multi-dimensional matching framework with pre-marital investment. On the empirical aspect, this theory can potentially help to understand the saving rate puzzle in China on both macroeconomic and microeconomic levels.

[3]Scholars have offered explanations for the saving rate puzzle from perspectives such as life cycle consideration (Modigliani and Cao, 2004), precautionary saving with borrowing constraints (Wen, 2010; Chamon et al., 2013), demographic change (Wei and Zhang, 2011; Du and Wei, 2013; Choukhmane et al., 2014), and pension and education reforms (Chamon and Prasad, 2010). See Bussiere et al. (2013) for a review of literature on savings in China.

Figure 1. Age-Saving Rate Profile in 2002



3. Stylized Facts

In this section, I show that the concerns for marriage prospects play an important role in forming the downward-sloping age-saving profile in China documented in earlier works (e.g. Chamon and Prasad, 2010; Song and Yang, 2010).

I compute the age profile of the saving rates using the 2002 round of the urban sample in the China Household Income Project (CHIP) survey[4]. Following Chamon and Prasad (2010), income and consumption profile at each age are smoothed by a three-year moving average which includes the age cohorts immediately above and below the given age. Saving rate is defined as 1 minus the ratio of consumption over disposable income. The resulting age profile is plotted in Figure 1 which features a U-shaped age profile of the saving rates[5].

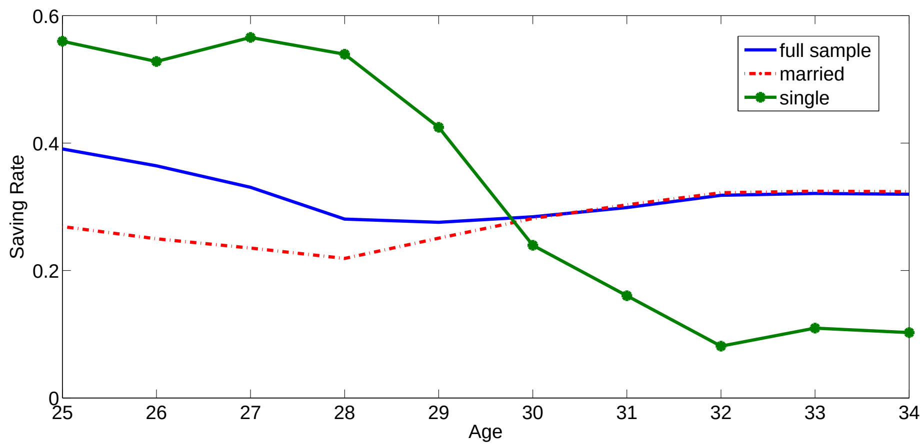
In Figure 2, I divide the whole sample into two sub-samples with respect to their marital status: single or married, and plot the age-saving profiles for each group. The age profile of the whole sample is replicated here for the ease of comparison[6]. Two observations stand

[4]The CHIP survey samples are sub-samples of the NBS annual urban household survey sample. The 2002 CHIP urban samples contains about seven thousand households from 12 provinces which represent major regions in China. Detailed explanation of the data can be found in Li et al. (2008).

[5]Chamon and Prasad (2010) and Song and Yang (2010) document similar results using the annual Urban Household Survey (UHS) conducted by the National Bureau of Statistics. The dataset they use is not publicly available.

[6]Only the age profile of people younger than 34 is plotted because there are few observations of single households with age above 35 (0.39% of the case), making the estimates of average saving rates in that range high inaccurate.

Figure 2. Age-Saving Rate Profile with Different Marital Status



out. First, at earlier ages, single households save more than married households at each age. Second, the downward-sloping pattern is more pronounced in the single sub-sample. There is actually no such pattern in the married group. The age profile of the whole sample is a weighted average of the age profiles of the two sub-samples and is very close to the profile of married group after age 32 when most people are married.

Although these patterns cannot be used directly as evidence for the competitive saving motive as an explanation to the saving rate puzzle, since people with different socioeconomic characteristics may choose their marriage ages differently, they do suggest a link between the saving rates and marital status in the younger cohorts. In the following sections, I model how people's marriage ages and saving behaviors are jointly determined by the marriage market conditions.

4. The Benchmark Model

To illustrate how people's saving behavior can be affected by mating consideration, I construct an economy populated by overlapping generations of two-period lived men m and women w whose ages are denoted respectively as young and old. Each generation is characterized by a *sex ratio at birth* R^* defined as the ratio of men to women. Since the focus of this paper is to show how people's savings are affected by an increase in the sex ratio, I assume $R^* \geq 1$. Population in each generation is normalized to one with measures of young men and young women as $M = \frac{R^*}{1+R^*}$ and $W = \frac{1}{1+R^*}$ respectively.

Documented empirical patterns on mate preference indicate that males and females have different priorities about traits of potential partners. Men usually choose appearance and personality as their top priority while women on the other hand choose occupation and income[7]. To be consistent with these empirical findings, men and women are treated differently in the model.

Each young man is endowed with a level of wealth $x_{m,1}$ at birth which is an independent draw from an exogenous distribution $H_{m,1}(x)$. $H_{m,1}(x)$ has positive support $[\underline{x}, \overline{x}]$ and density function $h_{m,1}(x) \in (0, +\infty)$, $\forall x_{m,1}$. In contrast, wealth endowments for young women are identical and are normalized to zero. In the meantime, women are distinguished by another trait: a match quality indicator z drawn from a distribution $H_z(z)$ with positive support $[\underline{z}, \overline{z}]$ and density function $h_z(z) \in (0, +\infty), \forall z$. z represents a woman's ability to generate emotional utility in marriage, which may include attractiveness, personality, intelligence, health and ability for domestic work. Besides, everyone earns the same labor income wn at each age where w is the market wage rate and n is a constant labor endowment.

A marriage only happens between a man and a woman at the end of age one if they mutually agree to get married. There are two benefits associated with marriage. First, consumption in a married household may have public good feature such that individuals consume more in total than aggregate household consumption. For example, couples can share the same couch. The second benefit is happiness from marriage which is determined by the match quality indicator z of the wife.

[7] Using data from a Korean online dating website, Lee (2009) finds around 80% of males choose appearance and personality as their top priority. In contrast, most often (55.6% in the sample) females choose occupation and income. Similar patterns are found in Fisman et al. (2006) and Hitsch et al. (2010). Although we haven't seen much empirical results about mate preference in China, demand for brideprice (usually in the form of housing) is a common practice, while dowry payment is voluntary and not known to the groom's family before marriage.

4.1. Utility Function

In the following part of this paper, I label the marital status of an agent by superscript **a** if (s)he is married, or by superscript **s** if (s)he is single.

4.1.1. Utility Function of Men

A single man allocates his liquid wealth $x_{m,1}+wn$ in the first period between consumption $c_{m,1}$ and saving $s_{m,1}$, taking other agents' saving choices as given. In the next period, if he remains single, his consumption is the sum of his saving from the first period and labor income in the second period which is

$$c^{\mathbf{s}}_{m,2} = (1+r)s_{m,1} + wn$$

in which r is the interest rate of saving.

If he is married, he and his wife pool their pre-marital savings and income and thus his second-period consumption is

$$c^{\mathbf{a}}_{m,2} = \kappa[(1+r)(s_{m,1}+s_{w,1}) + 2wn] \tag{1}$$

in which $s_{w,1}$ is the pre-marital saving of his wife, and parameter $\kappa \in [\frac{1}{2}, 1]$ captures the public good feature of consumption in a married household.

The optimization problem for a young man with endowed wealth $x_{m,1}$ is:

$$\begin{aligned} V_{m,1}(x_{m,1}) = \max_{s_{m,1}} \{ & \ln c_{m,1} + \beta \mathbb{E}(\ln c_{m,2} + z)\} \\ & s.t. \\ & c_{m,1} + s_{m,1} \leq x_{m,1} + wn;\ \ s_{m,1} \geq 0 \\ & c_{m,2} = \kappa[(1+r)(s_{m,1}+s_{w,1}) + 2wn], \quad \text{if he is married} \\ & c_{m,2} = (1+r)s_{m,1} + wn, \quad \text{if he remains single} \end{aligned} \tag{2}$$

In equation (2), β is the discount factor which satisfies the following assumption: $\beta(1+r) = 1$. z represents happiness from marriage which equals the match quality of his wife if he is married or 0 if he remains single in the second period. Notice that a man's life-time utility depends on whether and with whom he is matched. The matching pattern will be clear when we come to the description of marriage market.

4.1.2. Utility Function of Women

Women's utility function can be described similarly except for that their wealth endowment is normalized to 0. I assume each female's match quality z is realized after choice of saving $s_{w,1}$ at age 1. Besides, $s_{w,1}$ is not publicly observable and thus there's no strategic saving consideration on the women's side. Thus all women have the same optimization problem:

$$\begin{aligned} V_{m,1} = \max_{s_{w,1}} & \{\ln c_{w,1} + \beta \mathbb{E}(\ln c_{w,2} + z)\} \\ & s.t. \\ & c_{w,1} + s_{w,1} \leq wn; \;\; s_{w,1} \geq 0 \\ & c_{w,2} = \kappa[(1+r)(s_{m,1} + s_{w,1}) + 2wn], \quad \text{if she is married} \\ & c_{w,2} = (1+r)s_{w,1} + wn, \quad \text{if she remains single} \end{aligned} \tag{3}$$

4.2. Marriage Market

Marriage is monogamous and formed under mutual consent. Everyone wants to achieve higher utility after marriage, i.e., $\ln\{\kappa[(1+r)(s_{m,1} + s_{w,1}) + 2wn]\} + z$. Consequently, men are ranked by pre-marital saving $s_{m,1}$ while women are ranked by the realized match quality indicator z since all women's saving $s_{w,1}$ are identical in equilibrium.

Assumption 1. *The lower bound of match quality indicator $\underline{z} \geq -\log \kappa$.*

Lemma 1. *Given Assumption 1, all matches are acceptable. All agents prefer getting married with any agent of the opposite gender rather than remaining single.*

Proof. A man with saving $s_{m,1}$ compares utility from marriage $\ln\{\kappa[(1+r)(s_{m,1} + s_{w,1}) + 2wn]\} + z$ and remaining single $\log[(1+r)s_{m,1} + wn]$. It is easy to check that the utility from marriage is the larger one. Comparison for women's utilities is similar. □

I assume the matching process follows the Gale-Shapley deferred acceptance procedure (Gale and Shapley, 1962; Roth and Sotomayor, 1992), and without search frictions, the stable matching is in principle an assignment game.

Lemma 2. *Matching is positively assortative between men's saving $s_{m,1}$ and women's match quality indicator z.*

Proof. Since utility after marriage is increasing in men's pre-marital saving and women's match quality, if we take two men with pre-marital saving levels of $s_{m,1}$ and $s_{m,1}^{'}$ such that $s_{m,1}^{'} > s_{m,1}$, and two women with match quality indicators z and $z^{'}$ such that $z^{'} > z$, the man with $s_{m,1}^{'}$ is preferred by both women and the woman with $z^{'}$ is preferred by both men. As a result, the man with $s_{m,1}^{'}$ matches with woman with $z^{'}$, and the other two match with each other. □

Denote stationary distribution of men's saving $s_{m,1}$ in marriage market as $G_s(s)$ with support $[\underline{s}, \overline{s}]$ which is determined endogenously by young men's saving choices. Matching happens along the distributions of $G_s(s)$ and $H_z(z)$ from top to bottom. If $G_s(s)$ has a mass point at s_0, men with saving s_0 matches with equal probabilities to women with match quality z in the corresponding interval (See Appendix A for the description of the matching pattern).

If $R^* > 1$, men at the bottom of distribution $G_s(s)$ cannot get matched. Denote the marginal matched saving level

$$s_{m,1}^{*} = G_s^{-1}(1 - \frac{1}{R^*})$$

[8]Men who choose saving below $s_{m,1}^{*}$ are expected to remain single at age 2. I assume saving behavior of men is *symmetric* such that men with the same wealth endowment $x_{m,1}$ choose the same level of saving $s_{m,1}$. In other words, there is a saving function $s_{m,1} = s(x_{m,1})$.

Definition 1. *An **equilibrium** is a saving function of men $s(x_{m,1})$, a distribution of men's saving in marriage market $G_s(s)$, women's saving choice $s_{w,1}$ as well as a matching rule between men's per-marital saving $s_{m,1}$ and women's realized match quality indicator z such that:*

[8] $G_s^{-1}(p)$ is the quantile function of $G_s(s)$ such that $G_s^{-1}(p) = \inf\{p \leq G_s(s),\ \forall s \in [\underline{s}, \overline{s}]\}$

1. *Under the matching rule, matching is positively assortative between $s_{m,1}$ and z. The detailed matching pattern is given in Appendix A.*

2. *Saving function $s(x_{m,1})$ is optimal for men such that given $G_s(s)$, $s_{w,1}$ and assortative matching rule, $s(x_{m,1})$ solves men's optimization problem in equation (2), $\forall x_{m,1}$.*

3. *Given $G_s(s)$ and matching rule, $s_{w,1}$ solves women's optimization problem in equation (3).*

4. *$G_s(s)$ is consistent with men's saving choice and the distribution of endowment such that $G_s(s^*) = prob\{s(x) \leq s^* | x \sim H_{m,1}(x)\}$.*

For the moment, let's assume there is no mass point in $G_s(s)$. I show this is the case in Lemma 4. Based on Part 1 of Definition 1, if a woman draws match quality indicator z, she matches with a young man with saving

$$s_{m,1}(z) = G_s^{-1}[1 - \frac{1}{R^*}(1 - H_z(z))] \tag{4}$$

On the other hand, a man with saving $s_{m,1}$ matches to a woman with z such that

$$z(s_{m,1}) = H_z^{-1}[1 - R^*(1 - G_s(s_{m,1}))] \tag{5}$$

A competitive saving motive arises in this situation and $s_{m,1}$ becomes a "positional good" in the sense that a man not only cares about his absolute value of saving, but also his saving relative to others because whether he can get married and his match quality are determined by his ranking in the saving distribution[9].

Lemma 3. *With sex ratio $R^* \geq 1$, women choose first period saving $s_{w,1} = 0$.*

Proof. Based on Lemma 1, all women are matched no matter what their realized z is. Thus equation (3) reduces to $\max\{\ln(wn - s_{w,1}) + \beta E_z[\ln \kappa[(1+r)(s_{m,1}(z) + s_{w,1}) + 2wn] + z]\}$ in which $s_{w,1}$ is restricted to be nonnegative. First-order condition to $s_{w,1}$ shows 0 saving is optimal for women. $\square$

[9] I use the term "positional good" in the same way as in Cole et al. (1992) such that agents do not value position itself, but treat it as an instrument for better consumption or marriage opportunities.

No women save since they have constant labor income at each age so there is no need to smooth consumptions by saving. In addition, they can take advantage of the pre-marital savings of future husbands.

4.3. Equilibrium Saving Function

Lemma 4. *Equilibrium saving function $s(x_{m,1})$ is strictly increasing. Besides, it is continuous and differentiable except for a discrete jump at point $x^*_{m,1} = H^{-1}_{m,1}(1 - \frac{1}{R^*})$.*

Proof. Proof is given in Appendix B. □

Saving function is increasing since richer people can always achieve the same level of saving with less utility loss in the first period. With increasing saving function, a man's ranking in the marriage market is the same as his ranking in endowment:

$$G_s[s(x_{m,1})] = H_{m,1}(x_{m,1}) \tag{6}$$

Using words from Hopkins and Kornienko (2004), "it takes all the running to keep in the same place." Consequently, men with endowments larger than $x^*_{m,1}$ are matched by matching rule in equation (5), while men with endowments below $x^*_{m,1}$ remain single for life. In particular, men whose endowment is exactly $x^*_{m,1}$ match with women of match quality $\underline{z}$. For men expected to be single, their optimization problem is solved by imposing $\delta_m = 0$ in equation (2) and we can get single men's saving function and lifetime utility as

$$\begin{aligned} s^{\mathbf{s}}(x_{m,1}) &= \frac{\beta}{1+\beta} x_{m,1} \\ V^{\mathbf{s}}_{m,1}(x_{m,1}) &= (1+\beta)\log\left(\frac{x_{m,1}}{1+\beta} + wn\right) \end{aligned} \tag{7}$$

For $x_{m,1} \geq x^*_{m,1}$, the reason $s(x_{m,1})$ is strictly increasing and continuous follows the standard "no atoms, no holes" argument in labor search theory such that any mass point or discontinuity in $G_s(s)$ for $x_{m,1} > x^*_{m,1}$ creates profitable deviation for some men and thus should be ruled out in equilibrium. Lastly, the saving function is discontinuous at $x^*_{m,1}$ because competition forces a man with wealth $x^*_{m,1}$ to raise his saving high enough such that he is indifferent between matching with women $\underline{z}$ and remaining single.

Based on Lemma 4 and Condition 4 in Definition 1, $G_s(s)$ is differentiable too. I denote its density function as $g_s(s)$.

Incorporating matching rule $z(s_{m,1})$ in equation (5), Young men with wealth $x_{m,1} \geq x^*_{m,1}$ solve the optimization problem

$$\max_{s_{m,1}}\{\log(x_{m,1} + wn - s_{m,1}) + \beta \log[\kappa((1 + r)s_{m,1} + 2wn)] + \beta z(s_{m,1})\} \tag{8}$$

Differentiating with respect to $s_{m,1}$, we have the following first-order condition:

$$\frac{1}{x_{m,1} + wn - s_{m,1}} = \frac{1}{(1 + r)s_{m,1} + 2wn} + \frac{\beta R^* g_s(s_{m,1})}{h_z[1 - R^*(1 - G_s(s_{m,1}))]} \tag{9}$$

After plugging in equation (6), we have the following first-order differential equation for the saving function $s(x_{m,1})$:

$$s^{'}(x_{m,1}) = \frac{\beta\varphi(x_{m,1})}{\frac{1}{x_{m,1}+wn-s_{m,1}} - \frac{1}{(1+r)s_{m,1}+2wn}} \tag{10}$$

where $\varphi(x_{m,1}) = \frac{R^* h_{m,1}(x_{m,1})}{h_z[1-R^*(1-H_{m,1}(x_{m,1}))]}$ denotes the ratio of the marginal increase in the endowment position to the marginal change in the corresponding matching quality.

Proposition 1. *The unique equilibrium saving function $s(x_{m,1})$ takes the following form: $\forall x_{m,1} < x^*_{m,1}$, $s(x_{m,1}) = s^s(x_{m,1}) = \frac{\beta}{1+\beta}x_{m,1}$; $\forall x_{m,1} \geq x^*_{m,1}$, $s(x_{m,1})$ is solved by differential equation (10) with the following boundary condition:*

1. *If $R^* = 1$, everyone gets matched and the boundary condition is $s(\underline{x}) = \max[\frac{1}{2+r}(\underline{x} - wn), 0]$.*
2. *If $R^* > 1$, $s(x^*_{m,1})$ is pinned down by the following equation:*

$$\begin{aligned}&(1 + \beta) \log(\frac{x^*_{m,1}}{1 + \beta} + wn)\\ &= \log[x^*_{m,1} + wn - s(x^*_{m,1})] + \beta \log[\kappa((1 + r)s(x^*_{m,1}) + 2wn)] + \beta \underline{z}\end{aligned}$$

Proof. Proof is given in Appendix C. □

Proposition 1 shows that differential saving equation (10) is indeed optimal saving for any single man, given other people adopt this saving strategy as well. With balanced sex ratio $R^* = 1$, everyone gets matched. In particular, a man with the lowest endowment $\underline{x}$ is determined to match with a woman of the lowest match quality $\underline{z}$. No matter how much he saves, other men can trump him by saving more as long as it is profitable to do so. As a result, men with $\underline{x}$ simply take their position and matching prospects as given, and choose saving according to equation (12) as described below. Otherwise if $R^* > 1$, only men with endowment higher than $x^*_{m,1}$ get matched. Due to competition, saving of the marginal guy with $x^*_{m,1}$ is raised to the level such that he is indifferent between matching with $\underline{z}$ and remaining single. Major findings in this section are summarized by Proposition 2.

Proposition 2. *The marriage market equilibrium with pre-marital investment* ***exists*** *and is* ***unique****, in which young women do not save while young men's saving choices are described by Proposition 1, and matching is positively assortative between men's saving and match quality of women.*

Proof. The existence of a marriage market equilibrium with pre-marital investment is proved by Lemma 2, Lemma 3 as well as Proposition 1. Since any equilibrium saving function is described by the differential equation (10), uniqueness simply follows from the fundamental theorem of differential equation with boundary condition provided in Proposition 1. □

For the sake of comparison, I also compute the *cooperative saving function* $s^{\mathbf{c}}(x_{m,1})$ when each young man takes his endowed position $H_{m,1}(x_{m,1})$ as given. In this case, men with endowment below $x^*_{m,1}$ cannot get matched and thus $s^{\mathbf{c}}(x_{m,1}) = s^{\mathbf{s}}(x_{m,1})$, $\forall x_{m,1} < x^*_{m,1}$. For men with wealth higher than $x^*_{m,1}$, his match quality is $\hat{z}(x_{m,1}) = H_z^{-1}[1 - R^*(1 - H_{m,1}(x_{m,1}))]$ and his optimization problem is

$$\max_{s^{\mathbf{c}}_{m,1}} \{\log(x_{m,1} + wn - s^{\mathbf{c}}_{m,1}) + \beta \log[\kappa((1+r)s^{\mathbf{c}}_{m,1} + 2wn)] + \beta \hat{z}(x_{m,1})\}$$

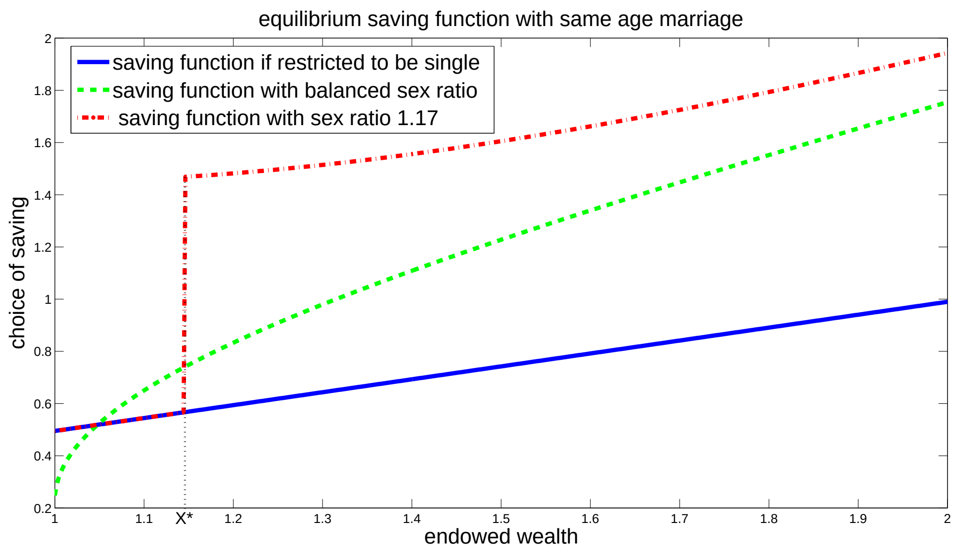


Figure 3. equilibrium saving functions with same age marriage

with first order condition

$$\frac{1}{x_{m,1} + wn - s^{\mathbf{c}}_{m,1}} = \frac{1}{(1+r)s^{\mathbf{c}}_{m,1} + 2wn} \tag{11}$$

and

$$s^{\mathbf{c}}(x_{m,1}) = \max[\frac{1}{2+r}(x_{m,1} - wn), 0] \tag{12}$$

Comparing equation (9) and (11), the last term in (9) clearly shows how the competitive saving motive arises when a man's status is affected by his saving. However, all those extra savings are "wasted" in the sense that no men's position in the marriage market changes from their positions in wealth endowment. But we cannot claim that cooperative saving is Pareto improvement to the competitive saving case since all the women benefit from men's saving and they strictly prefer men to save more.

4.4. Numerical Example with Same Age Marriage

An illustrative example is given in Figure 3. $H_{m,1}(x)$ and $H_z(z)$ are both uniform distributions over the interval $[1, 2]$, labor income wn is set to 0.5 and the discount factor β

is chosen to be 0.98. Differential equation (10) is solved numerically by the implicit Euler method[10]. The blue solid line is the saving function for single men $s^{\mathbf{s}}(x)$ and its expression is given by equation (7). With consumption smoothing considerations only and a logarithmic utility function, $s^{\mathbf{s}}(x)$ is proportional to endowed wealth. The green dashed line is the equilibrium saving function with $R^* = 1$ in which everyone gets matched. The man with the lowest wealth endowment chooses the cooperative level of saving since he won't attempt to improve his position. His saving level is lower than $s^{\mathbf{s}}(x)$ since he can partly free ride on his wife's future income. However, as men's wealth increases, the competitive saving motive arises in interval $(1, 2]$ and the equilibrium saving rate is thus higher than cooperative saving in general.

The red dotted line in Figure 3 represents equilibrium saving function with $R^* = 1.17$. With an unbalanced sex ratio, there is a jump in the saving function at $x^* = (1 - \frac{1}{R^*})(\overline{x} - \underline{x}) + \underline{x}$. Men at the bottom of the wealth distribution cannot get matched and thus choose $s^{\mathbf{s}}(x)$. At wealth level x^* representing threshold wealth of getting matched, saving jumps to a point where men with wealth x^* are indifferent between getting matched with match quality $\underline{z}$ and remaining single. The part with $x_{m,1} > x^*$ is characterized by differential equation (10).

One key observation here is that a rising sex ratio does not necessarily lead to an increase in aggregate saving, since two opposing effects might arise. As R^* increases from 1 to 1.18[11], on the one hand, men staying in the marriage market raise their saving in a competitive manner; on the other hand, men with the lowest wealth can't get matched in equilibrium and thus quit the saving competition. That's why we see that part of the saving function with an unbalanced sex ratio lies below the saving function with balanced sex ratio. Whether the net effect on aggregate saving is positive or negative depends on the chosen distributions and parameters. I'll address this question in section 6 where the model is calibrated to the Chinese economy.

[10]See Page 343 of Judd (1998) for a description of this method.

[11]China's sex ratio at birth in 2010, data from National Bureau of Statistics

5. Marriage Model with Endogenous Age Gap

Section 4 illustrates how men's saving choices can be affected if their marriage prospects are determined by their saving relative to others. However, the marriage market considered in the previous section is in essence static and reactions other than changes in savings are not considered. Many studies in economics and demography indicate that the marriage market exhibits great flexibility to the marriage squeeze problem and different responses emerge with marriage market imbalances, such as changes in the age gap between spouses, changes in remarriage rates of divorced and widowed people as well as gender biased immigration (Edlund and Lee, 2009; Ebenstein and Sharygin, 2009; Abramitzky et al., 2011). All these responses are likely to reduce the pressure of sex ratio imbalances.

5.1. Some Data Patterns on Pre-marital Saving and Age of Marriage

I incorporate the possibility of an age gap between spouses into the model for the following two reasons. First, a change in the age gap is considered to be one of the most important reactions to the marriage squeeze problem. For example, Anderson (2007) studies a similar topic and shows that after age gap responses are considered, an excess of women in the marriage market actually causes dowry deflation instead of inflation. As described in Figure 4, the age gap at marriage in China has indeed been increasing since the mid 1990s which is consistent with the changes in sex ratio in the marriage market. Second, the downward-sloping age profile of saving rates in the pre-marital cohorts naturally requires consideration about marriage timing. Results from estimating the following regression show there is a negative correlation between wealth and marriage age in China such that men delaying marriage are mostly those with low economic status[12][13].

[12] Existing literature offers two contrasting hypotheses about the relation between men's earnings (wealth) and their marital age. On the one hand, Keeley (1977) argues the correlation between men's income and their marital age should be negative since men with higher earnings should benefit more from specialization of household production. On the other hand, Bergstrom and Bagnoli (1993) suggest men with high productivity will delay marriage until their productivities are revealed to the public at later ages. Zhang (1995) and Danziger and Neuman (1999) empirically test these two hypothesis and find mixed results.

[13] The regression in Equation 13 is similar to the one used in Zhang (1995) except for that I use the age gap between spouses as the dependent variable instead of marriage age of the husband.

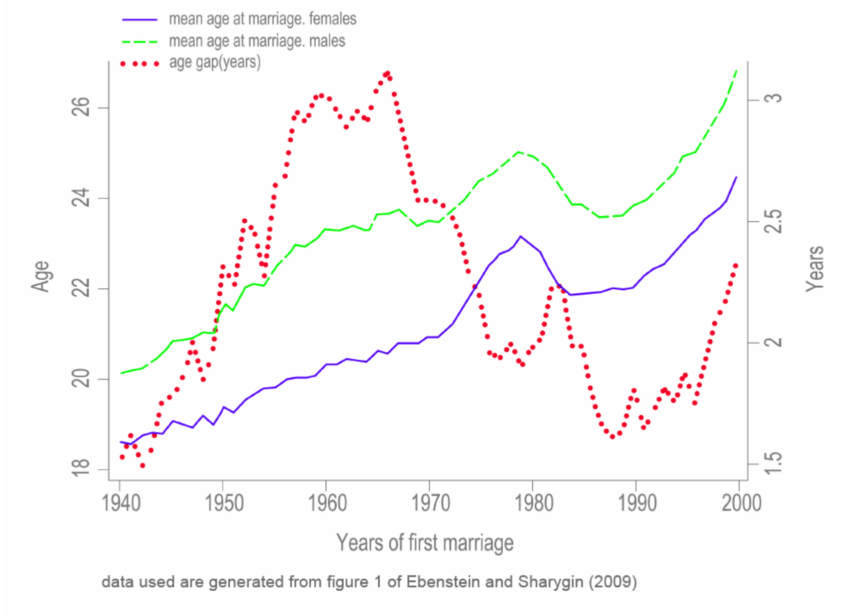


Figure 4. marriage age and age gap in China

$$\text{age gap of spouse} = \alpha_0 + \alpha_1 \text{wage} + \alpha_2 \text{education} + X\alpha_3 + \epsilon \qquad (13)$$

In the regression equation above, X is a list of marriage year and province dummies. Using data from all 9 waves of China Health and Nutrition Survey[14] with observations restricted to married men aged 18-60 years, I find α_1 is significantly negative. In other words, men with higher earnings marry earlier and thus have smaller age gap to the ages of their wives. This result is robust with and without an education dummy which takes value 1 if a man has above high-school education and 0 otherwise. Estimations of different specifications are reported in Table 1, with numbers in brackets as the absolute values of the corresponding t-statistics. Consequently, a model with a competitive saving motive incorporating the age gap between spouses should be able to generate a similar relationship between the age gap of spouses and the pre-marital wealth (income) of the husband.

[14]This dataset is described in detail in Subsection 7.1.

Table 1: men's age of marriage in China

Variable	**with Education**	**without Education**
Wage (in 2009 dollars)	**-0.101**	**-0.110**
	(5.62)	(5.74)
Marriage Year	**0.556**	**0.550**
	(68.29)	(68.08)
Education	**-1.005**	–
	(5.78)	
R^2	0.29	0.288
F	456.77	498.49

5.2. Model

I modify the framework in Section 4 as follows. Each agent lives for 3 periods: young, middle-aged and old with ages as $j_g = 1, 2, 3\ \forall g = m, w$. A person may choose to enter the marriage market at either age1 or age 2 but not both. Each young man allocates his liquid wealth $x_{m,1} + wn$ between consumption $c_{m,1}$ and saving $s_{m,1}$ and then decides his marital age. If he postpones marriage to age 2, next period he becomes middle-aged, allocates liquid wealth $x_{m,2} = (1 + r)s_{m,1} + wn$ between age 2 consumption $c_{m,2}$ and saving $s_{m,2}$, and then joins the marriage market. Given the same wealth endowment, middle-aged single men are potentially wealthier than their young counterparts since they have one more period to save. Single women choose their savings and marital age in the same way as men do except that their level of savings, $s_{w,1}$ and $s_{w,2}$ are not observable before marriage, and their wealth endowment is normalized to 0. Women choose their marital age before their matching qualities are realized. Unmarried women cannot enjoy match quality z. But after marriage, a woman starts to enjoy z every period even in widowhood. Since the focus here is the marital age of the first marriage, divorce and remarriage are not considered here.

5.2.1. Utility Function of Married Household

I first describe the utility functions of a married household which single agents use to decide when and with whom to get married. With an endogenous age gap, there are 4 potential age combinations for married couples: $(j_m, j_w) \in \{(2, 2), (2, 3), (3, 2), (3, 3)\}$. For the sake of argument, I use $A^{\mathbf{a}}_{g,j_m j_w}$ to denote a variable A of husband $(g = m)$ or wife $(g = w)$ in a married household with the age combination (j_m, j_w).

Couples who are both of age 3 simply consume all their family liquid wealth $x^{\mathbf{a}}_{33} = (1+r)s^{\mathbf{a}}_{22} + 2wn$, which is the sum of saving from previous period and current labor incomes. Newly wedded couples still pool their pre-marital savings such that $s^{\mathbf{a}}_{22} = s^{\mathbf{s}}_{m,2} + s^{\mathbf{s}}_{w,2}$. Thus their utility functions are

$$\begin{aligned} U^{\mathbf{a}}_{m,33}(s^{\mathbf{a}}_{22}) = U^{\mathbf{a}}_{w,33}(s^{\mathbf{a}}_{22}) &= \ln(\kappa x^{\mathbf{a}}_{33}) + z \\ s.t. & \\ x^{\mathbf{a}}_{33} &= (1+r)s^{\mathbf{a}}_{22} + 2wn \end{aligned} \tag{14}$$

For couples who are both of age 2, given family wealth $x^{\mathbf{a}}_{22} = (1 + r)s^{\mathbf{a}}_{11} + 2wn$, their maximization problem is

$$\begin{aligned} U^{\mathbf{a}}_{m,22}(s^{\mathbf{a}}_{11}) &= U^{\mathbf{a}}_{w,22}(s^{\mathbf{a}}_{11}) \\ &= \max_{s^{\mathbf{a}}_{22}} \{\ln[\kappa(x^{\mathbf{a}}_{22} - s^{\mathbf{a}}_{22})] + z + \beta[\ln(\kappa x^{\mathbf{a}}_{33}) + z]\} \\ s.t &: \\ x^{\mathbf{a}}_{22} &= (1+r)s^{\mathbf{a}}_{11} + 2wn \\ x^{\mathbf{a}}_{33} &= (1+r)s^{\mathbf{a}}_{22} + 2wn;\ s^{\mathbf{a}}_{22} \geq 0 \end{aligned}$$

and from the first-order condition we get

$$\begin{aligned} s^{\mathbf{a}}_{22} &= \frac{1}{1+\beta} s^{\mathbf{a}}_{11} \\ U^{\mathbf{a}}_{m,22}(s^{\mathbf{a}}_{11}) &= U^{\mathbf{a}}_{w,22}(s^{\mathbf{a}}_{11}) \\ &= (1+\beta)\ln[\kappa(\frac{s^{\mathbf{a}}_{11}}{\beta+\beta^2} + 2wn)] + (1+\beta)z \end{aligned} \tag{15}$$

The optimization problem for the other 2 age combinations require further assumptions since if one spouse is older than the other, the younger one will be a widow(er) in the next period and thus the optimal level of saving for one spouse is different from the optimal level for the other one. For simplicity I assume that if at least one spouse reaches age 3, the family will not save into the future, and thus the widow(er) can only live on his or her labor income wn next period[15]. Consequently, for a family with an age 3 husband and an age 2 wife, the

[15]This assumption is relaxed in Section (6) in which couples jointly maximize the household value function which is a weighted sum of value functions of each spouse. Under this setup, the stable matching patterns derived in this section still hold.

value functions for each spouse are:

$$U^{\mathbf{a}}_{m,3,2}(s^{\mathbf{a}}_{21}) = \ln[\kappa((1+r)s^{\mathbf{a}}_{21} + 2wn)] + z \tag{16}$$
$$U^{\mathbf{a}}_{w,3,2}(s^{\mathbf{a}}_{21}) = \ln[\kappa((1+r)s^{\mathbf{a}}_{21} + 2wn)] + \beta\ln(wn) + (1+\beta)z \tag{17}$$

Similarly, for a household with age combination $(2,3)$, the husband is younger and will be a widower next period. The corresponding value functions are:

$$U^{\mathbf{a}}_{m,2,3}(s^{\mathbf{a}}_{12}) = \ln[\kappa((1+r)s^{\mathbf{a}}_{12} + 2wn)] + z + \beta\ln(wn) \tag{18}$$
$$U^{\mathbf{a}}_{w,2,3}(s^{\mathbf{a}}_{12}) = \ln[\kappa((1+r)s^{\mathbf{a}}_{12} + 2wn)] + z \tag{19}$$

5.2.2. Stable Matching in the Marriage Market

At period t, for ages $j = 1,2$, I denote the measure of single men and women in the marriage market as $M_{j,t}$ and $W_{j,t}$, and the distributions of men's saving as $G_{s,j,t}(\cdot)$, $s_{m,j,t} \in [\underline{s_{j,t}}, \overline{s_{j,t}}]$. Here $M_{1,t} = \frac{R^*}{1+R^*}$ and $W_{1,t} = \frac{1}{1+R^*}$ are exogenous, while $M_{2,t}$, $W_{2,t}$ and the distributions of saving are endogenous resulting from equilibrium choices of saving and marriage age.

Each agent in the marriage market has 2 characteristics: saving and age $\{s_m, j_m\}$ for men, and match quality indicator and age $\{z, j_w\}$ for women. Notice that unlike what is typically assumed in the matching literature, the two characteristics of men and women are not independent.

Definition 2. *Given measures of single agents and distributions of men's saving at each age at period t, a **matching** at period t is a measure μ on the set $[\underline{z}, \overline{z}] \times \{1,2\} \times [\underline{s_t}, \overline{s_t}] \times \{1,2\}$ such that its marginal distributions equal to the distributions of the corresponding groups.*

Intuitively, for set X of women $\subseteq [\underline{z}, \overline{z}] \times \{1,2\}$ and set Y of men $\subseteq [\underline{s_t}, \overline{s_t}] \times \{1,2\}$, $\mu(X,Y)$ denotes the probability that a woman in set X marries a man in set Y. A match is *stable* if no matched agent prefers remaining single to staying with his or her current match, nor do a man and woman both prefer to leave their current situation to form a new match. I first show that if a match is stable, matching is positively assortative between men's saving and women's match quality in each age combination of spouses.

At period t, I denote men's choice of marital age as $\delta_{m,j,t}(s_{m,j,t}), \forall j = 1, 2$. $\delta_{m,1,t}(s_{m,1,t}) = 1$ if a young man with saving $s_{m,1,t}$ gets married this period, and 0 otherwise. $\delta_{m,2,t}(s_{m,2,t}) = 1$ if a single middle-aged man with saving $s_{m,2,t}$ gets married this period, and 0 otherwise. Marital indicators for women $\delta_{w,j,t}$, $\forall j = 1, 2$ are defined similarly. Since all women are ex ante identical before the marriage market and make the same choice on marital age, $\delta_{w,1,t}$ and $\delta_{w,2,t}$ are the same for all women in each age cohort. Before we proceed, we can define an equilibrium in the marriage market with pre-marital investment.

Definition 3. *An* ***equilibrium with age gap between spouses*** *at period t is measures of singles $\{M_{j,t}, W_{j,t}\}$, $\forall j = 1, 2$; distributions of single men's saving $G_{s,j,t}(\cdot)$, $\forall j = 1, 2$ and liquid wealth at age 2 $G_{x,2,t}(\cdot)$; men's saving and marriage timing functions $\{s_{j,t}(x_{m,j,t}), \delta_{m,j,t}(s_{m,j,t})\}$, $\forall j = 1, 2$; women's marital age $\delta_{w,j,t}$, $\forall j = 1, 2$; and a matching rule μ_t such that:*

1. *Saving functions are consistent with individual men's optimal choices. Choices of marital age $\delta_{m,j,t}(s_{m,j,t})$ and $\delta_{w,j,t}$ are also optimal.*
2. *Measures and distributions in the marriage market are consistent with agents' optimal choices and the endogenous distribution of endowment, such that:*

$$
\begin{aligned}
G_{s,1,t}(s_{m,1,t}) &= prob\{s_{1,t}(x_{m,1,t}) \leq s_{m,1,t}\}, \forall s_{m,1,t} \\
G_{x,2,t}(x_{m,2,t}) &= prob\{(Rs_{m,1,t-1} + wn \leq x_{m,2,t}) \wedge (\delta_{m,1,t-1}(s_{m,1,t-1}) = 0)\}, \forall x_{m,2,t} \\
G_{s,2,t}(s_{m,2,t}) &= prob\{s_{2,t}(x_{m,2,t}) \leq s_{m,2,t}\}, \forall s_{m,2,t} \\
M_{1,t} &= \frac{R^*}{1+R^*};\ W_{1,t} = \frac{1}{1+R^*} \\
M_{2,t} &= M_{1,t-1} \int [1 - \delta_{m,1,t-1}(s_{m,1,t-1})] dG_{s,1,t-1}(s_{m,1,t-1}) \\
W_{2,t} &= W_{1,t-1}(1 - \delta_{w,1,t-1})
\end{aligned}
$$

3. *The matching distribution μ_t is stable.*

I first introduce some qualitative properties of equilibrium which hold true irrespective of the measures and distributions in the marriage market, namely, positive assortative matching in each age combination of couples.

Proposition 3. *In each possible age combination among married couples, take two couples with pre-marital savings and match quality indicators as* (s_m, z) *and* (s'_m, z') *respectively. Then* $s_m \geq s'_m$ *if and only if* $z \geq z'$.

Proof. Positively assortative matching results from utilities after marriage (equations (15) to (19)) which strictly increase in the other person's trait. As a result, men with higher saving and women with higher match quality would always prefer each other. □

Now we come to properties under stationary distributions. Since measures of young agents and distributions of wealth endowment and match quality are constant in each period, we can expect to have a *long-run stationary equilibrium* in which each generation behaves in the same way as previous generations and measures, distributions and saving functions are also the same. Consequently the time subscript t can be dropped. In the remaining part of this Section, I assume the economy is at its long-run stationary equilibrium.

Proposition 4. *In long-run stationary equilibrium, all women marry at age 1, and choose pre-marital saving* $s_{w,1} = 0$.

Proof. See Appendix D. □

Women choose to marry at the earliest marriageable age for two reasons. First, based on utilities after marriage, young men strictly prefer young women while middle-aged men are indifferent about women's ages with the same z. In other words, women can't gain and may fall in ranking if they postpone marriage. Second, remaining single for one additional period causes a utility loss. Thus all women marry at age 1. Since women's income is wn at all ages and $\beta R = 1$, they would choose 0 saving even if there were no marriage market. When the prospect of marriage is also considered, women have a motivation to borrow since they can take advantage of their future husband's pre-marital saving. Since saving is restricted to be nonnegative, $s_{w,1} = 0$.

If the sex ratio $R^* > 1$, some men cannot marry at age 1, so there are both young and middle-aged men in the marriage market. Since all women in the marriage market are of age 1, we can show women agree in their ranking of men.

Proposition 5. *All men in the marriage market are ranked in the same order by women.*

Proof. This can be seen by comparing utilities from marriage in equations (15) and (17). Since match quality z is additive in the utility functions, all women agree on their ranking of $\{s_m, j_m\}$ combinations in the marriage market. In particular, a young man with saving $s_{m,1}$ and a middle-aged man with saving $s_{m,2}$ are of the same rank if and only if they offer the same level of utilities to women after marriage, i.e.,

$$(1+\beta)\ln(\frac{s_{m,1}}{\beta+\beta^2}+2wn)=\ln(\frac{s_{m,2}}{\beta}+2wn)+\beta\ln(\frac{wn}{\kappa})$$

$\square$

By Proposition 5, we can use women's indirect utility after marriage as a single index to summarize single men's 2-dimensional characteristics of saving and age. In particular, I define men's quality function $q(s_m, j_m)$ as

$$q(s_m,j_m)=\begin{cases}(1+\beta)\ln[\kappa(\frac{s_m}{\beta+\beta^2}+2wn)], \text{if } j_m=1\\ \ln[\kappa(\frac{s_m}{\beta}+2wn)]+\beta\ln(wn), \text{if } j_m=2\end{cases} \tag{20}$$

Men with the same q have the same rank in the marriage market. I denote the equilibrium distribution of q in the marriage market as $H_q(\cdot)$.

In the same spirit as Lemma 2, it is easy to show that matching is positively assortative between women's match quality z and men's quality q. With both young and middle-aged single men in the marriage market, let's define the total measure of single men and the sex ratio in marriage market as

$$M=M_1+M_2;\ \overline{R}=\frac{M}{W_1},$$

I denote the lowest quality of a man who can get matched by $q^*=H_q^{-1}(1-\frac{1}{\overline{R}})$. Men with quality $q<q^*$ cannot be matched and so remain single this period. Otherwise, a man with quality higher than q^* matches with a young woman of match quality

$$\phi(q)=H_z^{-1}[1-\overline{R}(1-H_q(q))] \tag{21}$$

For now I take q^* and $H_q(\cdot)$ as given, and explain how they are formed after I describe the utility functions of single men[16].

5.2.3. *Utility Function of Single Men*

Middle-aged Single Men

A middle-aged single man with current liquid wealth $x_{m,2} = (1+r)s_{m,1} + wn$ maximizes the following:

$$\begin{aligned} V_{m,2}(x_{m,2}) &= \max_{s_{m,2},\delta_{m,2}} \{\ln(x_{m,2} - s_{m,2}) + \beta(1-\delta_{m,2})\ln[(1+r)s_{m,2} + wn] \\ &\quad +\beta\delta_{m,2}[\ln[\kappa((1+r)s_{m,2} + 2wn)] + \phi(q(s_{m,2}, 2))]\} \end{aligned} \tag{22}$$

subject to quality function (20) and matching rule (21). Based on Assumption 1, that is, the lowest value of women's match quality $\underline{z} \geq -\ln\kappa$, a middle-aged man prefers marrying any woman to remaining single. Based on matching rule (21), the matching threshold saving for middle-aged men is

$$s_2^* = \frac{\beta}{\kappa}\exp[q^* - \beta\ln(wn)] - 2\beta wn$$

If $s_{m,2} < s_2^*$, $\delta_{m,2}(s_{m,2})$ takes the value 0 and this man will remain single for life. His saving rate and value function will be

$$\begin{aligned} s_2^{\mathbf{s}}(x_{m,2}) &= \frac{\beta}{1+\beta}(x_{m,2} - wn) \\ V_{m,2}^{\mathbf{s}}(x_{m,2}) &= (1+\beta)\ln(\frac{1}{1+\beta}x_{m,2} + \frac{\beta}{1+\beta}wn) \end{aligned}$$

Otherwise, $s_{m,2}$ is given by the following first-order condition:

$$\begin{aligned} &\frac{1}{(1+r)s_{m,1} + wn - s_{m,2}} - \frac{1}{(1+r)s_{m,2} + 2wn} \\ = &\frac{\beta}{s_{m,2} + 2\beta wn}\frac{\overline{R}h_q(q)}{h_z[1-\overline{R}(1-H_q(q))]} \end{aligned}$$

with $q = \ln[\kappa(\frac{s_{m,2}}{\beta} + 2wn)] + \beta\ln(wn)$.

Young Men

[16] Using a similar argument as in Lemma 4, we can show that $H_q(q)$ is continuous and strictly increasing $\forall q \geq q^*$.

A young single man with endowed wealth $x_{m,1}$ maximizes the following:

$$\begin{aligned} V_{m,1}(x_{m,1}) &= \max_{s_{m,1},\delta_{m,1}} \{\ln(x_{m,1}+wn-s_{m,1})+\beta(1-\delta_{m,1})V_{m,2}[(1+r)s_{m,1}+wn] \\ &\quad +(\beta+\beta^2)\delta_{m,1}[\ln(\frac{s_{m,1}}{\beta+\beta^2}+2wn)+\ln\kappa+\phi(q(s_{m,1},1))]\} \end{aligned} \tag{23}$$

The threshold level of saving for young men to marry is

$$s_1^* = (\beta+\beta^2)[\frac{1}{\kappa}\exp(\frac{q^*}{1+\beta})-2wn]$$

In equilibrium, distribution of single men's quality $H_q(\cdot)$ should be consistent with men's choice of marital ages and pre-marital savings who would in return take $H_q(\cdot)$ as given and choose savings and marital ages to maximize equations (22) and (23). The stationary distribution of $H_q(\cdot)$ and men's pre-marital saving choices cannot be easily characterized, since compared to the model in Section 4, now we have mixed matching such that woman with the same match quality z may match with different {wealth, age} combinations in stable matching. Thus I extend this part into a multi-period model in Section 6 to solve the model numerically.

6. Multi-period Model

In previous sections, I showed how a competitive saving motive arises when wealth is a status good in the marriage market and how marriage age and saving behavior can be jointly affected by the marriage market imbalances. In order to see whether this model can offer an explanation for the saving rate puzzle in China, I extend the model into a multi-period framework in a small open economy to show how the age profile of saving rates and marital ages are affected by changing the sex ratio empirically.

6.1. *Demographics*

Both men and women live for J periods, and people can keep working until retirement age J^{ret}. In contrast to the baseline model, newly born agents don't receive a wealth endowment; that is, the only way to generate wealth is by accumulating saving from labor income each

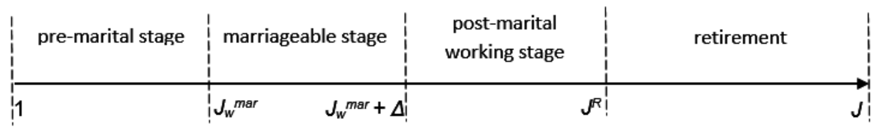


Figure 5. Life-cycle of a Representative Woman

period. In particular, a man receives a labor endowment in each period before retirement depending on his working experience and a transitory shock ε^m :

$$\begin{aligned} \ln n_j^m &= \gamma_0 + \gamma_1 \text{experience} + \gamma_2 \text{experience}^2 + \varepsilon^m \\ \varepsilon^m &\sim N(0, \sigma^2_{\varepsilon^m}) \end{aligned} \tag{24}$$

in which γ_0, γ_1 and γ_2 are constants to be calibrated, and experience $= j_m - 1$. The Mincerian equation (24) features a typical hump-shaped age profile of income which peaks at middle age.

In order to reduce the dimension of the matching problem, I assume women's labor endowment profile is deterministic over the life cycle:

$$\begin{aligned} \ln n_j^w &= \gamma_0 + \gamma_1 \text{experience} + \gamma_2 \text{experience}^2, \\ \text{experience} &= j_w - 1 \end{aligned} \tag{25}$$

Men can choose to get married at age $j_m \in [J_m^{mar}, J_m^{mar} + \Delta]$, in which J_m^{mar} is men's earliest marriageable age, and Δ controls the length of marital stage[17]. Marriageable ages for women are similarly defined as $[J_w^{mar}, J_w^{mar} + \Delta]$[18]. The life-cycle of a typical woman is summarized in Figure 5.

[17] I assume $\Delta = 2$ years in calibration

[18] I assume women can get married earlier than men. According to China's marriage law, women can be legally married at age 20. The age is 22 for men.

6.2. *Production*

At time t, the production function for aggregate output takes the Cobb-Douglas form:

$$Y_t = AK_t^{\alpha}N_t^{1-\alpha},$$

where Y_t is total output, K_t is aggregate capital, and N_t is the total labor endowment for all working-age people. Factor markets are competitive and capital and labor are paid their marginal products:

$$\begin{aligned} r_{K,t} &= \alpha AK_t^{\alpha-1}N_t^{1-\alpha} \\ w_t &= (1-\alpha)AK_t^{\alpha}N_t^{-\alpha} \end{aligned}$$

In this small open economy, interest rate r is fixed at the world interest rate. Capital flows freely among countries, and thus by the non-arbitrage condition, we have $r_K = r + \delta$, and the equilibrium wage and output are

$$\begin{aligned} w &= (1-\alpha)A(\frac{r+\delta}{\alpha A})^{\frac{\alpha}{\alpha-1}} \\ Y_t &= A(\frac{r+\delta}{\alpha A})^{\frac{\alpha}{\alpha-1}}N_t \end{aligned}$$

Notice that in a small open economy, the interest rate and wage are constant.

National private saving rate is defined as the ratio of aggregate private savings to GDP:

$$\begin{aligned} s_t^P &= \frac{Y_t + rNFA_{t-1} - C_t}{Y_t + rNFA_{t-1}} \qquad (26) \\ NFA_{t-1} &= S_{t-1} - K_t \end{aligned}$$

where households' disposable income is the sum of output Y_t and interest payments from net foreign assets accumulated in the previous period, NFA_{t-1}.

6.3. Period Utility

The choice of period utility is the same as in Section 5 with minor modification to the budget constraints. At age j, a single or widowed agent's period utility comes from consumption (c_j):

$$u^{\mathbf{s}}(c_j) = \ln(c_j)$$

subject to the following budget constraint:

$$c_j + s_j \leq wn_j + (1+r)s_{j-1} = x_j$$

where current liquid wealth x_j is the sum of labor income wn_j and wealth accumulated from the previous period. The expression for labor endowment n_j is derived from equation (24) for men or equation (25) for women before retirement age J^{ret}, or 0 otherwise.

Married couples enjoy the same period utility after marriage from consumption and direct utility from marriage z based on the wife's match quality:

$$u^{\mathbf{a}}(C_{j_m j_w}, z) = \ln(\kappa C_{j_m j_w}) + z$$

with household budget constraint

$$C_{j_m j_w} + s_{j_m, j_w} \leq w(n^m_{j_m} + n^w_{j_w}) + (1+r)s_{j_m-1, j_w-1}$$

where $s_{j_m-1,j_w-1} = s_{m,j_m-1} + s_{w,j_w-1}$.

6.4. Marriage Market

Most of the specification of the marriage market that was developed in Section 5 still holds here. Women have to choose a marital age ex ante between J^{mar}_w and $J^{mar}_w + \Delta$, and draw match quality z from distribution $H_z(z)$ after entering the marriage market. Pre-marital saving is not observable and thus there is no competitive saving motive for women. With a deterministic age-labor endowment profile, all women face the same optimization problem before the marital stage, and thus their choice of pre-marital saving and marriage timing are also the same. As in Section 5, in equilibrium women choose to marry at their earliest marriageable age J_w^{mar}[19].

Unlike women, single men in the marriage market are of different ages when each generation's sex ratio $R^* > 1$ and not all men can marry at age J_m^{mar}. I denote the measures of single men at age j_m as M_{j_m} and their saving distributions as G_{s,j_m}, $\forall j_m \in [J_m^{mar}, J_m^{mar} + \Delta]$. Consequently men are characterized by combinations $\{s_m, j_m\}$ in the marriage market. As in Proposition 4, all men in the marriage market are ranked in the same order by women and their qualities are summarized by the indicator $q(s_m, j_m)$ based on utilities after marriage. (See equation (31) below.) With a total measure of single men and a sex ratio in marriage market of

$$M = \sum_{J_m^{mar}}^{J_m^{mar}+\Delta} M_{j_m};\ \overline{R} = \frac{M}{W_1}$$

and an equilibrium distribution of q of $H_q(q)$, matching is positively assortative between q and z.

I denote the lowest value of q among men who can be matched as $q^* = H_q^{-1}(1 - \frac{1}{\overline{R}})$. Men with quality $q < q^*$ cannot be matched and remain single this period. Otherwise, a man with quality higher than q^* matches with a young woman of match quality

$$z(q) = H_z^{-1}[1 - \overline{R}(1 - H_q(q))] \tag{27}$$

On the other hand, the quality q that a woman of type z matches with is

$$q(z) = H_q^{-1}[1 - \frac{1}{\overline{R}}[1 - H_z(z)]] \tag{28}$$

6.5. Value Function

6.5.1. Single Agents in Post-marital Stage

Single men beyond the age of marriage maximize life-time utility by allocating current liquid wealth between consumption and saving. If the man is still working, he maximizes

$$\begin{aligned} U^{\mathbf{s}}_{m,j_m}(x_{j_m}) &= \max_{s_{j_m}} \ln c_{j_m} + \beta \mathbb{E}_{n_{j_m+1}} U^{\mathbf{s}}_{m,j_m+1}(x_{j_m+1}) \\ & \quad s.t. \\ c_{j_m} + s_{j_m} &= x_{j_m};\ s_{j_m} \geq 0 \\ x_{j_m+1} &= (1+r)s_{j_m} + w n_{j_m+1} \end{aligned} \tag{29}$$

[19]In contrast to Proposition 4 in Section 5, I make this result a conjecture here and then verify that it holds in equilibrium.

and labor income n_{j_m+1} is subject to income risk according to equation (24). After retirement, his labor income will be 0 and all his liquid wealth is generated by saving from previous period. At age J, he'll consume all wealth and set $s_J = 0$.

The optimization problem for a single woman after the age of marriage is similar but since her labor income is deterministic, the expectation operator is dropped from the maximization problem above.

6.5.2. Married Household

Couples with husband's age j_m and wife's j_w make the consumption-saving decision jointly. If both couples are working, they maximize:

$$U^{\mathbf{a}}_{j_m j_w}(x_{j_m j_w}, z) = \max_{s_{j_m j_w}} \ln(\kappa C_{j_m j_w}) + z + \beta \mathbb{E}_{n_{j_m+1}} U^{\mathbf{a}}_{j_m+1, j_w+1}(x_{j_m+1, j_w+1}, z) \tag{30}$$

with household budget constraint

$$\begin{aligned} C_{j_m j_w} + s_{j_m j_w} &\leq w(n^m_{j_m} + n^w_{j_w}) + (1+r)s_{j_m-1, j_w-1} = x_{j_m j_w} \\ x_{j_m+1, j_w+1} &= w(n^m_{j_m+1} + n^w_{j_w+1}) + (1+r)s_{j_m j_w} \end{aligned}$$

The household's liquid wealth is the sum of saving from the previous period and current labor incomes from both spouses. The expectation operator is included in equation (30) if the husband will still be working in the next period and thus subject to labor income risk. Otherwise, the operator is dropped.

If both couples are the same age, when they both reach age J they'll choose to consume all their wealth and save no more: $s_{JJ} = 0$. Otherwise, if only the husband reaches age J while wife's age $j_w < J$, the optimal level of saving for husband is different from optimal saving for wife. Clearly the husband would like to choose to consume all the family wealth today, but by doing so the wife will have no wealth in the next period. In such cases, the optimization problem is

$$\begin{aligned} U^{\mathbf{a}}_{J, j_w}(x_{J, j_w}, z) &= \max_{s_{J, j_w}} \ln(\kappa C_{J, j_w}) + z + \mu\beta U^{\mathbf{s}}_{w, j_w+1}(x_{j_w+1}) \\ & \quad s.t. \\ C_{J, j_w} + s_{J, j_w} &\leq x_{J, j_w} \\ x_{j_w+1} &= (1+r)s_{J, j_w} \end{aligned}$$

where the constant $\mu \in (0,1)$ characterizes wife's Pareto weight.

Since in equilibrium women marry earlier than men, the general case is that $j_m > j_w$. The husband's life-time utility is:

$$\begin{aligned} U^{\mathbf{a}}_{j_m j_w,m}(x_{j_m j_w}, z) \quad &= \quad \ln(\kappa C_{j_m j_w}) + z + \beta \mathbb{E}_{n_{j_m+1}} U^{\mathbf{a}}_{j_m+1,j_w+1,m}(x_{j_m+1,j_w+1}, z) \\ &= \mathbb{E}_{n_{j_m+1}}\{ \sum_{t=0}^{J-j_m} \beta^t \ln(\kappa C_{j_m+t,j_w+t})\} + (\sum_{t=0}^{J-j_m} \beta^t) z \end{aligned}$$

and the wife's is

$$\begin{aligned} U^{\mathbf{a}}_{j_m j_w,w}(x_{j_m j_w}, z) &= \ln(\kappa C_{j_m j_w}) + z + \beta \mathbb{E}_{n_{j_m+1}} U^{\mathbf{a}}_{j_m+1,j_w+1,m}(x_{j_m+1,j_w+1}, z) \\ &= \mathbb{E}_{n_{j_m+1}}\{ \sum_{t=0}^{J-j_m} \beta^t \mathbb{E}_{j_m j_w} \ln(\kappa C_{j_m+t,j_w+t})\} + (\sum_{t=0}^{J-j_w} \beta^t) z \qquad (31) \\ &\quad + \mathbb{E}_{n_{j_m+1}}\{ \sum_{t=J-j_m+1}^{J-j_w} \beta^t \mathbb{E}_{j_m j_w} \ln(c^{\mathbf{s}}_{j_w+t})\} \end{aligned}$$

6.5.3. Single Agents at Marriageable Stage

A man of marital age $j_m \in [J_m^{mar}, J_m^{mar} + \Delta]$ with liquid wealth $x_{j_m} = (1+r)s_{j_m-1} + wn_{j_m}$ takes the women's saving $s^w_{J_w^{mar}}$, the quality function $q(s_m, j_m)$, its distribution $H_q(q)$ as well as the assortative matching rule (27) as given, and allocates his wealth between consumption and saving optimally by solving the following:

$$\begin{aligned} U^{\mathbf{s}}_{m,j_m}(x^m_{j_m}) \quad &= \quad \max_{s^m}\{\ln c^m + \\ &\quad \max[\beta \mathbb{E}_{n^m_{j_m+1}} U^{\mathbf{a}}_{j_m+1,J_w^{mar}+1,m}(x', z[q(s^m, j_m)]), \\ &\quad \beta \mathbb{E}_{n^m_{j_m+1}} U^{\mathbf{s}}_{j_m+1,m}(x^m_{j_m+1})]\} \end{aligned}$$

subject to:

$$\begin{aligned} c^m + s^m &= x^m_{j_m} \\ x' &= (1+r)(s^m + s^w_{J_w^{mar}}) + w(n^m_{j_m+1} + n^w_{J_w^{mar}+1}) \\ x^m_{j_m+1} &= (1+r)s^m + wn^m_{j_m+1} \end{aligned}$$

Upon meeting with a woman with match quality $z[q(s^m, j_m)]$, a man decides whether to marry by comparing the expected life-time utility from marriage $U^{\mathbf{m}}_{j_m+1,J_w^{mar}+1,m}$ to the utility of remaining single for one more period $U^{\mathbf{s}}_{j_m+1,m}$. A single man of age $J^{mar}_m + \Delta$ who fails to get matched this period remains single for life.

A single woman with wealth x_w at age J^{mar}_w solves

$$U^{\mathbf{s}}_{w,J^{mar}_w}(x_w) = \max_{s^w}\{\ln c_w + \int_z \max[\beta[q(z) + (\sum_{t=0}^{J-j_w} \beta^t)z], \beta U^{\mathbf{s}}_{w,J^{mar}_w+1,}(x'_w)]dG_z(z)\}$$

subject to

$$\begin{aligned} c_w + s_w &\leq x_w;\ s_w \geq 0 \\ x'_w &= (1+r)s_w + wn^w_{j_w+1} \end{aligned}$$

and assortative matching function (28) which is the expected utility from marriage.

6.5.4. *Single Agents at Pre-marital Stage*

The optimization problem for single men of age $j_m < J^{mar}_m$ is

$$\begin{aligned} U^{\mathbf{s}}_{m,j_m}(x^m_{j_m}) &= \max_{s^m_{j_m}}\{\ln(x^m_{j_m} - s^m_{j_m}) + \beta\mathbb{E}_{n_{j_m+1}} U^{\mathbf{s}}_{j_m+1,m}(x^m_{j_m+1})]\} \\ & \quad s.t. \\ x^m_{j_m+1} &= (1+r)s^m_{j_m} + wn^m_{j_m+1} \end{aligned}$$

With the competitive saving motive, they keep accumulating wealth to improve their ranking at marital age.

The optimization problem for single women of age $j_w < J^{mar}_w$ is

$$\begin{aligned} U^{\mathbf{s}}_{w,j_w}(x^w_{j_w}) &= \max_{s^w_{j_w}}\{\ln(x^w_{j_w} - s^w_{j_w}) + \beta U^{\mathbf{s}}_{w,j_w+1}(x^w_{j_w+1})]\} \\ & \quad s.t. \\ x^w_{j_w+1} &= (1+r)s^w_{j_w} + wn^w_{j_w+1} \end{aligned}$$

There is no competitive saving motive for women since their wealth is not publicly observable before marriage.

6.6. Equilibrium

Definition 4. *A **multi-period long-run equilibrium** consists of measures of singles in the marriage market* $\{M_{j_m}, W_{j_w}\}$, $\forall j_m = [J_m^{mar}, J_m^{mar} + \Delta]$, $j_w = [J_w^{mar}, J_w^{mar} + \Delta]$; *single men's saving function* $s_{j_m}(x_{m,j_m})$ *and the associated value functions* $U^{\mathbf{s}}_{m,j_m}(x^m_{j_m})$, *distributions of single men's saving* $G_{s,j_m}(\cdot)$ *and liquid wealth* $G_{x,j_m}(\cdot)$ *at age* j_m $\forall j_m$; *men's and women's conditional marital indicators* $\{\delta_{m,j_m}(s_{m,j_m}), \delta_{w,j_w}\}$, $\forall j_m = [J_m^{mar}, J_m^{mar} + \Delta]$, $j_w = [J_w^{mar}, J_w^{mar} + \Delta]$; *married household's saving functions* $s^{\mathbf{a}}_{j_m j_w}(x)$ *and the associated value functions* $\{U^{\mathbf{a}}_{j_m j_w, m}(x, z), U^{\mathbf{a}}_{j_m j_w, w}(x, z)\}$, *and a matching rule* μ *such that:*

1. *Given the measures and saving distributions in the marriage market, the value functions after marriage, and the value functions for remaining single, then the single men's saving functions* $s_{j_m}(x_{m,j_m})$ *and marital timing choices* $\delta_{m,j_m}(s_{m,j_m})$ *as well as women's marital timing choices* δ_{w,j_w} *are optimal.*

2. *The measures and distributions in the marriage market are consistent with agents' optimal choices and the distribution of endowments, such that:*

$$\begin{aligned}
G_{s,j_m}(s) &= prob\{s_{j_m}(x_{m,j_m}) \leq s\}, \forall s, j_m \qquad (32)\\
G_{x,j_m}(x) &= prob\{((1+r)s_{m,j_{m-1}} + wn^m_{j_m} \leq x) \wedge (\delta_{m,j_m-1}(s_{m,j_{m-1}}) = 0)\}, \forall x, j_m \geq 2\\
M_{j_m} &= M_1, \forall j_m < J_m^{mar}\\
M_{j_m} &= M_{j_m-1} \int [1 - \delta_{m,j_{m-1}}(s)] dG_{s,j_m-1}(s),\ \forall j_m \geq J_m^{mar}\\
W_{j_w} &= W_1, \forall j_w < J_w^{mar};\ W_{j_w} = W_{j_w-1}(1 - \delta_{w,j_w-1}),\ \forall j_w \geq J_w^{mar}
\end{aligned}$$

3. *The saving functions of married households* $s^{\mathbf{a}}_{j_m j_w}(x)$ *solve the optimization problem (30) given family wealth* x, $\forall j_m, j_w$.

4. *The matching distribution* μ *is stable.*

7. Numerical Results

Does a change in the sex ratio have a significant effect on the aggregate saving and on the age profile of saving of the pre-marital cohorts? In this section, I calibrate the multi-period model in Section 6 to answer this question quantitatively.

7.1. Calibration

The main dataset I use for the calibration here is the China Health and Nutrition Survey (hereafter CHNS) which offers detailed information covering economic, demographic and other major social variables on the household and individual levels. This paper uses all nine waves (1989, 1991, 1992, 1997, 2000, 2004, 2006, 2009, 2011) provided by the survey. Most of the waves cover 9 provinces that vary considerably in geography and economic development[20]. Counties in these provinces were stratified by income and 4 counties were selected randomly within each province. In addition, the provincial capital and a lower income county are also included.

The samples used here are limited to male adults in the labor force between the ages of 18 to 60. The CHNS data include both urban and rural samples. I only use the urban part because their income process is more consistent with the purpose of this study. I exclude the samples with annual working hours smaller than 400 or larger than 4000. Individuals reporting as self-employed are also dropped. This leaves us with 8296 observations.

Income process: Parameters γ_0, γ_1 and γ_2 in the labor endowment equation (24) are derived by estimating the following equation:

$$y_{i,t} = \gamma_0 + \gamma_1 experience_{i,t} + \gamma_2 experience^2_{i,t} + d_t + \varepsilon^m_{i,t}$$

where the outcome variable $y_{i,t}$ is the logarithm of real annual labor income of person i in year t. I use annual labor income instead of wage as the dependent variable here because the self-reported data about annual working hours in CHNS are very noisy which reduces the quality of the calculated wage rate. Hence, I only use regressions with wage as the outcome variable in a few specifications to test the robustness of my estimation of $\{\gamma_0, \gamma_1, \gamma_2\}$. Experience is defined as the age of person i minus (7+years of schooling). The set of yearly dummies d_t is normalized to sum to zero: $\sum_t d_t = 0$. The standard deviation of transitory shocks σ_ε in equation (24) matches the standard deviation of the error term $\varepsilon^m_{i,t}$.

Demographics: Demographic parameters are set using a model period of 2 years. An

[20]The 9 provinces are Liaoning, Heilongjiang, Jiangsu, Shandong, Henan, Hubei, Hunan, Guangxi, Guizhou. Liaoning participated in all the waves except Wave 1997, and Heilongjiang was not surveyed until Wave 1997. I exclude the observations from Beijing, Shanghai and Chongqing since they are only available in Wave 2011.

agent lives from real-life age of 18 to real-life age of 80 so that $J = 32$. $J^{ret} = 22$ corresponds to a real-life retirement age of 60. Since I don't have good information about parental savings which constitute a majority part of brideprice in China, I choose to let single men save for themselves in the model, and set the marital ages relatively late in life. In particular, $J_m^{mar} = 10$ and $J_w^{mar} = 9$ corresponding to real-life age of 36 and 34. I vary the duration of the marital stage Δ from 0 to 3 to empirically evaluate to what extent the pressure on the aggregate saving rate can be tempered by the endogenous age gap between couples. According to the National Bureau of Statistics (NBS) in China, the sex ratio at birth is 1.18 boys per girl in 2011, which is set as the benchmark sex ratio in the model. In cases in which the husband reaches the final age J while the wife is younger, the choice of Pareto weight of the wife μ is taken from Fernandez and Wong (2013) and set as 0.3.

Interest rate and production: Song et al. (2011) computed the average one-year real deposit rate in China during 1998-2005 and suggest it takes value 0.0175. Since one model period corresponds to 2 years in this paper, I choose $r = 1.0175^2 - 1 = 3.53\%$, and the discount factor β is set as $\frac{1}{1+r} = 0.966$. The production function parameters α and A are chosen to be 0.4 and 1 respectively. Annual depreciation rate of capital in Song et al. (2011) is set to 0.1. Since one model period here corresponds to 2 years, I set $\delta = 1-(1-0.1)^2 = 19\%$.

Other parameters including the distribution of match quality $H_z(\cdot)$ and the parameter for the public good feature of marriage κ are from Du and Wei (2013). $H_z(\cdot)$ is a Normal distribution with mean 1 and standard deviation 0.05, and $\kappa = 0.8$. Parameter values are summarized in Table 2.

7.2. *Results*

Using these parameters, I find the following results:

7.2.1. *Competitive Saving Motive in the Stationary Equilibrium*

The stationary saving functions of newly born men corresponding to different sex ratios R^* and marriage market durations Δ are plotted in Figure 6. It also plots the wealth

Table 2: Parameter Values

Parameters	**Meaning**	**Value**	**Source**
J, J^{ret}	lifespan and retirement age	32, 22	CHNS
J_m^{mar}, J_w^{mar}	marital ages	10, 9	CHNS
R^*	sex ratio at birth	1.1778	Population Census 2010
γ_0, γ_1, γ_2	labor endowment parameter	8.9; 0.03; -0.005	CHNS
σ_ε	variance of transitory shock	0.73	CHNS
α, A	production function parameters	0.4; 1	Du and Wei (2013)
δ	depreciation rate	19%	Song et al. (2011)
r, β	interest rate, discount factor	0.035; 0.97	Song et al. (2011)
κ	household consumption factor	0.8	Du and Wei (2013)
μ	wife's Pareto weight	0.3	Fernandez and Wong (2013)
$H_z(\cdot)$	distribution of match quality	$N(1, 0.05)$	Du and Wei (2013)

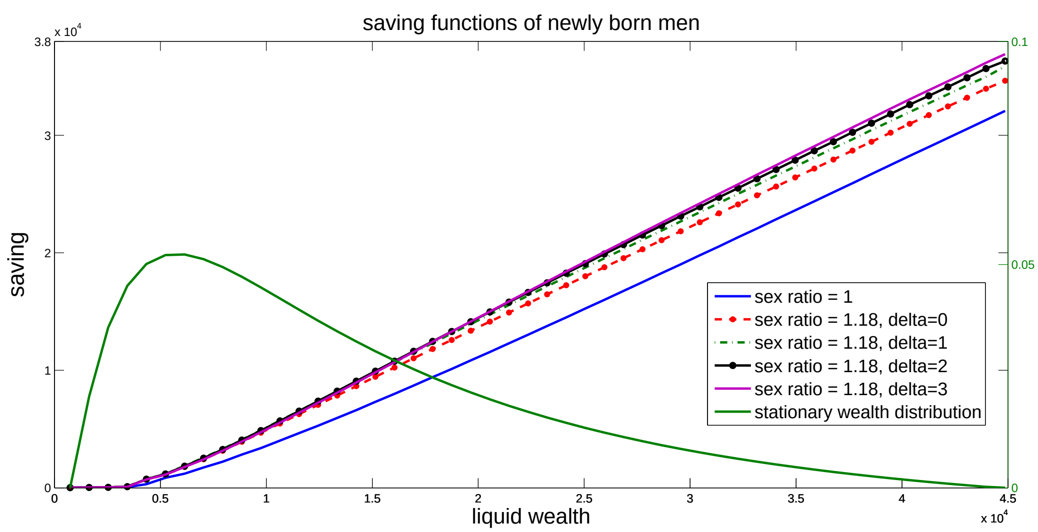


Figure 6. Saving Functions for Newly Born Men

distribution of newly born men labeled on the right y-axis. I find that the saving functions are strictly increasing with wealth level except for men with little wealth. Since their labor incomes are subject to the lowest transitory shocks this period according to equation (24), and their expected age-income profiles in the future are as good as those of any other men of the same age, zero saving is optimal for them to smooth consumption over their life-cycle. Except for these men, the saving function is strictly increasing with wealth.

One major observation from Figure 6 is, with a balanced sex ratio ($R^* = 1$), increasing the duration of the marital stage Δ has no effect on single men's savings. Since everyone prefers to getting married to remaining single and people prefer to get married as early as possible, in the stationary equilibrium every man gets married at the earliest marital age $J_m^{mar} = 10$ even when $\Delta > 0$ and they have the option to delay marriage. Thus every single man chooses saving as if the marriage market would open once in lifetime ($\Delta = 0$).

Secondly, when R^* increases, men choose to save more at each wealth level because they anticipate more intense mating competition at the marital stage. As a result, they accumulate more pre-marital saving starting at age 1. This is the reason that the aggregate saving rate is affected by R^*. Furthermore, as the value of Δ increases, men would increase saving even more at each wealth level because this will further increase the marriage market tightness $\bar{R}$ when men who failed to be matched at the earliest marital age rejoin the market later.

To make this point clearer, let us consider the stationary equilibrium with unbalanced sex ratio ($R^* = 1.18$) and duration parameter $\Delta = 3$. Thus every man has the possibility of getting married between age J_m^{mar} and $J_m^{mar}+\Delta$. Wealth distributions and saving functions of single men at age $j_m \in [J_m^{mar}, J_m^{mar} + \Delta]$ are plotted in Figure 7. Threshold matching wealth at age j_m, $x^*_{j_m}$ are also labeled in the figure. These threshold values are generated such that an age j_m man with wealth $x^*_{j_m}$ is indifferent between marrying a woman with match quality $\underline{z}$ and remaining single for one more period. When a single man joins the marriage market for the first time at age J_m^{mar}, he can get married if his accumulated wealth is larger than $x^*_{J_{mar}}$, or he has to remain single this period. For a single man of age $J_m^{mar} + 1$, his liquid wealth is the sum of his saving from the previous period and his current labor income. If his current income draw is high enough and the resulting wealth level is higher than the required matching wealth $x^*_{J_{mar}+1}$, this man gets married at age $J_m^{mar} + 1$, or otherwise he

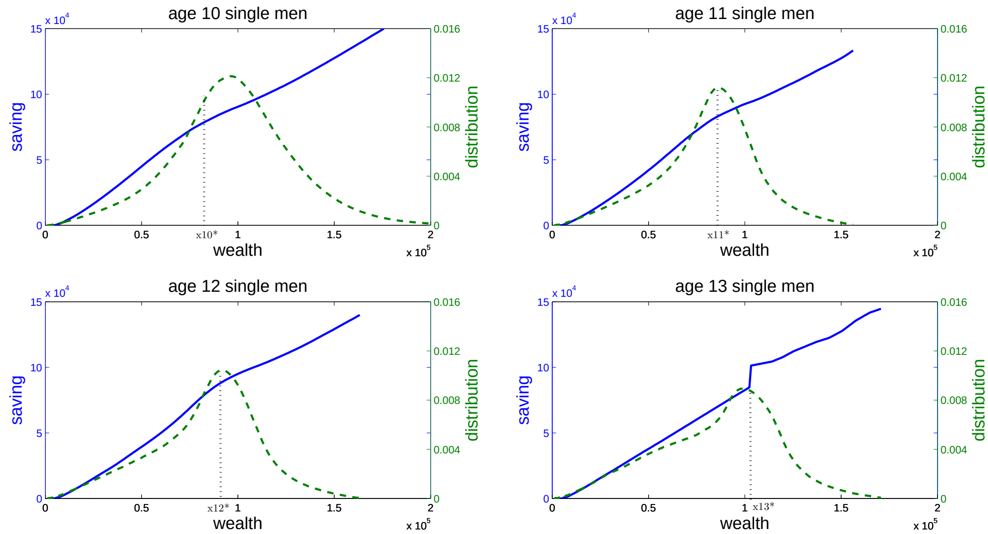


Figure 7. Wealth Distributions and Matching Pattern at the Marital Stage

stays single again and rejoin the market in the next period. This process stops when a man reaches age $J_m^{mar} + \Delta$. If he fails to marry at this age, he will remain single for life.

By increasing Δ, we actually invite extra competitors into the market who might be competent if they receive good draws of labor income. Fiercer competitions force single men to save more ex ante. In other words, in stationary equilibrium longer duration of marital stage actually increases the savings of the pre-marital cohorts.

7.2.2. *Effect of Sex Ratio on Aggregate Saving*

To test the effect of a rising sex ratio to the aggregate saving rate, I next consider the following experiment: At time 0, the sex ratios of all existing generations are 1, and the economy is at its long-run stationary equilibrium. Starting from time 1, the sex ratio of all newly-born generations switches to 1.18 unexpectedly. We'd like to see how the economy-wide saving rate would react to such a permanent shock. An algorithm for such transition dynamics is described in Appendix E, and the resultant dynamics of aggregate saving rates corresponding to different marital stage durations are given in Figure 8.

In Figure 8, I trace the evolution of aggregate savings when R^* changes from 1 to 1.18.

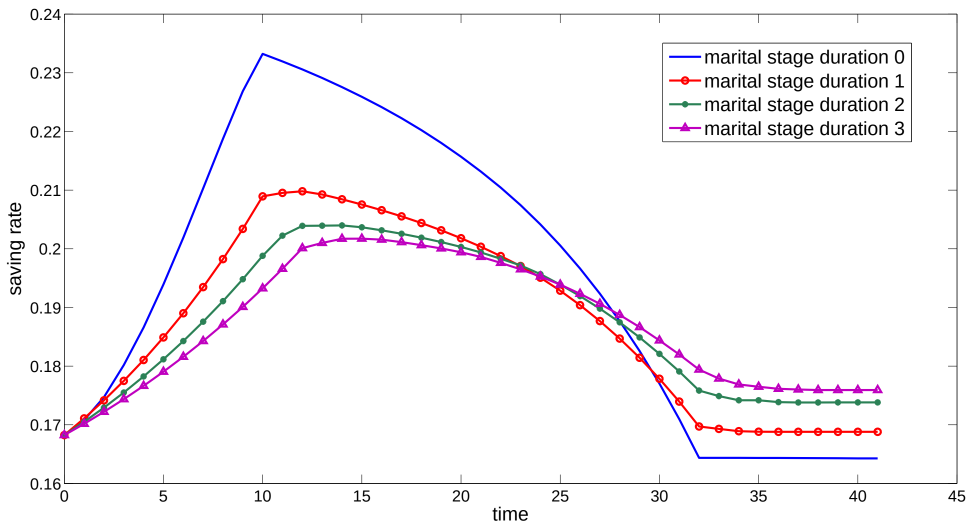


Figure 8. Dynamics of Saving Rate after Unexpected Sex Ratio Increase

The blue solid line corresponds to the dynamics of saving rate with $\Delta = 0$ and every single man can only get married at age J_m^{mar}. Over 32 periods (64 years), the saving rate transition path reaches its peak at time 10 when the generation born at time 1 reach its earliest marriageable age. After that, the increased dis-savings from married households start to offset the increase in saving rates in the pre-marital generations. At time 32, the economy converges to its new steady state with an unbalanced sex ratio of 1.18 in every living generation. The aggregate saving rate at the peak is 23.3% of GDP which is higher than the saving rate in the original stationary equilibrium by 6.5% of GDP. This result is similar to what Du and Wei (2013) find (6% of GDP).

Then I test how the dynamics of the aggregate saving rate can be affected if we allow an endogenous age gap between spouses. As we increase the duration of marital stage Δ, the dynamics of saving rates is dampened, generating lower peaks.When the duration parameter Δ is set to 3 corresponding to 8 years as the maximum age gap between couples, the peak of the saving rate dynamics is only 3.35% of GDP higher than the aggregate saving rate in the previous stationary equilibrium. In other words, not considering age gap responses can potentially inflate reactions in saving rate by 94% ((6.5%-3.35%)/3.35%). Such a reduction

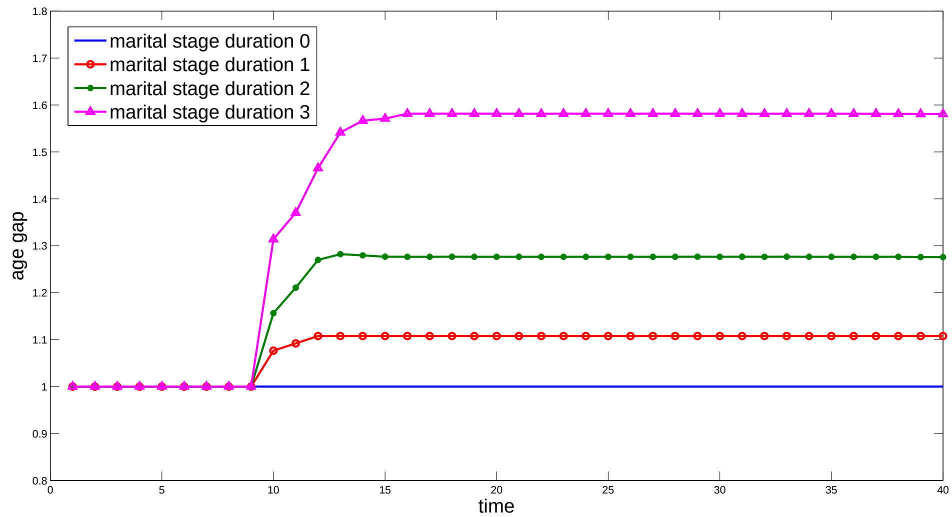


Figure 9. Dynamics of Age Gap after Unexpected Sex Ratio Increase

in the aggregate saving rates arises since men in the affected cohorts have the option to postpone marriage when the duration of marital stage is expanded. With more chances to get married, the competitive saving motive is tempered. The corresponding dynamics of the average age gap between newly wedded couples along the transitional path is reported in Figure 9.

Consequently, although a longer duration of the marital stage results in a higher aggregate saving rate in the steady state, it dampens the reaction of the aggregate saving rate to a permanent sex ratio shock along the transitional path. In other words, allowing a longer duration of marital stage has opposite effects on the aggregate saving rate in stationary equilibrium and during the transition periods. It dampens the reaction of the aggregate saving rate to a sex ratio increase along the transition path, but eventually results in a higher saving rate when the economy reaches its new stationary equilibrium.

7.2.3. Downward-sloping Age Profile of Saving Rates

As documented by Chamon and Prasad (2010) and Song and Yang (2010), the age profile of savings in China has had an unusual pattern in recent years with younger households

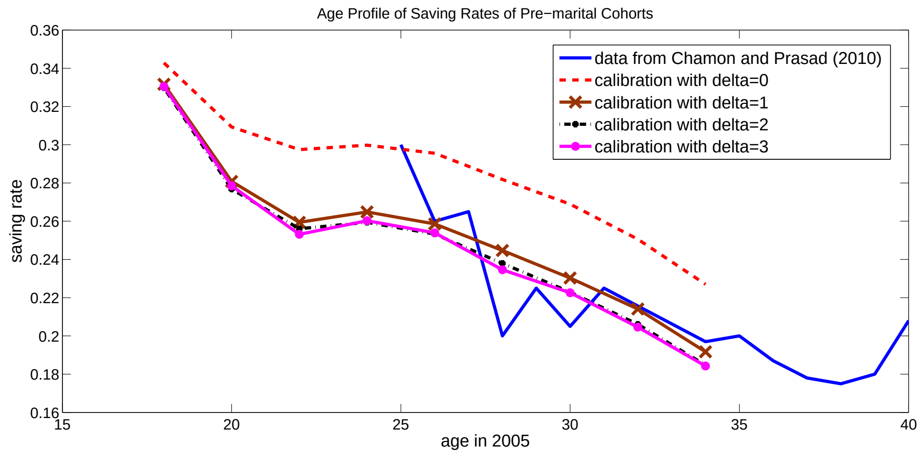


Figure 10. Age Profile of Saving Rates of Pre-marital Cohorts

having high savings rates relative to the middle-aged. This is the opposite to what classical life-cycle theory predicts because usually life-cycle saving is hump-shaped with young and old generations saving less than middle-aged generations. As depicted in Figure 10, the calibrated age profile of saving rates in 2005 is generated by matching each age cohort by the sex ratio in births at their birth year. For example, the cohort of age 26 in year 2005 was born in year 1980, and the sex ratio at birth in 1980 was 1.0782. Then I calibrated the model using $R^* = 1.0782$, and recorded the generated saving rate for this cohort at age 26. This process generates the downward-sloping age profile of saving rates seen in Figure 10 which is similar to the empirical data documented in Chamon and Prasad (2010).

The unusual pattern in the age-saving rate profile is consistent with a competitive saving motive among pre-marital cohorts. Without any wealth endowment, men of age 1 (real-life age 18) are quite similar to each other. Although they might earn different income this period according to equation (24), most uncertainties between this period and their earliest marital age (real-life age 36) are yet to be realized. The competitive saving motive is strongest when people are similar to each other because the return to saving is the highest as it is the easiest to overtake one's rivals. This leads to high saving for all newly born men. As men grow older and approach the age of marriage, the competitive saving motive gets weaker. On the one hand, these men become increasingly differentiated by wealth accumulated up to that age.

Table 3: Robustness Checks of Maximum Increase in the Aggregate Saving Rate

Checked Parameters	**Meaning**	**Exogenous Age Gap ($\Delta = 0$)**	**Endogenous Age Gap ($\Delta = 3$)**
Baseline Case		6.5%	3.35%
$\mu = 0.2$	Pareto weight of wives	6.5%	3.49%
$\mu = 0.5$	Pareto weight of wives	6.5%	3.24%
$\kappa = 0.5$	private consumption	5.75%	2.28%
$\kappa = 1$	complete public good	6.7%	3.72%
$E(z) = 0.5$	mean of dist. of z	5.68%	2.41%
$E(z) = 2$	mean of dist. of z	7.17%	3.04%
σ_z	z's standard deviation	4.84%	2.71%

On the other hand, uncertainties of labor income are gradually resolved and thus the chance that a man can significantly improve his position by receiving high income before marital stage becomes smaller. Eventually as the age of marriage gets nearer, a man's position is mostly determined by his wealth. That is when the competitive saving motive is weakest and the return to saving is low.

Compared to single men, women in equilibrium choose 0 saving in the pre-marital stage, because first, they face a deterministic upward-sloping age profile of income, and second, with sex ratio $R^* \geq 1$, they can get married for sure and take advantage of the savings of their future husbands.

7.3. Robustness Checks

I examine the robustness of my findings about the competitive saving motive by once again doing the experiment in Subsection 7.2.2 with an unexpected permanent change in the sex ratio at birth from 1 to 1.18[21]. The results of robustness checks are reported in Table 3.

[21] I also examined the robustness of the feature of the downward-sloping age profile of saving rates in the pre-marital cohorts under the parameters listed in this subsection, and found that under all these parameters, the simulated age-saving rate profiles are downward-sloping and comparable to the empirical data. Due to limitation of space, this part is not reported here. Al the results on robustness checks are available upon request.

7.3.1. *Pareto Weight of Wife μ*

μ is the Pareto weight of the wife in cases where couples are of different ages and the older spouse reaches age$J$, which takes value 0.3 in the benchmark numerical results. Instead of using the value 0.3 as listed in Table 2, I now examine a lower value case when $\mu = 0.2$ and a higher value case when $\mu = 0.5$. I find varying μ has very small effects on our results. With $\mu = 0.2$ and exogenous age gap ($\Delta = 0$), the peak of the saving rate path is 6.5% higher than the saving rate with a balanced sex ratio. With endogenous age gap ($\Delta = 3$), the different is reduced to 3.49%. When $\mu = 0.5$, the maximum changes in saving rate are 6.5% and 3.24% corresponding to exogenous and endogenous age gaps respectively. With both values of μ, the changes in the aggregate saving rate is very close to the results in the benchmark case, which is not surprising since the value of μ only affects the wife's Pareto weight in the last period of marriage if only the husband reaches that last age J. Except for this case, since couples consume the same amount of consumption goods each period, the Pareto weight of wives is implicitly set to 0.5.

7.3.2. *Public Good Feature of Consumption κ*

In our benchmark case, $\kappa = 0.8$. Now I examine two different values of it. In the first case, $\kappa = 0.5$ corresponding to completely private consumption in a married household. In the second case, $\kappa = 1$ meaning consumption in a married household is a public good without congestion. I find with a larger κ, the response in the aggregate saving rate is also larger with an increase in the sex ratio, because the higher benefit from marriage encourages single men to compete more fiercely by saving more aggressively. When $\kappa = 0.5$, peak increases in aggregate saving rate are 5.75% and 2.28% corresponding to different age gap setups. Both numbers are smaller than the results in the benchmark case but not by much. Besides, allowing endogenous age gap dampens the reaction of the aggregate saving rate by a larger fraction $\frac{5.75\%-2.28\%}{5.75\%} \approx 60\%$ than the benchmark case. This is because when marriage becomes less attractive, single men are more willing to postpone marriage. When $\kappa = 1$, the changes in the saving rate are 6.7% and 3.72% corresponding to different age gap setups. Both numbers are higher but close to the results in the benchmark case.

7.3.3. *Mean of the Distribution of Match Quality z*

The distribution of match quality z is important for our results since a large part of the benefit of marriage is from the match quality of wives. The mean of z takes value 1 in the benchmark case. I examine the cases in which the mean can take a higher value ($E(z) = 1$) or a lower value ($E(z) = 0.5$). Similar to the result of changing κ, higher value of $E(z)$ results in a higher benefit from marriage, and thus the competitive saving motive will be stronger. The changes in the saving rate with both values chosen are reported in Table 3. I find that changing the value of $E(z)$ to 0.5 or 1 can increase or decrease the responses of the aggregate saving rate in the benchmark model by 15%.

7.3.4. *Standard Deviation of the Match Quality z*

As a robustness check, I increase the standard deviation of z from 0.05 to 0.2 so the distribution of z is more dispersed, and find that the changes in the saving rate are smaller than the results in the benchmark case. This is opposite to what Hopkins and Kornienko (2010) find. By using a one-period tournament model, these authors claim that greater dispersion of rewards forces people into greater efforts since the payoff of differentiating oneself from others is larger. Why the result here changes in the opposite direction is interesting and left for future research. One candidate explanation is, in my multi-period model, although a more dispersed z increases the return to saving for single men, it also increases the risk of return to saving ex ante since single men's marriage prospects are more uncertain. It seems that the risk part dominates in the numerical results and thus the response in the aggregate saving rate is reduced.

To summarize, changing the parameter values used in the benchmark model doesn't change the responses in the aggregate saving rate by much. With an unexpected permanent increase of the sex ratio at birth from 1 to 1.18 and an exogenous age gap between spouses, the peak increase in the aggregate saving rate is around 6% of GDP which is similar to the empirical finding in Wei and Zhang (2011). Allowing single men to postpone their marriage besides increasing their pre-marital savings at given ages can potentially reduce the response in the saving rate to 3% of GDP which is about half the size in the exogenous age gap case.

8. Concluding Remarks and Future Research

This paper builds a life-cycle matching model with pre-marital investment. It is a multi-dimensional matching model with non-transferable utilities. I show that under certain conditions, I can summarize the multi-dimensional characteristics of men into a single dimension, thus the stable matching patterns can be characterized. With this model framework, I analyze how a competitive saving motive arises and how people's saving behavior and marital ages will react to marriage market imbalances.

By calibrating this model to China's economy, I analyze whether a competitive saving motive can explain the saving rate puzzle in China, i.e., a rising aggregate saving rate and a downward-sloping age profile of saving rates among young cohorts. The quantitative result is consistent with empirical findings. After the sex ratio rises, single men increase their saving in a competitive way in order to improve their ranking in the marriage market, and the resultant increase in the aggregate saving rate is economically significant. Although tempered by some men's delaying marriage, the aggregate saving rate still increase by 3.35% of GDP, which is around 30% of the total increase in the saving rate from 1995 to 2007. In addition, the competitive saving motive also generates a downward-sloping age profile of saving rates of the pre-marital cohorts.

The theory can be extended in a number of directions. Adding some randomness in the matching process in the marriage market should generate more realistic patterns for the ages of marriage. In that case, we may best fit the empirical distribution of marital ages and age gaps between spouses.

References

Abramitzky, R., A. Delavande, and L. Vasconcelos (2011). Marrying up: The role of sex ratio in assortative matching. *American Economic Journal: Applied Economics 3*(3), 124–157. 1, 2, 5

Anderson, S. (2007). Why the marriage squeeze cannot cause dowry inflation. *Journal of Economic Theory 137*, 140 C 152. 1, 2, 5.1

Bergstrom, T. and D. Lam (1989a). The effects of cohort size on marriage markets in twentieth century sweden. Working Paper. 2

Bergstrom, T. and D. Lam (1989b). The two-sex- problem and the marriage squeeze in an equilibrium model of marriage markets. CREST Working Paper. 2

Bergstrom, T. C. and M. Bagnoli (1993). Courtship as a waiting game. *Journal of Political Economy 101*(1), 185–202. 12

Bhaskar, V. (2013). The demographic transition and the position of women: A marriage market perspective. Working Paper. 2

Bhaskar, V. and E. Hopkins (2013). Marriage as a rat race: Noisy pre-marital investments with assortative matching. Working Paper. 2

Brandt, L., A. Siow, and C. Vogel (2008). Large shocks and small changes in the marriage market for famine born cohorts in china. Working Paper. 2

Bussiere, M., Y. Kalantzis, R. Lafarguette, and T. Sicular (2013). Understanding household savings in china: the role of the housing market and borrowing constraints. MPRA Paper No. 44611. 3

Chamon, M., K. Liu, and E. Prasad (2013). Income uncertainty and household savings in china. *Journal of Development Economics 105*, 164–177. 3

Chamon, M. and E. Prasad (2010). Why are saving rates of urban households in china rising? *American Economic Journal: Macroeconomics 2*(1), 93–130. 1, 3, 3, 5, 7.2.3

Chiappori, P.-A., M. Iyigun, and Y. Weiss (2009). Investment in schooling and the marriage market. *The American Economic Review 99*(5), 1689–1713. 2

Chiappori, P.-A., R. J. McCann, and L. P. Nesheim (2010). Hedonic price equilibria, stable matching, and optimal transport: equivalence, topology, and uniqueness. *Economic Theory 42*(2), 317–354. 2

Chiappori, P.-A., S. Oreffice, and C. Quintana-Domeque (2012). Multidimensional matching with a potential handicap: Smoking in the marriage market. Working Paper. 2

Choukhmane, T., N. Coeurdacier, and K. Jin (2014). The one-child policy and household savings. Working Paper. 3

Cole, H. L., G. J. Mailath, and A. Postlewaite (1992). Social norms, savings behavior, and growth. *Journal of Political Economy 100*(6), 1092–1125. 2, 9

Cole, H. L., G. J. Mailath, and A. Postlewaite (2001). Efficient non-contractible investments in large economies. *Journal of Economic Theory 101*, 333–373. 2

Danziger, L. and S. Neuman (1999). On the age at marriage: theory and evidence from jews and moslems in israel. *Journal of Economic Behavior & Organization 40*, 179C193. 12

Du, Q. and S.-J. Wei (2013). A theory of the competitive saving motive. *Journal of International Economics 91*, 275C289. 1, 2, 3, 7.1, 2, 7.2.2

Ebenstein, A. Y. and E. J. Sharygin (2009). The consequences of the "missing girls" of china. *The World Bank Economic Review*. 5

Edlund, L. and C. Lee (2009). Son preference, sex selection and economic development: Theory and evidence from south korea. Working Paper. 5

Fernandez, R. and J. C. Wong (2013). Divorce risk, wages, and working wives: A quantitative life-cycle analysis of female labor force participation. Working Paper. 7.1, 2

Fisman, R., S. S. Iyengar, E. Kamenica, and I. Simonson (2006). Gender differences in mate selection: Evidence from a speed dating experiment. *The Quarterly Journal of Economics 121*(2), 673–697. 7

Gale, D. and L. S. Shapley (1962). College admissions and the stability of marriage. *American Mathematical Monthly 69*, 9–14. 4.2

Hitsch, G. J., A. Hortacsu, and D. Ariely (2010). Matching and sorting in online dating. *The American Economic Review 100*(1), 130–163. 7

Hopkins, E. and T. Kornienko (2004). Running to keep in the same place: Consumer choice as a game of status. *The American Economic Review 94*(4), 1085–1107. 2, 4.3, B, C

Hopkins, E. and T. Kornienko (2010). Which inequality? the inequality of endowments versus the inequality of rewards. *American Economic Journal: Microeconomics 2*(3), 106–137. 7.3.4

Iyigun, M. and R. P. Walsh (2007). Building the family nest: Premarital investments, marriage markets, and spousal allocations. *Review of Economic Studies 74*, 507–535. 2

Judd, K. L. (1998). *Numerical Methods in Economics*. The MIT Press. 10

Keeley, M. (1977). The economics of family formation. *Economic Inquiry 15*(2), 238–250. 12

Lee, S. (2009). Marriage and online mate-search services: Evidence from south korea. Working Paper. 7

Li, S., C. Luo, Z. Wei, and X. Yue (2008). *Appendix: The 1995 and 2002 Household Surveys: Sampling Methods and Data Description. In B. Gustafsson, S. Li and T. Sicular, eds., Inequality and Public Policy in China.* Cambridge University Press. 4

Modigliani, F. and S. L. Cao (2004). The chinese saving puzzle and the life-cycle hypothesis. *Journal of Economic Literature 42*(1), 145–170. 3

Peters, M. and A. Siow (2002). Competing premarital investments. *Journal of Political Economy 110*(3), 592–608. 2

Roth, A. E. and M. A. O. Sotomayor (1992). *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge University Press. 4.2

Song, Z., K. Storesletten, and F. Zilibotti (2011). Growing like china. *American Economic Review 101*(1), 196–233. 7.1, 2

Song, Z. and D. Yang (2010). Life cycle earnings and the household saving puzzle in a fast-growing economy. Working Paper. 1, 3, 5, 7.2.3

Wei, S.-J. and X. Zhang (2011). The competitive saving motive: Evidence from rising sex ratios and saving rates in china. *Journal of Political Economy 119*(3), 511–564. 1, 3, 7.3.4

Wen, Y. (2010). Saving and growth under borrowing constraints: Explaining the "high saving rate" puzzle. Working Paper. 3

Zhang, J. (1995). Do men with higher wages marry earlier or later? *Economics Letters 49*, 193–196. 12, 13

Appendix A. Positively Assortative Matching Rule

1. If there is no mass point in $G_s(s)$, a man with saving $s_{m,1}$ and a woman with match quality indicator z match with each other if and only if $R^*[1 - G_s(s_{m,1})] = 1 - H_z(z)$ with sex ratio R^*. Men with $s_{m,1} < s^*_{m,1}$ are not matched.

2. If there is a mass point in $G_s(s)$ at s_0, then the matches for men with saving s_0 are independently random draws from the corresponding interval in $H_z(z)$. In particular: (i) if $G_s^-(s_0) \geq 1 - \frac{1}{R^*}$,[22] men with saving s_0 match with women with $z \in [\underline{z}_0, \overline{z}_0]$ with probability $\frac{h_z(z)}{H_z(\overline{z}_0) - H_z(\underline{z}_0)}$, in which $\underline{z}_0 = H_z^{-1}[1 - R^*(1 - G_s^-(s_0))]$, and $\overline{z}_0 = H_z^{-1}[1 - R^*(1 - G_s(s_0))]$. (ii) If $G_s(s_0) \geq 1 - \frac{1}{R^*}$ but $G_s^-(s_0) < 1 - \frac{1}{R^*}$, then for any man with s_0, with probability $p_0 = \frac{1 - R^*[1 - G_s(s_0)]}{R^* prob\{s_{m,1} = s_0\}}$ he matches with a woman, and with probability $1 - p_0$ he is not matched. Conditional on matching, the probability of marrying a woman with match quality z is $\frac{h_z(z)}{H_z(\overline{z}_0) - H_z(\underline{z})}$.

Appendix B. Proof of Lemma 4

Part of the proof here is adapted from similar proofs in Hopkins and Kornienko (2004).

If s^* is the optimal choice for a man with $x_{m,1}$, then $s^* \geq s^{\mathbf{c}}(x_{m,1})$ the cooperative saving level, or otherwise it is strictly dominated by $s^{\mathbf{c}}(x_{m,1})$ due to consumption smoothing consideration. If the relation holds with equality, then naturally s^* is increasing (maybe weakly) in $x_{m,1}$ since $s^{\mathbf{c}}(x_{m,1}) = \max[\frac{1}{2+r}(x_{m,1} - wn), 0]$. Otherwise if $s^* > s^{\mathbf{c}}(x_{m,1})$, then for the lifetime utility $v(x_{m,1}, s^*) = \ln(x_{m,1} + wn - s^*) + \beta \ln[\kappa((1 + r)s^* + 2wn)] + \beta z(s^*)$, $\frac{\partial v^2}{\partial x_{m,1} \partial s^*} = \frac{1}{(x_{m,1} - s^*)^2} > 0$, which implies an increase in wealth leads to an increase in the marginal return on saving, and thus the optimal choice of saving $s(x_{m,1})$ increases.

Next, I show $s(x_{m,1})$ is strictly increasing. Suppose $s(x_{m,1})$ is not strictly increasing in an interval $[x_1, , x_2]$ such that $s(x) = s_0, \forall x \in [x_1, x_2]$. Then $G_s(s_0) > G_s^-(s_0)$, i.e., there's a mass point in the distribution of $G_s(s)$. But in this interval, if any man increases his saving by a small amount ε, he gains a discrete increase in his ranking by the matching rule in equation (5) with marginal loss of utility from consumption. The existence of a profitable deviation

[22] $G_s^-(s_0)$ is the left limit of $G_s(s_0)$ at s_0: $G_s^-(s_0) = \lim_{s \to s_0^-} G_s(s_0)$

rules out mass points in distribution $G_s(s)$. As a result, the optimal saving function $s(x)$ must be strictly increasing.

By a similar argument, $s(x_{m,1})$ must be continuous $\forall x \geq x^*_{m,1}$. Otherwise, there is a jump at wealth level x_0 such that $\lim_{x\to x_0^-} s(x) = \widetilde{s} < s(x_0)$. According to equation (5), all men choosing saving $s \in [s(x), \widetilde{s}]$ have the same ranking. As a result, if a man with wealth x_0 decreases his saving to $\widetilde{s} - \varepsilon$, he gains direct utility from consumption without affecting his ranking; thus reducing saving would be a profitable deviation making a discontinuous saving function not sustainable in equilibrium.

The last step is to show that the saving function $s(x)$ is differentiable. Consider two wealth levels x_1 and x_2 such that $x_1, x_2 \geq x^*_{m,1}$ and $x_2 = x_1 + \Delta$. Denote their saving choice as $s(x_1)$ and $s(x_2)$ respectively. By optimality, we have

$$\begin{aligned}
& \ln[x_1 + wn - s(x_1)] + \beta\ln[\kappa((1+r)s(x_1) + 2wn)] + H_z^{-1}[1 - R^*(1 - H_{m,1}(x_1))] \\
\geq\ & \ln[x_1 + wn - s(x_2)] + \beta\ln[\kappa((1+r)s(x_2) + 2wn)] + H_z^{-1}[1 - R^*(1 - H_{m,1}(x_2))]
\end{aligned}$$

and

$$\begin{aligned}
& \ln[x_2 + wn - s(x_2)] + \beta\ln[\kappa((1+r)s(x_2) + 2wn)] + H_z^{-1}[1 - R^*(1 - H_{m,1}(x_2))] \\
\geq\ & \ln[x_2 + wn - s(x_1)] + \beta\ln[\kappa((1+r)s(x_1) + 2wn)] + H_z^{-1}[1 - R^*(1 - H_{m,1}(x_1))]
\end{aligned}$$

By the mean value theorem, $\exists\ s_1, s_2 \in [s(x_1), s(x_2)]$ such that

$$\begin{aligned}
& H_z^{-1}[1 - R^*(1 - H_{m,1}(x_1))] - H_z^{-1}[1 - R^*(1 - H_{m,1}(x_2))] \\
\geq\ & (\frac{1}{(1+r)s_1 + 2wn} - \frac{1}{x_1 + wn - s_1})[s(x_2) - s(x_1)]
\end{aligned}$$

and

$$\begin{aligned}
& H_z^{-1}[1 - R^*(1 - H_{m,1}(x_2))] - H_z^{-1}[1 - R^*(1 - H_{m,1}(x_1))] \\
\geq\ & (\frac{1}{(1+r)s_2 + 2wn} - \frac{1}{x_2 + wn - s_2})[s(x_1) - s(x_2)]
\end{aligned}$$

After rearranging the terms and dividing each term by Δ, we have

$$\begin{aligned}
& \frac{H_z^{-1}[1-R^*(1-H_{m,1}(x_2))]-H_z^{-1}[1-R^*(1-H_{m,1}(x_1))]}{\Delta(\frac{1}{x_1+wn-s_1}-\frac{1}{(1+r)s_1+2wn})} \\
\geq & \frac{s(x_2)-s(x_1)}{\Delta} \\
\geq & \frac{H_z^{-1}[1-R^*(1-H_{m,1}(x_2))]-H_z^{-1}[1-R^*(1-H_{m,1}(x_1))]}{\Delta(\frac{1}{x_2+wn-s_2}-\frac{1}{(1+r)s_2+2wn})}
\end{aligned}$$

As Δ goes to 0, the upper bound and lower bound for $\frac{s(x_2)-s(x_1)}{\Delta}$ converge to the same expression, and we get the derivative of $s(x)$ which converges to equation (10).

Appendix C. Proof of Proposition 1

In Lemma 4, we've showed that the equilibrium saving function must be strictly increasing and thus men with wealth endowment $x_{m,1} < x^*_{m,1}$ cannot be matched, and their saving function is described by equation (7). For men with wealth $x_{m,1} \geq x^*_{m,1}$, we need to check the differential equation (10) resulting from first-order condition 9, and matching rule 6 actually represents optimal choice of any single man given other people also use the same saving strategy. Following Hopkins and Kornienko (2004), it is sufficient to show that the utility function from equation (8) $U(x_{m,1}, s)$ exhibits pseudo-concavity such that it is decreasing in s for $s > s(x_{m,1})$ and increasing in s for $s < s(x_{m,1})$. Suppose a man with endowment $x > x^*_{m,1}$ chooses saving $s_0 < s(x)$ instead of $s(x)$, then there might be two possible cases.

In the first case $s_0 < s(x^*_{m,1}) = s^*$ such that this man will not be matched. From the optimal saving rule of single men from equation (7), we have $s_0 = \frac{\beta}{1+\beta}x$, and his corresponding life-time utility is $V_0 = (1+\beta)\log(\frac{x}{1+\beta} + wn)$. Notice that we should restrict $x < \frac{1+\beta}{\beta}s^*$ otherwise this man gets matched naturally. But such a choice is strictly dominated by choosing s^* and matching with $\underline{z}$. If he chooses s^*, his life-time utility is

$V_1 = \ln(x + wn - s^*) + \beta \ln((1 + r)s^* + 2wn) + \beta(\kappa + \underline{z})$. Then we have

$$
\begin{aligned}
&V_1 - V_0 \\
=& \ln(x + wn - s^*) + \beta \ln((1 + r)s^* + 2wn) + \beta(\kappa + \underline{z}) \\
&- (1 + \beta) \log(\frac{x}{1 + \beta} + wn) \\
=& \ln(x + wn - s^*) + (1 + \beta) \log(\frac{x^*_{m,1}}{1 + \beta} + wn) - \ln(x^*_{m,1} + wn - s^*) - (1 + \beta) \log(\frac{x}{1 + \beta} + wn) \\
=& [\ln(x + wn - s^*) - (1 + \beta) \log(\frac{x}{1 + \beta} + wn)] - [\ln(x^*_{m,1} + wn - s^*) - (1 + \beta) \log(\frac{x^*_{m,1}}{1 + \beta} + wn)] \\
>& 0
\end{aligned}
$$

In the calculation above, the second equality comes from the boundary condition stated in Proposition 1, and the last inequality is base on

$$
\begin{aligned}
&\frac{\partial}{\partial x}[\ln(x + wn - s^*) - (1 + \beta) \log(\frac{x}{1 + \beta} + wn)] \\
=& \frac{s^* - \frac{\beta}{1+\beta}x}{(x + wn - s^*)(\frac{x}{1+\beta} + wn)} > 0
\end{aligned}
$$

In the second possible case, $s^* \leq s_0 < s(x)$. Then we can find another $x_0 < x$ such that $s(x_0) = s_0$. From equation (8), conditional on other men adopting the saving function strategy and $\frac{\partial U}{\partial s} = 0$, we have $\frac{\partial U^2}{\partial x \partial s} > 0 \ \forall x > x^*_{m,1}$. Hence $\frac{\partial}{\partial s} U(x, s_0) > \frac{\partial}{\partial s} U(x_0, s_0) = 0$. As a result, s_0 cannot be optimal saving for men with wealth x. Thus $U(x_{m,1}, s)$ is increasing for $s < s(x_{m,1})$. The proof for $U(x_{m,1}, s)$ decreasing for $s > s(x_{m,1})$ is similar.

We are left of showing that differential equation (10) uniquely pins down the equilibrium saving function. This just follows from the fundamental theorem of differential equation with the initial condition given in Proposition 1. With the balanced sex ratio $R^* = 1$, essentially every man gets matched. Men with the lowest wealth endowment $\underline{x}$ is determined to match with $\underline{z}$. As a result, this man will not attempt to change his position, and will simply adapt the cooperative saving based on equation (12). Otherwise, if $R^* > 1$, only men with endowment above $x^*_{m,1}$ get matched, and the saving rate of men with $x^*_{m,1}$ is raised to a level such that they are indifferent between getting matched with $\underline{z}$ and remaining single. Thus, there is exactly one solution satisfying boundary conditions mentioned in Proposition

1, and that constitutes the equilibrium saving function.

Appendix D. Proof of Proposition 4

A representative woman doesn't save when she is single for the following reasons: Since her income is wn at all ages and $\beta(1+r)=1$, she would choose 0 saving even if there were no marriage market due to the consumption smoothing consideration. Now with a marriage market, her motivation to save is further reduced since she can take advantage of her future husband's pre-marital saving. Besides, there is no strategic saving motive since her saving is not observable in marriage market.

Given that women's saving is 0 in equilibrium, for a young man with saving s marrying a young woman with z, based on equation (15), his utility after marriage is

$$u_{m,22} = (1+\beta)\ln[\kappa(\frac{s}{\beta+\beta^2}+2wn)] + (1+\beta)z$$

and from equation (18) if he marries a middle-aged woman with same z, he gets

$$u_{m,23} = \ln[\kappa(\frac{s}{\beta}+2wn)] + \beta\ln(wn) + z$$

Under assumption 1, we can see that $u_{m,22} > u_{m,23}$, and thus young men prefer marrying young women. Similarly, if we compare equation (14) with equation (16), we find middle-aged men consider women of different ages as perfect substitutes. As a result, for all realized z, a woman has no gain and may lose her ranking in the marriage market if she postpones marriage.

On the other hand, for a woman marrying at age 1, based on equations (15) and (17), her expected lifetime utilities if she marries a young man or a middle-aged man are respectively

$$\begin{aligned} u_{w,22} &= \ln(wn) + (\beta+\beta^2)\int[\ln(\frac{s_{11}(z)}{\beta+\beta^2}+2wn) + \ln\kappa + z]dH_z(z) \\ u_{w,32} &= (1+\beta^2)\ln(wn) + \beta\int[\ln(\frac{s_{21}(z)}{\beta}+2wn) + \ln\kappa + (1+\beta)z]dH_z(z) \end{aligned}$$

in which $s_{11}(z)$ and $s_{21}(z)$ denote the savings of young and middle-aged men she matches with if her realized match quality is z. For a woman marrying at age 2, based on equations (14) and (19), her expected lifetime utilities if she marries a young man or a middle-aged man are respectively

$$\begin{aligned} u_{w,23} &= (1+\beta)\ln(wn) + \beta^2 \int [\ln(\frac{s_{12}(z)}{\beta} + 2wn) + \ln\kappa + z] dH_z(z) \\ u_{w,33} &= (1+\beta)\ln(wn) + \beta^2 \int [\ln(\frac{s_{22}(z)}{\beta} + 2wn) + \ln\kappa + z] dH_z(z) \end{aligned}$$

in which $s_{12}(z)$ and $s_{22}(z)$ denote the savings of young and middle-aged men she matches with if her realized match quality is z. By the argument in the first paragraph of this proof, we know $s_{11}(z) > s_{12}(z)$, and $s_{21}(z) = s_{22}(z)$. Consequently,

$$\begin{aligned} & u_{w,22} - u_{w,23} \\ = \; & (\beta+\beta^2) \int [\ln(\frac{s_{11}(z)}{\beta+\beta^2} + 2wn) + \ln\kappa + z] dH_z(z) \\ & -\beta\ln(wn) - \beta^2 \int [\ln(\frac{s_{12}(z)}{\beta} + 2wn) + \ln\kappa + z] dH_z(z) \\ > \; & (\beta+\beta^2) \int [\ln(\frac{s_{11}(z)}{\beta+\beta^2} + 2wn) + \ln\kappa + z] dH_z(z) \\ & -\beta\ln(wn) - \beta^2 \int [\ln(\frac{s_{11}(z)}{\beta} + 2wn) + \ln\kappa + z] dH_z(z) \end{aligned}$$

$$\begin{aligned} = \; & \int [(\beta+\beta^2)\ln(\frac{s_{11}(z)}{\beta+\beta^2} + 2wn) - \beta\ln(wn) - \beta^2\ln(\frac{s_{11}(z)}{\beta} + 2wn)] dH_z(z) \\ & +\beta(z + \ln\kappa) \\ > \; & (\beta+\beta^2) \int [\ln(\frac{s_{11}(z)}{\beta+\beta^2} + 2wn) - \ln(\frac{\beta s_{11}(z) - \beta wn}{\beta+\beta^2} + 2wn)] dH_z(z) \qquad (33) \\ & +\beta(z + \ln\kappa) \\ > \; & 0 \end{aligned}$$

Here step (33) relaxes the borrowing constraint for the woman and allows her to smooth consumption between age 2 and 3. Similarly,

$$
\begin{aligned}
& u_{w,32} - u_{w,33} \\
= \; & (1+\beta^2)\ln(wn) + \beta \int [\ln(\frac{s_{21}(z)}{\beta} + 2wn) + \ln\kappa + (1+\beta)z] dH_z(z) \\
& -(1+\beta)\ln(wn) - \beta^2 \int [\ln(\frac{s_{22}(z)}{\beta} + 2wn) + \ln\kappa + z] dH_z(z) \\
= \; & (\beta - \beta^2) \int [\ln(\frac{s_{21}(z)}{\beta} + 2wn) - \ln(wn) + \ln\kappa + z] dH_z(z) \\
& +\beta z \\
> \; & 0
\end{aligned}
$$

As a result, a woman will lose ranking in the marriage market and will suffer a utility loss is she postpones marriage. That is, rational women choose to marry at age 1.

Appendix E. Algorithm for Quantitative Model

I first describe how to solve for the long-run stationary equilibria under a constant sex ratio R^*. The key step is to solve measures of singles and distributions of saving in the marriage market.

1. Solve the optimization problems of married households and single agents in post-marital stages, equations (31) and (27), and derive their saving functions and value functions $s^{\mathbf{a}}_{j_m j_w}(x)$, $\widetilde{U}^{\mathbf{a}}_{j_m j_w,m}(x)$, $\widetilde{U}^{\mathbf{a}}_{j_m j_w,w}(x)$, $\forall j_m \geq J^{mar}_m + 1$, $\forall j_w \geq J^{mar}_w + 1$; $s^{\mathbf{s}}_{m,j_m}(x)$, $s^{\mathbf{s}}_{w,j_w}(x)$, $U^{\mathbf{s}}_{m,j_m}(x)$, $U^{\mathbf{s}}_{w,j_w}(x)$, $\forall j_m \geq J^{mar}_m + \Delta + 1$, $\forall j_w \geq J^{mar}_w + \Delta + 1$ as functions of current liquid wealth. In particular, $\widetilde{U}^{\mathbf{a}}_{j_m j_w,m}(x)$,$\widetilde{U}^{\mathbf{a}}_{j_m j_w,w}(x)$ are defined as part of the lifetime utility without the additive happiness terms such that:

$$
\begin{aligned}
U^{\mathbf{a}}_{j_m j_w,m}(x) &= \widetilde{U}^{\mathbf{a}}_{j_m j_w,m}(x) + (\textstyle\sum_{t=0}^{J-j_m} \beta^t) z \\
U^{\mathbf{a}}_{j_m j_w,w}(x) &= \widetilde{U}^{\mathbf{a}}_{j_m j_w,w}(x) + (\textstyle\sum_{t=0}^{J-j_w} \beta^t) z
\end{aligned}
$$

Since match quality terms are additive to utilities after marriage, it doesn't affect the married households' saving choice.

2. Conjecture measures of singles $\{M_{j_m}, W_{j_w}\}$ $\forall j_m \geq J_m^{mar}+1, \forall j_w \geq J_w^{mar}+1$ and stationary distributions of saving $G_{j_m}^s(\cdot)$, $\forall j_m \geq J_m^{mar}$. In particular, $W_{j_w}=0$ $\forall j_w \geq J_w^{mar}+1$. Consequently, men $\{s_m, j_m\}$ are ranked by expected utilities of women after marriage, $E_{n_{j_m+1}^m}\widetilde{U}_{j_m+1,J_w^{mar}+1,w}^{\mathbf{a}}[R(s_{m,j_m}^{\mathbf{s}}+s_{w,j_w}^{\mathbf{s}})+w(n_{j_m+1}^m+n_{j_m+1}^w)]$. Define quality of men as

$$q(s_m, j_m) = E_{n_{j_m+1}^m}\widetilde{U}_{j_m+1,J_w^{mar}+1,w}^{\mathbf{a}}[(1+r)(s_m+s_{w,j_w}^{\mathbf{s}})+w(n_{j_m+1}^m+n_{j_m+1}^w)] \qquad (34)$$
$$\forall j_m \in [J_m^{mar}, J_m^{mar}+\Delta]$$

and the distribution of q as

$$\begin{aligned} H_q(q_0) &= prob\{q \leq q_0\} \\ &= \sum_{j_m=J_m^{mar}}^{J_m^{mar}+\Delta} \frac{M_{j_m}}{M} G_{j_m}^s[s_m \leq q^{-1}(q_0, j_m)] \end{aligned}$$

3. Based on $H_q(q_0)$, solve the optimization problems of single agents at the marital stage and pre-marital stage, and derive their saving functions $s_{m,j_m}^{\mathbf{s}}(x)$, $s_{w,j_w}^{\mathbf{s}}(x)$ and marital timing choices. $\delta_{m,j_m}(s)$, $\delta_{w,j_w}(s)$.

4. Update the distributions $G_{j_m}^s(\cdot)$ and measures of singles $\{M_{j_m}, W_{j_w}\}$ in the marriage market by equilibrium condition equations (32). Go back to step 2 and iterate until convergence.

Here are the steps to solve for transitional dynamics after an unexpected permanent increase in the sex ratio R^* from 1 to 1.18.

1. Draw a long history of T periods. In period 1, the economy receives the unexpected population shock in which M_1 and W_1 change from 0.5 to $\frac{R^*}{1+R^*}$ and $\frac{1}{1+R^*}$ respectively. Assume in the last period, this economy reaches its new long-run stationary equilibrium with the unbalanced sex ratio. Conjecture measures of singles and distributions of saving in the marriage market at every date along this history: $\{M_{j_m,t}, W_{j_w,t}\}$ $\forall t$, $\forall j_m \geq J_m^{mar}+1$, $j_w \geq J_w^{mar}+1$; and $G_{j_m,t}^s(\cdot)$, $\forall t$, $\forall j_m \geq J_m^{mar}$.

2. Use the definition of $q(s_m, j_m)$ in equation (34). Generate distributions of q, $H_{q,t}(q)$ on each date along this transition path by

$$\begin{aligned} H_{q,t}(q_0) &= prob\{q_t \leq q_0\} \\ &= \sum_{j_m=J_m^{mar}}^{J_m^{mar}+\Delta} \frac{M_{j_m,t}}{M_t} G^s_{j_m,t}[s_m \leq q^{-1}(q_0, j_m)] \end{aligned}$$

3. Given $H_{q,t}(q)$ $\forall t$, solve the optimization problems of single agents at the marital stage and pre-marital stage, and derive their saving functions $s^{\mathbf{s}}_{m,j_m,t}(x)$, $s^{\mathbf{s}}_{w,j_w,t}(x)$ and marital timing choices. $\delta_{m,j_m,t}(s)$, $\delta_{w,j_w,t}(s)$ $\forall t$.

4. Update measures and distributions in the marriage market at each period t by the following equations.

$$\begin{aligned} G_{s,j_m,t}(s) &= prob\{s_{j_m,t}(x_{m,j_m,t}) \leq s\}, \forall s, j_m, t \\ G_{x,j_m,t}(x) &= prob\{((1+r)s_{m,j_{m-1},t-1} + wn^m_{j_m,t} \leq x) \wedge (\delta_{m,j_m-1,t-1}(s_{m,j_{m-1},t-1}) = 0)\}, \forall x, t, j_m \geq 2 \\ M_{j_m,t} &= M_{1,t-j_m+1}, \forall j_m < J_m^{mar} \\ M_{j_m,t} &= M_{j_m-1,t-1} \int [1 - \delta_{m,j_{m-1},t-1}(s)] dG_{s,j_m-1,t-1}(s),\ \forall j_m \geq J_m^{mar} \\ W_{j_w,t} &= W_{1,t-j_w+1}, \forall j_w < J_w^{mar};\ W_{j_w,t} = W_{j_w-1,t-1}(1 - \delta_{w,j_w-1,t-1}),\ \forall j_w \geq J_w^{mar} \end{aligned}$$

5. Go back to step 1. Iterate until convergence.