Piracy, Awareness and Welfare in a Required Aftermarket*

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Abstract

Many industries have two sales stages: the primary market and the aftermarket. Existing research shows consumers are routinely unaware of aftermarkets (Cruickshank, 2000; Hall, 2003); and due to legal or structural restrictions, firms commonly have monopoly power (Borenstein et al., 2000; Adelmann, 2010). However, the primary market could be a great deal more competitive. Examples of this sales process include products with service agreements, software with in-app purchases, and durable goods with required replacement parts. But in many of these aftermarkets, the consumer has the option to obtain the aftermarket product through non-traditional means (e.g. "piracy"). We model such an environment by combining the two most common travel cost models: A Salop circle (Salop, 1979) for the primary market and a Hotelling linear city (Hotelling, 1929) for the aftermarket. We find that firms with more competition in the primary market will spend more on "enforcement" (disincentivising non-traditional acquisitions) and reduce prices in the primary market so they may exhibit more market power in the aftermarket. This is in direct contradiction with the common belief that anti-piracy efforts are the domain of "big business" (Tan, 2002; Kwong et al., 2003; Lysonski and Durvasula, 2008). Further, we find that it is social welfare enhancing for "enforcement" spending to be as effective as possible.

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1 Introduction and Review of Literature

Are consumer durable goods manufacturers in danger of having their replacement part market destroyed by 3D printers? What is the expected behavior of software producers when the customer tries to perform their own maintenance or acquire add-ons by alternative means? In this paper, we examine a market where the customer is required to obtain a supplementary aftermarket good to continue use of the product. While the aftermarket product is produced only by the manufacturer, it can be acquired by non-traditional means (such as "piracy"). However, the consumer might not be aware of the aftermarket when buying the original product.

The firms choose both pricing and "enforcement" – an amount of money spent by the firm on punishing the customers who acquire the aftermarket good through alternative means. In the course of our examination, we will show that such an environment results in more enforcement and more market power (in the aftermarket) in industries that are more competitive in the primary market. This finding contradicts the common belief that anti-piracy efforts are the domain of "big business" (Tan, 2002; Kwong et al., 2003; Lysonski and Durvasula, 2008). Further, awareness of the aftermarket results in less non-traditional acquisitions – but despite selling more units, this is profit decreasing for the firm.

In many industries there are required "add-ons" or parts that the consumer encounters after they have purchased a good. While similar in concept to a two-part tariff, these additional payments can be unknown to the buyer and aren't always related to intensity of use. An example would be a proprietary replacement part that breaks with time and not use (and can be "pirated" with a 3D printer) and numerous software examples.

Consider enterprise software. Firms sell a primary product at an un-shrouded price, but there are also maintenance packages. The primary product producer is generally the only one legally allowed and/or capable of providing the maintenance package and is able to charge relatively high prices. Producers might also make the use of the primary product, without the maintenance package, as difficult as possible if new potential customers are unaware of the aftermarket. Perhaps the consumer is fully aware of these costs in some cases; this paper explores a spectrum of awareness. However, consumers have some alternative means to acquire the aftermarket good (e.g. "piracy") and the firm can take fixed cost actions (enforcement) to prevent non-traditional acquisition of the aftermarket good.

Enterprise software is not the only example in the software space. Recently, a trend has emerged in "app-store" environments to shroud part of the product's price. Made possible by the "in-app" purchase mechanism in many app stores, the producer creates a product that is functional for some period of time, but then effectively requires an add-on product. This assumes users have invested some amount of time such that switching to another product is non-trivial[1]. Just as quickly as this behavior appeared, attempts by consumers to find a way to pirate in-app purchases found their way onto the web (Protalinski, 2012).

There is a long history of businesses profiting from uninformed consumers. About half of consumers in the U.K. don't know what their bank fees are (Cruickshank, 2000) and only 3% of inkjet owners know the ink cost at time of purchase (Hall, 2003). Gabaix and Laibson (2006) show that this might be the result of a shrouding game where the producer attempts to hide information from the consumers.

Existing research in the piracy space has focused on counterfeit goods (Chaudhry and Walsh, 1996; Jacobs et al., 2001) or piracy in a primary market (Banerjee et al., 2005; De Castro et al., 2008; Husted, 2000; Hui and Png, 2003; Jaisingh, 2009), but, to the knowledge of the author, no paper has considered piracy in an aftermarket.

Aftermarkets are common in the durable goods literature. For instance, Mann (1992) showed that a monopolist might manipulate consumer maintenance decisions to affect the used good market and Kinokuni (1999) showed that producers might raise prices in the replacement part market inefficiently high to encourage new sales. In some ways, Carlton and Waldman (2010), who considered a monopolized aftermarket and concludes that the monopolization of the aftermarket is sometimes efficiency enhancing, is the most similar to this work: they come to similar counterintuitive results, but for differing reasons.

[1]It is not the intent of the author to suggest that all in-app products fall into this category, many are simply targeting differentiated preferences

In the next section, we use game theory to develop a theoretical model for "piracy" in an aftermarket. The product in this aftermarket is obligatory, but can be "pirated[2]."

2 The Aftermarket Piracy Game

2.1 Introduction to the Game

Our model describes a world where a set of firms produce a good in a primary, and monopolistically competitive, market. This "traps" consumers into purchasing some future good from the primary product producer or to "pirate" it. Consumers may or may not be aware of the aftermarket when purchasing the primary good. However, we will assume that utility from the good (v) is sufficiently high that the consumers always chooses to acquire the aftermarket good, and switching cost are high enough that the aftermarket does not result in a purchase of a new primary good. We also consider the impact on social welfare as firms enter the market until profit is zero.

Definition 1. *"Piracy" is any action taken by the consumer that results in acquiring the aftermarket good without paying the producer in the aftermarket for the good. It may or may not be illegal depending on the context.*

Clearly our definition of piracy is broader than the legal definition. Traditional examples, such as pirating software add-ons and printing copyrighted/patented replacement parts (with a 3D printer) still fall under this definition. However, many compulsory maintenance packages (such as the enterprise software example in the introduction), where the consumer services their own capital purchase, would fall under this definition as well.

General Assumptions of the Model

We make the following assumptions:

1. All consumers in the model obtain sufficient utility from the primary market product that they all buy a single unit from one of the firms.

[2]It is of note, that the model in this paper, does not intend to model intensity of use. The additional payments in our model are assumed to be unrelated to use.

2. All consumers either purchase or pirate the aftermarket good.

3. Consumers are uniformly distributed around a unit-length Salop circle (Salop, 1979). Location on the circle represents product characteristics such that the ideal product (for that consumer) would exist at their current location. Travel cost (t), therefore, represents the cost multiplier of deviating from that ideal product.

4. Consumers are uniformly distributed *in ability to pirate* across a unit-length Hotelling (Hotelling, 1929) linear city. The travel cost (s) in the Hotelling city represents the cost multiplier to pirate the good.

5. The pirated good and manufactured aftermarket good are identical. Some results are slightly different when there is a quality difference between the goods. Those results are available in appendix C.

Stages of the Game

The game has the following stages:

1. In the long run, firms choose to enter into the market until there is zero profit for each firm (optional stage)[3]

2. Firms choose a pricing policy for both markets and the amount of money to spend on "enforcement"

3. Consumers choose a product based on a unit-length Salop circle (Salop, 1979) and expected/perceived disutility from the next stage

4. Finally, consumers choose whether or not to pirate the aftermarket good in a unit-length Hotelling linear city (Hotelling, 1929). This stage can repeat G times.

[3]We describe this as an optional stage because some industries have other barriers to entry besides a constant fixed cost, such as political or legal barriers. Therefore, it is useful to consider the outcome both when firms are fixed and firms expand with profit.

Using backwards induction we solve for pricing, "enforcement", social welfare and number of firms.[4]

Definition 2. *"Enforcement" is any fixed cost action by the firm that results in disutility for the pirating consumer.*

Examples of enforcement might be more complex parts (to prevent 3D printing), creating obfuscation to prevent do-it-yourself maintenance, suing consumers for pirating the product – not for the financial benefit from the individual sued but for the "shock value" to the market – and digital rights management.

2.2 Calculating the Indifference Point of Awareness

Assumption 1. *The firm has no influence on the awareness of the consumers*

In motivating this assumption, I will setup a very simple illustrative model. In principal there is a preliminary stage that occurs for the consumer at some early point in time. Consumers are uniformly distributed across a linear city representative of the cost of being informed about some product. This models the idea that it is much easier for a car mechanic (for instance) to acquire information about a future part failure than most other people. Similarly, it is "cheaper" for a software engineer to determine if a piece of enterprise software will require "maintenance packages." For many products, there are similar examples. Therefore, there is continuum of costs associated with the previous experiences (learning) and knowledge of the customers (expertise). Given this continuum of costs, we can say that the indifferent consumer is located at the position such that *perceived* future savings or benefit of acquiring the information (which may or may not be accurate) equals the cost of acquiring that information.

$$\begin{gathered}\hat{S}(\bar{S},\varepsilon) = C_I A_W \\ A_W^* = \frac{\hat{S}(\bar{S},\varepsilon)}{C_I}\end{gathered} \tag{1}$$

A_W is the customer's position, between 0 and 1, indicating how easy it is for that consumer to acquire the information. C_I is the information cost multiplier and $\hat{S}(\bar{S},\varepsilon)$ is the *perceived* future

[4]Work for this paper is available as an online appendix at http://bensresearch.com/piracy/

savings or benefit of acquiring the information which is a function of actual future savings ($\bar{S}$) and some unknown factor (ε). Therefore, A_W^* (hereinafter A_W) is the proportion of consumers who decide to acquire the information about a product. We will call this "awareness", though both learning and expertise are involved.

In our model, actual future savings ($\bar{S}$) for any individual consumer are zero, because there are n identical firms and no individual consumer will change the firm solution. So, whatever the value of $\hat{S}(\bar{S}, \varepsilon)$, it is determined by the unknown factor (ε). However, this result is driven by the assumptions of the model. When there is non-identical firms, there maybe savings to be had. Further, ε could be influenced by psychological and/or marketing factors such as advertising.

Given that we know that unaware consumers are common (Cruickshank, 2000; Hall, 2003), the focus of this paper's model is industries' *response* to a given level of "awareness." We will therefore assume that the firm has no influence on A_W and it is exogenous to the model.

Assumption 2. *An infinite number of consumers are uniformly, and independently, distributed around the primary market circle, aftermarket linear city and awareness linear city*

Assumption 2 states that the consumers on any portion of the Salop circle (Salop, 1979), are expected to be no more or less informed. Further, their expected ability to pirate the good is identical. Additionally, a particular consumer's awareness has no influence on their ability to pirate. This assumption is necessary for the model to be reasonably solvable, however it may not always be reasonable.

Consider a software product where number of "features" are distributed around the Salop circle. In this case, it is probably unlikely that the ability to pirate is identical regardless of preference for number of features (which is likely why we see differing responses to piracy based on the sophistication of software). Nonetheless, this model is still likely representative of many situations in the market.

2.3 Calculating the Indifference Point of the Aftermarket

Once the consumer has chosen a primary product, they must decide between the aftermarket good provided by the producer, a monopolist in the aftermarket, and pirating the good. Consumers have

Table 1: List of Model Variables and Definitions

Variable	Meaning
v	Initial utility of obtaining the primary market product
P_1	Price in the primary market
A_W	Representative awareness of the aftermarket when buying the primary good
t	Travel cost in the primary market
G	Number of times the aftermarket repeats
C_1	Cost of the good in the primary market
n	Number of firms
x_1	Proportion of the Salop circle captured between the firm and its neighbor
P_2	Price in the aftermarket
s	Piracy travel cost
E	Enforcement chosen by the firm
r	Effectiveness of E
C_2	Cost of the good in the aftermarket
x_2	Proportion of the aftermarket product purchased from the firm
f	Fixed cost for the firm

varying abilities to pirate the good based on their location and incur a utility cost based on their position and the piracy cost multiplier (s).

The firm chooses both the price of the aftermarket good (P_2) and enforcement (E, r is effectiveness of enforcement). Piracy has a utility penalty to the pirating consumer, which is a function of the amount of money the company spends on enforcement. In equation 2, we set the utility cost of buying the aftermarket good equal to the utility cost of pirating the aftermarket good and solve for the indifferent consumer[5].

$$\begin{aligned} -P_2 &= -(1-x_2)s - rE \\ x_2 &\rightarrow \frac{Er - P_2 + s}{s} \end{aligned} \tag{2}$$

Therefore, x_2 describes the *proportion* of consumers who will buy the aftermarket good of the firm's primary market consumers. This condition is subject to $0 \leq x_2 \leq 1$.

Using this information we can also describe $\bar{U}_C$: the expected utility cost of one occurrence of the aftermarket for one consumer. Because of assumption 2 we can use expected utility as any section of the Salop circle produces an identical spectrum of consumers with regard to their ability to pirate the good. This is calculated by integrating over the utility cost outcomes using the solution points.

[5]This assumes that the pirated good and the good from the manufacturer are identical. There is only minor differences in the model outcomes when there is a quality difference between the pirated and manufactured good, this is explored in appendix C.

$$
\begin{aligned}
\bar{U}_C &= \int_0^{\frac{Er-P_2+s}{s}} P_2 dx_2 + \int_{\frac{Er-P_2+s}{s}}^1 ((1-x_2)s + rE)dx_2 \\
&= \frac{P_2(Er-P_2+s)}{s} + \frac{(P_2-Er)(Er+P_2)}{2s}
\end{aligned}
\tag{3}
$$

2.4 Calculating the Indifference Point of the Primary Market

With the solution to the aftermarket known, we can solve for the primary market. Consumers are uniformly distributed around a single unit length Salop circle. In addition to prices and travel cost, the consumers also considering, at least partially, the future expected utility cost of the aftermarket because the use of the primary market good depends on acquiring the aftermarket good. In equation 4, we set the consumer utility of buying the primary market good from a given firm equal to the utility of buying the good from the neighboring firm (which is located at $\frac{1}{n}$ under the maximal differentiation principle), we then solve for the indifferent consumer.

$$
\begin{aligned}
v - P_1 - x_1 t - A_W G\bar{U}_C &= v - P_N - (\frac{1}{n} - x_1)t - A_W G\bar{U}_{NC} \\
x_1 &\rightarrow \frac{n(GA_W(\bar{U}_{NC} - \bar{U}_C) + P_N - P_1) + t}{2nt}
\end{aligned}
\tag{4}
$$

P_1 is the price, t is the travel cost, $\bar{U}_C$ is the expected utility cost from the aftermarket and G is how many times the aftermarket stage is expected to repeat. P_N and $\bar{U}_{NC}$ are the price and utility cost of the firm's neighbor on the Salop circle.

It is noteworthy that the solution to x_1 is identical if you model two Salop circle: one where the consumers are fully informed of the aftermarket (high type of proportion A_W) and one where they are completely uninformed (low type with proportion $1 - A_W$) and the firm maximizes over both markets ($x_1^* = A_W x_{1_H}^* + (1 - A_W)x_{1_L}^*$). Therefore it is acceptable to interpret A_W as either the proportion of consumers that are aware of the aftermarket *or* a representative level of awareness.

2.5 Producers' Problem

Profit for the firm comes in the following form where x_1^* and x_2^* are the solutions to equations 4 and 2:

$$\pi = 2x_1^*(P_1 - C_1) + G2x_1^*x_2^*(P_2 - C_2) - f - E \tag{5}$$

Like any Salop circle, the firm captures consumers from both its right and left side (therefore the "2"). The size of the aftermarket is some proportion of the primary market multiplied by the number of times that market repeats (G).

From equation 5 we can take first order conditions with respect to P_1, P_2 and E and then replace all "neighbor" variables (as all firms are identical). This results in:

$$\frac{C_1 - P_1}{t} + \frac{1}{n} = \frac{G(P_2 - C_2)(Er - P_2 + s)}{st} \tag{6}$$

$$\begin{aligned}&\frac{G(nA_W(Er - P_2 + s)(C_1 s + G(P_2 - C_2)(P_2 - Er) - s(-C_2 G + GP_2 + P_1))}{nst} \\ &+ \frac{st(C_2 + Er - 2P_2 + s))}{nst} = 0\end{aligned} \tag{7}$$

$$\begin{aligned}&\frac{1}{2}(-\frac{2Gr(nA_W(P_2 - Er)(s(-C_1 - C_2 G + GP_2 + P_1)}{ns^2 t} \\ &+ \frac{G(C_2 - P_2)(P_2 - Er)) + st(C_2 - P_2))}{ns^2 t} - 2) = 0\end{aligned} \tag{8}$$

With three equations and three unknowns, this results in the following solutions:

$$P_1 \to \underbrace{C_1 + \frac{t}{n}}_{S_m} + \frac{s(A_W - 1)(GA_W r + n)^2}{Gr^2} \tag{9}$$

$$P_2 \to \underbrace{C_2 + \frac{s}{r}}_{H_m} * \frac{(1 - A_W)(GA_W r + n)}{G} \tag{10}$$

$$E \to \underbrace{\frac{C_2}{r} - \frac{s}{r} + \frac{2s}{r^2}}_{H_m} * \frac{(1 - \frac{A_W}{2})(GA_W r + n)}{G} \tag{11}$$

Highlighted in the equations 9, 10 and 11 are solutions to related simpler models. S_m is the solution to the standard Salop circle model while H_m is the solution to a simple linear city model where there is a monopolist who competes with a pirated good and selects both price and enforcement[6] – solved for the readers' convenience in appendices A and B.

Proposition 1. *Piracy is decreasing in awareness and number of firms*

Proof. Using equations 10 and 11, we substitute the solutions for E and P_2, into the equation for x_2:

$$x_2 = A_W + \frac{n}{Gr} \tag{12}$$

Since pirating is $1 - x_2$, it is decreasing in awareness and number of firms.

$\square$

In the following discussion, we will see that this can be explained by higher rates of enforcement (E) and lower prices. This result is constrained by $A_W + \frac{n}{Gr} \leq 1$.

Taking a comparative static on enforcement spending results in:

$$\frac{\Delta E}{\Delta A_W} = -\frac{s(2G(A_W - 1)r + n)}{Gr^2} \tag{13}$$

Since all parameter values are assumed to be positive, examining the numerator implies that as long as $n \geq |2Gr(A_W - 1)|$ (or $n \geq 2Gr(1 - A_W)$) then equation 13 is negative. Using the constraint for an interior solution and substituting the solution points results in:

$$\begin{aligned} 0 \leq & x_2 \leq 1 \\ 0 \leq A_W + & \frac{n}{Gr} \leq 1 \end{aligned} \tag{14}$$

Combining the interior solution constraint and range of negative values constraint for equation 13 on n, we get the following:

$$2Gr(1 - A_W) \leq n \leq Gr(1 - A_W) \tag{15}$$

Which is impossible – which means that equation 13 is always positive. Enforcement increases with A_W until it reaches the corner solution. Initially, this may be a surprising result. However,

[6] If the aftermarket in this model was the only market

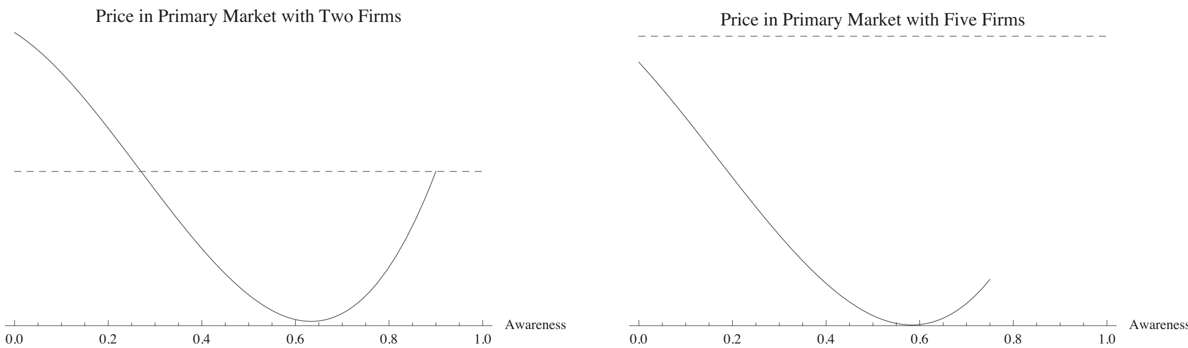

Figure 1: Price of primary good (P_1) with two and five firms in the industry. The dashed line is the marginal cost (C_1) of the primary good. The graphs stop when $x_2 = 1$

consider the behavior of the the other components – remembering that awareness is at the stage level:

$$\frac{\Delta P_2}{\Delta A_W} = s(\overbrace{1 - \frac{n}{Gr} - 2A_W}^{Sign\ Determination}) \tag{16}$$

P_2 is decreasing the majority of the awareness line unless enforcement in the aftermarket is very effective, and always decreasing when $A_W \geq \frac{1}{2}$ – see figure 2.

Consider the the comparative static of P_1 with respect to awareness:

$$\frac{\Delta P_1}{\Delta A_W} = \frac{s(GA_W r + n)\overbrace{(Gr(3A_W - 2) + n)}^{Sign\ Determination}}{Gr^2} \tag{17}$$

Which is positive in the range when $Gr(2 - 3A_W) \leq n \leq Gr(1 - A_W)$ and always positive when

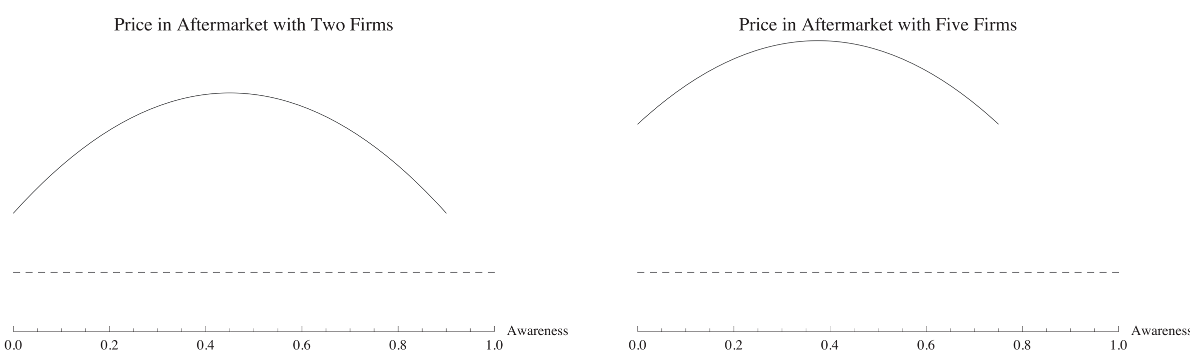

Figure 2: Price of aftermarket good (P_2) with two and five firms in the industry. The dashed line is the marginal cost (C_2) of the aftermarket good. The graphs stop when $x_2 = 1$

$A_W \geq \frac{2}{3}$. In essence, when the firm has little market power in the primary market, the firm will subsidize the aftermarket, where they can charge more (see figure 1) – Gabaix and Laibson (2006) saw similar results in a competitive market with add-ons (without piracy). This primary market subsidy is increasing in awareness for at least the first half of the awareness line, but is always decreasing by $\frac{2}{3}$. Looking at the comparative static $\frac{\Delta P_2}{\Delta A_W}$ (equation 16), this is explained by the price premium available to the firm in the aftermarket.

Proposition 2. *At least one of the prices decreases with higher values of awareness*

Proof. The sign of equation 17 ($\frac{\Delta P_1}{\Delta A_W}$) is positive if $G(3A_W - 2)r + n \geq 0$ and the sign of equation 16 ($\frac{\Delta P_2}{\Delta A_W}$) is positive if $1 - \frac{n}{Gr} - 2A_W \geq 0$. For both to be true, we rearrange, then combine the two constraints:

$$\begin{aligned}
&G(3A_W - 2)r + n \geq 0 \Rightarrow 3A_W - 2 + \frac{n}{Gr} \geq 0 \Rightarrow \frac{3}{2}A_W + \frac{n}{2Gr} \geq 1 \\
&1 - \frac{n}{Gr} - 2A_W \geq 0 \Rightarrow 1 \geq \frac{n}{Gr} + 2A_W \\
&\underbrace{\frac{3}{2}}_{\gamma_1} A_W + \underbrace{\frac{1}{2}}_{\beta_1} \frac{n}{Gr} \geq \underbrace{2}_{\gamma_2} A_W + \underbrace{1}_{\beta_2} * \frac{n}{Gr}
\end{aligned} \tag{18}$$

Comparing the coefficients of the component, γ_1 is smaller than γ_2. Further, β_1 is smaller than β_2, it is therefore impossible for $\frac{3}{2}A_W + \frac{1}{2}\frac{n}{Gr} \geq 2A_W + \frac{n}{Gr}$.

□

Proposition 3. *Firms with more competition in the primary market will exhibit more market power in the aftermarket and more heavily subsidize the primary market*

Proof. Start with the prices from the original solution:

$$\begin{aligned}
P_1 &\to C_1 + \overbrace{\frac{t}{n}}^{\lambda_1} + \underbrace{\frac{s\overbrace{(A_W - 1)}^{-}\overbrace{(GA_W r + n)^2}^{\lambda_3}}{Gr^2}}_{\lambda_2} \\
P_2 &\to C_2 + \underbrace{\frac{s}{r} * \frac{\overbrace{(1 - A_W)}^{+}\overbrace{(GA_W r + n)}^{\lambda_5}}{G}}_{\lambda_4}
\end{aligned} \tag{19}$$

Since t is positive, an increase in n reduces λ_1. A_W is bounded from zero to one, therefore $A_W - 1$ is negative. Because all parameter values are positive, λ_2 is negative. λ_3 increases in scale with n. Therefore λ_2 decreases in n. Because λ_1 and λ_2 are decreasing in n, P_1 is decreasing in n.

A_W is bounded between zero and one, therefore $1 - A_W$ is always positive. Since all parameter values are positive λ_4 is positive. Increases in n increase the scale of λ_5, λ_5 increases the scale of λ_4. Therefore, P_2 is increasing in n.

□

Proposition 4. *Firms in more competitive industries will exhibit more enforcement (E) and effective enforcement (rE)*

Proof. Taking the comparative statics of enforcement (E) and effective enforcement (rE) with respect to to number of firms (n) results in:

$$\begin{aligned} \frac{\Delta E}{\Delta n} &= \frac{(2 - A_W)s}{Gr^2} \\ \frac{\Delta rE}{\Delta n} &= \frac{(2 - A_W)s}{Gr} \end{aligned} \tag{20}$$

Since A_W is bounded between zero and one, $2 - A_W$ is always positive. Therefore both equations are positive.

□

The intuition here is simple, increasing enforcement is more attractive for more competitive industries (in the primary market) compared to those who have primary market power because the primary market alternative isn't as profitable.

Result 1. *Firms in more competitive industries will transfer more of the purchasing process to the aftermarket through both pricing and enforcement*

From proposition 3 we've shown that firms in more competitive industries will exhibit more market power in the aftermarket and subsidize the primary market (charge less). They also sell a higher number of goods in the aftermarket (proposition 1), which they accomplish at higher prices with higher levels of enforcement (proposition 4).

2.6 Firm Profit

Substituting in the solutions from equations 9, 10 and 11 into the profit maximization equation (5) results in the following:

$$\pi = \underbrace{\frac{t}{n^2} - f}_{S_m} + (A_W - 1)^2 * \underbrace{\frac{s}{r} - \frac{C_2}{r} - \frac{s}{r^2}}_{H_m} * \frac{n(2 - A_W)}{G} \tag{21}$$

Much like the solutions to prices and enforcement, the profit function can be seen as a combination of simpler models. We see the firm receives the same profit from the Salop circle (S_m) plus some additional payment based on market power (n), how aware the consumer is (A_W) and the solution to the piracy model (H_m).

Proposition 5. *Firm and industry profit are lower at higher values of awareness, but the impact is larger for more competitive industries*

Proof. Taking the comparative static of profit (equation 21) as well as industry profit ($n\pi$) with respect to awareness (A_W) results in:

$$\begin{aligned} \frac{\Delta\pi}{\Delta A_W} &= \frac{s(2G(A_W - 1)r + n)}{Gr^2} \\ \frac{\Delta n\pi}{\Delta A_W} &= \frac{ns(2G(A_W - 1)r + n)}{Gr^2} \end{aligned} \tag{22}$$

We know for an interior solution $n \leq Gr(1 - A_W)$. However, for either comparative static to be positive $n \geq |2Gr(A_W - 1)|$ (or $n \geq 2Gr(1 - A_W)$). Clearly this would violate the interior solution. Therefore, the above comparative statics are negative for an interior solution.

Taking a further comparative static with respect to number of firms results in:

$$\frac{\Delta n\pi}{\Delta A_W \Delta n} = \frac{2s(G(A_W - 1)r + n)}{Gr^2} \tag{23}$$

Like the previous equations, for this to be positive $n \geq 2Gr(1 - A_W)$, which would violate the interior solution. Therefore, equation 23 is negative for an interior solution.

$\square$

Result 2. *At least one of the prices will decrease and enforcement will increase with higher values of awareness. There is less industry profit with more awareness, but industry profit will be impacted more in more competitive industries.*

As shown by proposition 2 and equation 13, at least one price will be lower while enforcement will be higher with higher values of awareness of the aftermarket. This results in less profit for the firm, but because more competitive industries transfer more of the sales process to the aftermarket, more competitive industries will be hurt more by higher awareness values (proposition 5).

2.7 Consumer Surplus and Social Welfare

When calculating consumer surplus, we define utility as actualized utility (i.e. $U_1 = v - P_1 - x_1 t - G\bar{U_C}$), not prospective utility. That is, we calculate the social welfare and consumer surplus after the game has finished. From this we can state that:

$$\begin{aligned} CS &= 2n \int_0^{\frac{1}{2n}} (v - P_1 - x_1 t - G\bar{U_C}) dx_1 \\ &= v - C_1 - C_2 G \\ &- \frac{2ns(G(A_W - 1)r + n)((2A_W - 3)(GA_W r + n) + Gr) + 5Gr^2 t}{4Gnr^2} \end{aligned} \tag{24}$$

$$\begin{aligned} SW &= CS + n\pi \\ &= \frac{1}{4}(-4C_1 - \frac{4C_2 Gnr(Gr + n) + 2ns(G^2(A_W - 1)^2(2A_W - 1)r^2}{Gnr^2} \\ &- \frac{2Gn(A_W - 1)^2 r + n^2 + Gr^2 t}{Gnr^2} - 4fn + 4v) \end{aligned} \tag{25}$$

Note, we do not consider the cost of being informed in the above calculations. Including information cost would simply deduct $\frac{\hat{S}^2}{2C_I}$ from consumer surplus, and therefore social welfare, as well as replace every A_W variable with $\frac{\hat{S}}{C_I}$. Exploring values of social welfare when considering information cost is available as part of the online appendix, but isn't necessary to support the conclusions of the paper.

Proposition 6. *Higher awareness values are not always welfare enhancing*

Proof. A comparative static of social welfare with respect to awareness results in:

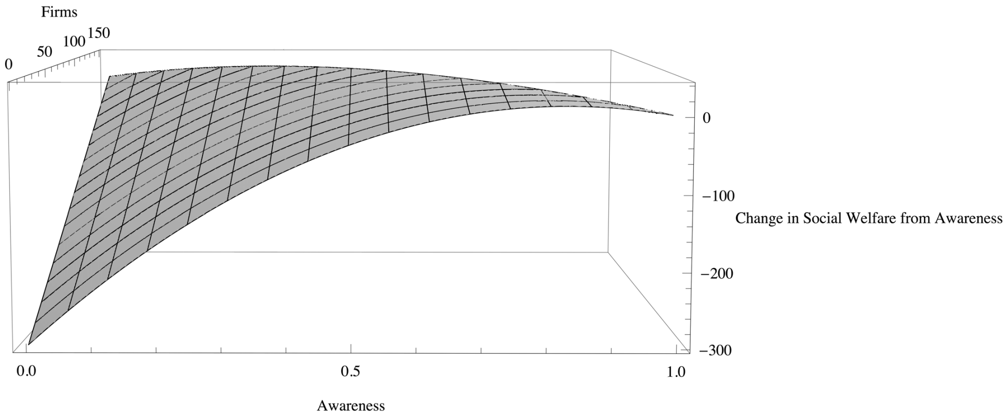

Figure 3: Social welfare change from increasing awareness as a function of firms and awareness. The graph stops when $x_2 = 1$

$$\frac{\Delta SW}{\Delta A_W} = \frac{(1 - A_W)s\overbrace{(G(3A_W - 2)r + 2n)}^{Sign\ Determination}}{r} \tag{26}$$

For equation 26 to be positive $A_W \geq \frac{2}{3}$ or $|G(3A_W - 2)r| \leq 2n$ which can be rearranged as $G\left(1 - \frac{3}{2}A_W\right) r \leq n$. However, when both of these statements are false, higher awareness values are social welfare decreasing (figure 3).

□

Remember, enforcement (E) – which is in no part a transfer – increases with awareness (A_W), but the effect isn't as dramatic with many firms (n).

$$\frac{\Delta E}{\Delta A_W \Delta n} = -\frac{s}{Gr^2} \tag{27}$$

Further, a smaller number of customers experience that enforcement (they buy the good instead – proposition 1) with higher awareness values – due to both higher enforcement and, in the higher half of the awareness line, decreases in price.

Proposition 7. *Higher piracy travel costs are not necessarily welfare decreasing*

Proof. Examine the comparative static of social welfare (with respect to piracy travel cost):

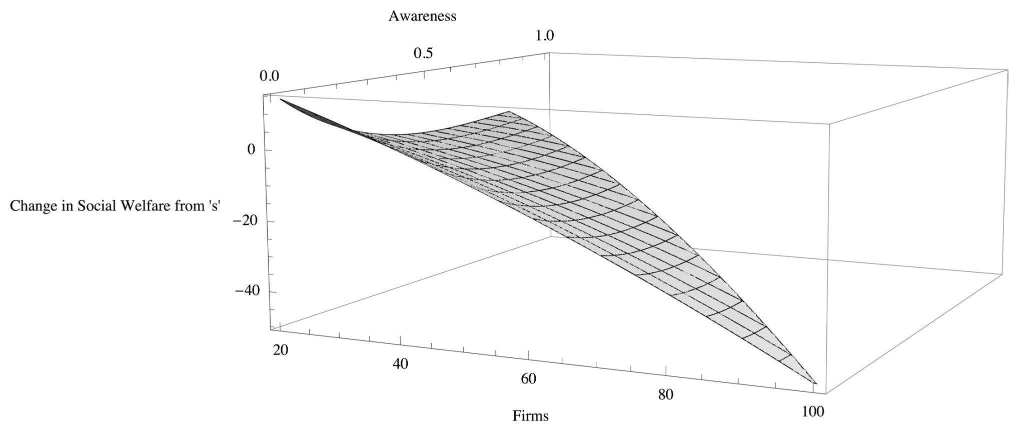

Figure 4: Social welfare change from increasing piracy travel cost as a function of firms and awareness. The graph stops when $x_2 = 1$

$$\frac{\Delta SW}{\Delta s} = -\frac{\overbrace{G^2(A_W-1)^2(2A_W-1)r^2}^{\eta_1} + \overbrace{2Gn(A_W-1)^2r+n^2}^{\eta_2}}{2Gr^2} \tag{28}$$

When A_W is less than $\frac{1}{2}$, η_1 is negative. If the absolute value of η_1 is greater than η_2 and η_1 is negative, then $\frac{\Delta SW}{\Delta s}$ is positive. Formally, the range for the positive comparative static can be expressed as:

$$\begin{gathered} G^2(A_W-1)^2(1-2A_W)r^2 \geq 2Gn(A_W-1)^2r+n^2 \\ G^2(1-2A_W)r^2 \geq 2Gnr + \frac{n^2}{(A_W-1)^2} \end{gathered} \tag{29}$$

When equation 29 is false (or A_W is more than $\frac{1}{2}$), then any increase in piracy travel cost (s) results in a decrease in social welfare.

□

It is of note that an enforcement decreasing solution in piracy travel cost (s) is easier in the early portion of the awareness/firm line:

$$\frac{\Delta E}{\Delta s} = -\frac{G(A_W-1)^2r+n(A_W-2)}{Gr^2} \tag{30}$$

But with larger industries that doesn't necessary hold as $\frac{\Delta E}{\Delta s}$ is decreasing when $n \leq \frac{G(A_W-1)^2r}{2-A_W}$ (rearranging the numberator of equation 30). Additionally, any increase in s decreases the consumer surplus for any customer who chooses to pirate the good. So, it is a trade off between these forces that result in an ambiguous result. We explore this trade off visually in figure 4.

Proposition 8. *With a fixed number of firms, higher enforcement effectiveness values are social welfare improving*

Proof. Taking the comparative static of social welfare with respect to enforcement effectiveness results in:

$$\frac{\Delta SW}{\Delta r} = \frac{n\left(Gr\left(C_2 + (A_W - 1)^2 s\right) + ns\right)}{Gr^3} \tag{31}$$

Since all parameter values are positive, $\frac{\Delta SW}{\Delta r}$ is always positive.

□

This is largely the same story as many of the conclusions above. Because E is in no part a transfer, reductions in E are often welfare enhancing.

$$\frac{\Delta E}{\Delta r} = \frac{\overbrace{Gr\left(\overbrace{(A_W - 1)^2 s - C_2}^{\vartheta_2}\right) + \overbrace{2n(A_W - 2)s}^{\vartheta_3}}^{\vartheta_1}}{Gr^3} \tag{32}$$

Since ϑ_3 is always negative, for equation 32 to be positive, ϑ_2 must be positive. For ϑ_2 to be positive, $s > \frac{C_2}{(A_W - 1)^2}$ – from a simple rearrangement of the section at issue. However, as that is a necessary, but not sufficient condition, a rearrangement of ϑ_1 to $G > \frac{2n(A_W - 2)s}{r(C_2 - (A_W - 1)^2 s)}$ must also hold for a positive increase.

In essence, increases in enforcement (E) with increasing enforcement effectiveness (r) only occur with relatively high piracy travel costs and relatively low numbers of firms. But from equation 30 and proposition 4, we know that is when enforcement is extremely low. Otherwise, increases in effectiveness result in less enforcement. It is therefore not surprising that increases in effectiveness are social welfare enhancing.

Result 3. *As shown by propositions 6, 7 and 8, social welfare can increase from additional utility costs to consumer which often coincides with less enforcement spending*

Since enforcement is in no part a transfer and is effectively spending that reduces another agent's welfare in hopes of changing their behavior, changes in the environment (changes in parameter values) that result in lower enforcement spending often coincide with higher social welfare.

2.8 Entry

In the long run, firms will continue to enter the market until firm profit is zero. Using the profit equation, we set the profit to zero followed by solving for n algebraically.

$$\begin{aligned}\pi &= \frac{t}{n^2} - f + (A_W - 1)^2 * \frac{s}{r} - \frac{C_2}{r} - \frac{s}{r^2} * \frac{n(2-A_W)}{G} \\ 0 &= \frac{t}{n^2} - f + (A_W - 1)^2 * \frac{s}{r} - \frac{C_2}{r} - \frac{s}{r^2} * \frac{n(2-A_W)}{G}\end{aligned} \tag{33}$$

Unfortunately, the solution to n is nearly a page long, so can't be interpreted, though it is included in the online appendix for the interested reader. However, we still want to explore the effects of entry.

One solution to this problem is to setup a set of components where the parameter values are substituted into the zero profit condition then solved. This is performed repeatedly to create graphs. Further, these sets of tools are generalized such that readers can trivially change the parameter values and see the outcome[7].

From these tools we can still draw some conclusions, particularly when those conclusions contain ambiguity. Let's call these conclusions *facts*.

Fact 1. *Higher awareness values are not always welfare enhancing*

Taking a look at figure 5, we see that welfare has sections of increasing and decreasing welfare as a function of awareness. We also see that the number of firms are decreasing with increased awareness. Please note that given various sets of parameter values, welfare can be strictly increase *or* decreasing in awareness – you can try this for yourself in the online appendix.

Intuitively, awareness eventually drives the aftermarket price to near marginal cost, so it is easy to see why for some set of values it would be welfare improving. However, increased profits results

[7] Available in the online appendix at http://bensresearch.com/piracy/

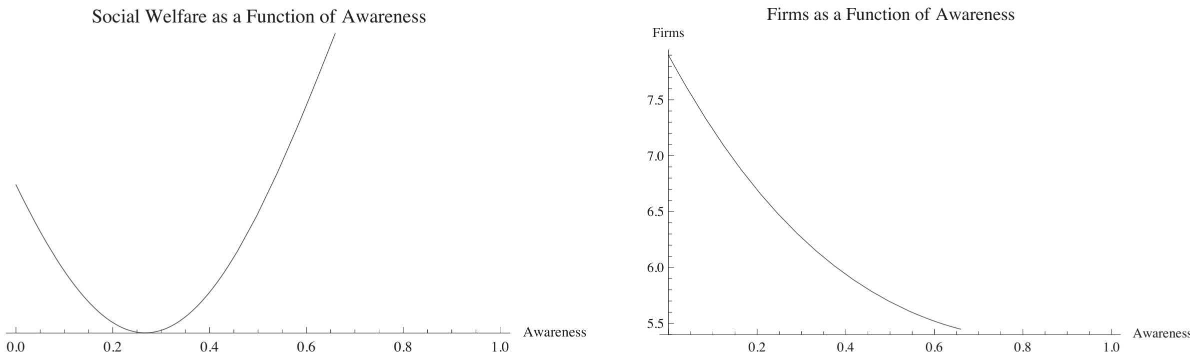

Figure 5: Social welfare and firms as a function of awareness (with entry). The graphs stop when $x_2 = 1$

in more firms. This reduces the travel costs in the primary market which then improves social welfare, but at a diminishing rate.

Fact 2. *Higher piracy travel cost values are not always welfare enhancing*

Examining figure 6 we see regions of both social welfare improvement and reduction with increases in piracy cost. Similar to the analysis of fact 1, piracy cost has two counteracting forces. Low piracy costs reduces the aftermarket price of the firm, but higher piracy costs results in more firms, which results in lower primary market travel costs. Note that given various sets of parameter values, welfare can be strictly increase *or* decreasing in piracy cost – you can try this for yourself in the online appendix.

In the case of both fact 1 and 2, the specific outcome is a function of the relationships between travel costs and the effectiveness of enforcement[8]. We also see that our conclusions about awareness,

[8] And, of course, G because it magnifies the aftermarket effects

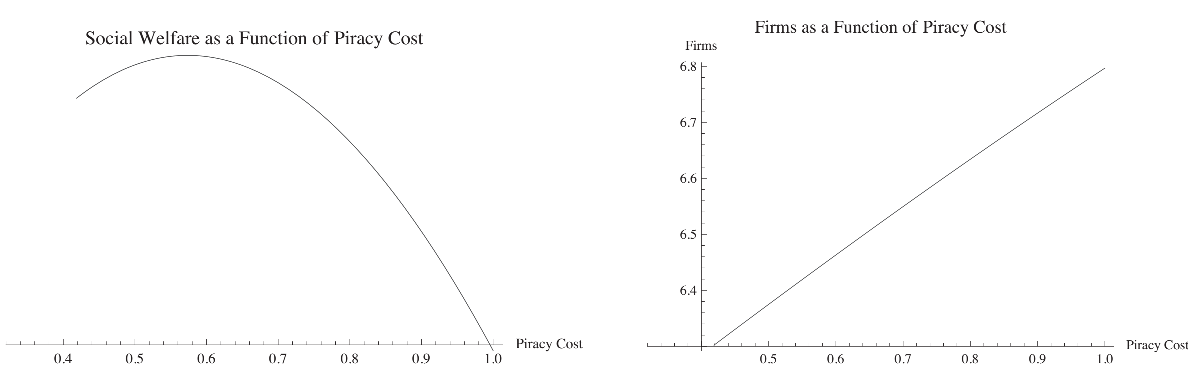

Figure 6: Social welfare and firms as a function of piracy travel cost (with entry).

piracy travel cost and social welfare still hold with entry.

3 Conclusion

In this game, we explore an environment where a product depends on aftermarket purchases, but the customer may or may not be aware of that when buying the primary product. We find that, for the firm, a lack of perception is always profit enhancing. However, it may not be welfare decreasing for the consumer. Higher profits results in more firms which result in less travel cost. Similarly, higher piracy travel costs are not necessarily of negative impact for society. Perhaps the most interesting for policy makers is the strict positive impact of increased enforcement effectiveness.

This area of research is of increasing interest because the number of products that can be pirated may expand from the digital to physical with the consumerization of 3D printers. Since the Xerox v. ISOs (203 F.3d 1322 - 2000) established that it was acceptable, in the view of the court, to have monopoly power in an aftermarket as long as the aftermarket products were protected by intellectual property rights (Borenstein et al., 2000), manufactures have enjoyed additional market power in their aftermarkets (Adelmann, 2010; Gleklen, 2012).

Because 3D printers can be used to duplicate nearly any product made out of plastic, and the materials are expanding, one might expect the threat of physical piracy to be as important as the digital variety noted in the paper. Further, one might expect the manufactures to respond with enforcement in the form of complex parts (so they are harder to print).

Enforcement comes in many forms, from increased complexity of replacement parts, obfuscation of technical details or copy protection. Nonetheless, one form of enforcement is legal – though this doesn't apply to all industries. This paper suggests that actions by the government that reduce the cost of enforcing intellectual property rights (assuming adjusting effectiveness is costless) are welfare improving. This can be seen as a reduction in a type of spending that is no part transferable, but simply trying to change others' behavior.

For many industries, the result that piracy is high when the consumer is uniformed, but uninformed consumers is always profit enhancing for the firm is of particular interest. This result suggests that despite having a lower proportion of sales (in the aftermarket), firms will still often subsidize the aftermarket and operate below marginal cost in the primary market. This is particularly true when the primary market is more competitive, thereby allowing the firm to shift part of the sales process to the less competitive aftermarket as well as increase enforcement. This is a contradiction of the common belief that anti-piracy efforts are the domain of large corporations.

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Appendices

A Solution to Simple Salop Model (S_m)

In this section we solve the simple Salop model. This is not original work, as this model as been solved many times since Salop (1979) and is provided here purely for the readers' convenience.

Set the utility of buying from the firm equal to buying it from its neighbor and solve for x_1:

$$\begin{aligned} v - P_1 - x_1 t &= v - P_N - (\frac{1}{n} - x_1)t \\ x_1 &\to \frac{n(P_N - P_1) + t}{2nt} \end{aligned} \tag{34}$$

The firm has the following profit equation (the firm captures customers from "both sides", thus the "2"):

$$\pi = 2x_1^*(P_1 - C_1) - f \tag{35}$$

Taking the first order condition with respect to P_1, then replacing P_N with P_1 because all firms are identical, results in:

$$\begin{aligned} 0 &= \frac{C_1 - P_1}{t} + \frac{1}{n} \\ P_1 &\to C_1 + \frac{t}{n} \end{aligned} \tag{36}$$

Substituting in equation 36 into equation 35 results in:

$$\pi = \frac{t}{n^2} - f \tag{37}$$

B Solution to Simple Linear City Piracy Model (H_m)

Assume you have a product produced by a monopoly, but the product can be pirated. In a Hotelling linear city model (Hotelling, 1929) this can be thought of as the consumer choosing between pirating the good and buying the good. Setting the utilities equal to each other we can find the indifferent consumer:

$$\begin{aligned} v - P_2 &= v - (1 - x_2)s - rE \\ x_2 &\to \frac{Er - P_2 + s}{s} \end{aligned} \tag{38}$$

The firm chooses the both the price (P_2) and the amount to spend on enforcement (E) from the following profit equation:

$$\pi = x_2^*(P_2 - C_2) - E \tag{39}$$

Taking the first order conditions results in:

$$\frac{C_2 r - P_2 r + s}{s} = 0$$
$$\frac{C_2 + Er - 2P_2 + s}{s} = 0 \tag{40}$$

Given two equations and two unknowns, this can be solved for P_2 and E:

$$P_2 \to C_2 + \frac{s}{r}$$
$$E \to \frac{C_2}{r} - \frac{s}{r} + \frac{2s}{r^2} \tag{41}$$

Substituting in the solutions in equation 41 into equation 39 results in:

$$\pi = \frac{s}{r} - \frac{C_2}{r} - \frac{s}{r^2} \tag{42}$$

C When Piracy isn't a Perfect Substitute

The model in the body of the paper assumes there is no difference between the aftermarket good produced by the monopolist and the good that is pirated. Suppose there is a "quality cost" of Q_c of the pirated good. If this was the case, finding the indifferent consumer would be reformulated as follows:

$$-P_2 = -(1 - x_2)s - rE \overbrace{-Q_c}^{New}$$
$$x_2 \to \frac{Er \overbrace{+Q_c}^{New} - P_2 + s}{s} \tag{43}$$

Such a setup only results in minor differences in the model's outcome. Pricing is identical, and enforcement is only different by $-\frac{Q_c}{r}$:

$$P_1 \to \underbrace{C_1 + \frac{t}{n}}_{S_m} + \frac{s(A_W - 1)(GA_W r + n)^2}{Gr^2} \tag{44}$$

$$P_2 \to \underbrace{C_2 + \frac{s}{r}}_{H_m} * \frac{(1 - A_W)(GA_W r + n)}{G} \tag{45}$$

$$E \to \underbrace{\frac{C_2}{r} - \frac{s}{r} + \frac{2s}{r^2}}_{H_m} * \frac{(1 - \frac{A_W}{2})(GA_W r + n)}{G} \underbrace{- \frac{Q_c}{r}}_{New} \tag{46}$$

Profits are only different by $\frac{Q_c}{r}$:

$$\pi = \underbrace{\frac{t}{n^2} - f}_{S_m} + (A_W - 1)^2 * \underbrace{\frac{s}{r} - \frac{C_2}{r} - \frac{s}{r^2}}_{H_m} * \frac{n(2 - A_W)}{G} \underbrace{+ \frac{Q_c}{r}}_{New} \tag{47}$$

And social welfare is only different by $n\frac{Q_c}{r}$:

$$\begin{aligned} SW = \frac{1}{4}(-4C_1 - \frac{4C_2Gnr(Gr+n) + 2ns(G^2(A_W-1)^2(2A_W-1)r^2}{Gnr^2} \\ - \frac{2Gn(A_W-1)^2r + n^2 + Gr^2t}{Gnr^2} - 4fn + 4v) \underbrace{+n\frac{Q_c}{r}}_{New} \end{aligned} \tag{48}$$

This means there is only a small change in two comparative statistics:

$$\frac{\Delta E}{\Delta r} = \frac{Gr\left((A_W-1)^2s - C_2 \overbrace{+Q_c}^{New}\right) + 2n(A_W-2)s}{Gr^3} \tag{49}$$

$$\frac{\Delta SW}{\Delta r} = \frac{n\left(Gr\left(C_2 \overbrace{-Q_c}^{New} + (A_W-1)^2s\right) + ns\right)}{Gr^3} \tag{50}$$

Conceptually, the introduction of Q_c is simply "built in enforcement". It therefore makes sense that the firm would reduce enforcement as a result. This obviously improves profits and because enforcement is no part a transfer, it improves social welfare.

Interpretations of $\frac{\Delta E}{\Delta r}$ and $\frac{\Delta SW}{\Delta r}$ can largely stay the same. $\frac{\Delta E}{\Delta r}$ can still be either positive or negative based on piracy travel costs and number of firms. $\frac{\Delta SW}{\Delta r}$ is no longer unambiguously positive. However, it remains positive as long as any of the following are true: production cost is higher than the quality difference ($C_2 \geq Q_c$), a reasonable piracy travel cost ($s \geq \frac{Q_c - C_2}{(A_W-1)^2}$) or a higher number of firms ($G \leq -\frac{ns}{r(C_2 - Q_c + (A_W-1)^2s)}$).

D The Corner Solution

Unfortunately, this model isn't as interesting when it comes to the corner solution. Performing a constrained optimization where $x_2 = 1$ results in the following three equations (once λ is eliminated):

$$\begin{aligned} P_1 &= C_1 + G(C_2 - P_2) + \frac{t}{n} \\ P_2 &= \frac{En}{G(1-A_W)} \\ 1 &= \frac{n}{Gr} + A_W \end{aligned} \tag{51}$$

Which, because the third condition does not contain any of the choice variables, is not solvable for a single unique solution. However, we also know that $\frac{E - P_2 + s}{s} = 1$.

This results in the following solutions for P_1, P_2 and E:

$$
\begin{aligned}
P_1 &\to C_1 + C_2 G + \frac{t}{n} \\
P_2 &\to 0 \\
E &\to 0
\end{aligned}
\tag{52}
$$

However, this result is also constrained by the $\frac{n}{Gr} + A_W \geq 1$. Final profit is:

$$
\pi = \frac{t}{n^2} - f \tag{53}
$$