Identifying What is Tempting

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Abstract

An individual with present bias is one who is particularly impatient for consumption now at the expense of consumption later, but less impatient between any two dates in the future. A hypothesis for the cause of present bias is that immediate consumption is subject to temptation, whereas future consumption is not. Under this hypothesis an individual's level of present bias is a combination of what she is tempted to do and the amount of self-control she uses to avoid succumbing to this temptation. I show that given a level of present bias what is tempting and how much self-control is used is not always identified: it could be that she is tempted to consume everything she has available right now, but she controls herself; that her temptation is more mild and she succumbs to it completely; or something in between. I then present an algorithm that is able to disentangle this combination by eliciting the maximum price she will pay for commitment and her present bias. This works because for a given level of present-bias commitment becomes more valuable as the effort required to control one's self increases.

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1 Introduction

An individual who exhibits a larger discount rate between immediate consumption and future consumption than she does between consumption in two similarly separated future dates is said to be present biased. Present bias has been correlated with a variety of behaviors, from holding credit card debt and microfinance loan uptake, to scholastic performance[1]. A hypothesis for the cause of present bias is that immediate consumption is subject to temptation, whereas choices about future consumption are not[2]. Under this hypothesis an individual's observed level of present bias is a combination of what she is tempted to do and the amount of self-control she uses to avoid succumbing to this temptation. I show that this combination is not well defined: it could be that she is tempted to consume everything she has available right now, but she successfully controls herself; or that her temptation is more mild, but that she succumbs to it completely; or something in between these two extremes.

Temptation that puts a greater weight on the future will be more mild than temptation that puts less[3]. A researcher who only elicits an individual's present bias will not know if it is myopic temptation and strong self-control or mild temptation and weak self-control that is correlated with other behavior, for example excessive credit card debt or opting into commitment savings devices. This knowledge has an effect on the types of policies that can be implemented. For instance, if temptation is myopic enough debt repayment contracts cannot be contingent on unobservable income. That is, a contract cannot specify that when an individual's realized income is above a certain level she must pay, and when it is below that level she doesn't have to. This is because when temptation is myopic the individual is always tempted to spend as much as possible, so she will always be tempted to claim that she received whatever amount of income allows her to spend the most now, though

[1]See literature reviews by Frederick et al. (2002); Bryan et al. (2010)

[2]Meier and Sprenger (2010); Ashraf et al. (2006); Bauer et al. (2012); Laibson (1997)

[3]Experiments in Trope and Fishbach (2000) and citations within present evidence that temptation is not totally myopic.

she may not actually succumb to this temptation. However, when temptation is farsighted separating contracts are feasible because the individual is not always tempted to consume as much as possible. Furthermore, when temptation is farsighted levying fees, such as early withdrawal fees for 401Ks, or charging high interest rates can make borrowing now less tempting. If this is the case then high interest rates on credit cards could increase welfare, while if temptation is myopic high interest rates are only predatory and decrease welfare. Additionally, if temptation is very myopic then commitment devices like credit card limits or mandatory payments do not have to be very precise since limiting any extreme behavior is helpful and easy to do. While if it is more farsighted then commitment must be more targeted so that it actually eliminates some tempting behavior[4].

While there is evidence that the ability to control oneself can be depleted[5], the identification algorithms presented here will be able to produce the first evidence about whether or not people find the actual *exertion* of self-control to be costly. Costly self-control allows for models of time-consistent agents who value commitment[6], or dual-self models with a long-term self with stable preferences[7]. Time consistency results in straightforward welfare analysis since there is only one agent to focus on, while it is more difficult with time-inconsistent preferences because it is unclear which set of preferences matter. In addition, when self-control is costly, eliminating tempting options that are never chosen is always beneficial because it reduces the amount of self-control that is exerted. However, if self-control is not costly, eliminating tempting options is valued only if they will be chosen when available. This means that only commitments that change behavior, binding commitments, are useful when self-control is not costly.

The common present-bias identification algorithm uses the $\beta\delta$ model of quasi-hyperbolic

[4]For a more complete discussion of how the myopia of temptation affects borrowing and saving behavior see Groves (2013)

[5]In the psychology literature this is referred to as willpower depletion. See Baumeister et al. (1998, 2000).

[6]As in Gul and Pesendorfer (2001); Noor (2007); Noor and Takeoka (2010); Dekel et al. (2009).

[7]Fudenberg and Levine (2006, 2011)

discounting. It presents an individual with a series of choices between a smaller reward, $\$x$, at date t and a larger reward, $\$y$, at date $t + \tau$, where $\$y$ is varied and $\$x$ is kept constant (or vice-versa). The individual makes decisions about these trade-offs in at least two time frames: $t_0 = 0$ (now) and τ days in the future; and for $t_1 > 0$ and $T = \tau + t_1$ days in the future. The combination of discount factor and present bias, $\beta\delta$, are measured with the future time frame. Present bias, β, is an effect between immediate and future rewards, so the decisions in the immediate time frame disentangle β and the discount factor, δ. The individual's present bias is a measure of how much her immediate decision misaligns with her normative preferences.

Temptation and self-control can be disentangled by eliciting the maximum price the individual is willing to pay for commitment: a high price means that her temptation is myopic but she controls herself, and a lower price means that her temptation is more farsighted but she is less successful in controlling herself. When temptation can be farsighted, and the effectiveness of self-control is linear in utilities, as in Gul and Pesendorfer (2001, 2004, 2005); Krusell et al. (2010), more data is needed to fully identify a model of temptation and self-control than is produced using the common present-bias algorithm. This is because the effectiveness of self-control and the myopia of temptation can be modulated in such a way that while the individual is observed to have a present bias of $\beta^\star$ she may have been tempted to consume everything but controlled herself, or her temptation was more mild but she succumbed completely to it. However, given a particular $\beta^\star$ the individual must exert more self-control if she is tempted to consume everything than if her temptation is more mild. Therefore commitment will be more valuable the more myopic her temptation is because she has a greater amount of self-control to avoid exerting. The new algorithm would run as follows: as before, first the individual would choose between rewards in the present and future time frames; the future frame is the commitment decision. Then the maximum price she would be willing to pay to enforce this commitment is elicited[8]. This algorithm will work well if we assume that utility is linear[9], which has been argued to be the case with most experimental rewards as in Rabin (2000).

If the utility function is nonlinear then its curvature needs to be estimated as well. This is done by inducing the individual to reveal her certainty equivalents for multiple lotteries that pay off immediately. The researcher can then measure the curvature of the utility functions using changes in the individual's decisions due to changes in reward levels and distributions.

Given the additional steps required to estimate a nonlinear utility function, one may be tempted to assume a linear utility function and use the simpler identification algorithm with the goal of estimating "bounds" on the parameter values. Unfortunately this will not always work since the linear approximation of a variety of popular utility functions will overestimate or underestimate parameter values depending on reward levels and other parameter values. In fact, this approach will not work even in the absence of temptation and self-control since the linear identification algorithm results in indifference conditions for levels of utility, not marginal utilities. The ratio of the utility level for consumption today over the utility level for consumption in the future will define the discount factor. Unlike the ratio of marginal utilities, the ratio of utility levels is not necessarily monotonic in wealth, even if the true utility function is continuous and concave. Introducing temptation and self-control complicate matters further because there are additional parameters to estimate. These concerns are analyzed and discussed in more detail in Section 3.3.

The identification algorithms presented in this paper are related to those used in Andersen et al. (2008) and Andreoni and Sprenger (2012). Neither attempt to estimate both temptation and self-control, only present bias. Bucciol (2012) estimates a model of myopic

[8] A price would be drawn at random and she would have to pay only if it were weakly less than the one she stated.

[9] Linear utility is assumed in a number of experiments, such as Ahlbrecht and Weber (1997); Coller and Williams (1999); Thaler (1981).

temptation and costly self-control using decisions about liquid and illiquid asset holdings and a life cycle consumption-savings model. He finds a small but significant strength of temptation. In other words, while individuals are tempted to spend everything they have, this temptation is not very strong and is relatively easy to avoid. However, since farsighted self-control is not analyzed it could be instead that people are tempted to do something much less extreme but are overcome by this temptation. Ameriks et al. (2007) use a survey with hypothetical questions about how the individual would allocate ten certificates for a "dream restaurant night" over the span of two years. The questions ask for the individual's ideal allocation, how she thinks she would actually allocate the certificates, and how she would be most tempted to allocate the certificates. The answers to the questions are then correlated to individual characteristics such as age, wealth, and education. Unfortunately the data for the survey participants' answers about what they were most tempted to do is not presented. If it were this would give evidence for or against the idea of farsighted temptation.

Models of temptation and self-control can be split into two categories depending on how they define what is tempting. First there are those in which only immediate consumption is tempting and the individual is always tempted to consume everything she has available today[10]. Then there are models in which temptation is more mild[11]. This accomplished by allowing future consumption to affect temptation now. This paper the model of self-control preferences from Gul and Pesendorfer (2005). This model is a generalized version of the model of temptation and costly self-control presented in Krusell et al. (2010) and Gul and Pesendorfer (2001), and allows for both farsighted and myopic temptation.

The remainder of this paper is organized as follows: in Section 2 the $\beta\delta$ model and the Gul and Pesendorfer (2005) model are described. In Section 3 the linear utility algorithms and comparative statics are then presented, followed by the nonlinear utility algorithm and

[10] Gul and Pesendorfer (2001, 2004); Fudenberg and Levine (2006, 2012)

[11] Noor (2007); Krusell et al. (2010). The $\beta\delta$ model falls in this second camp if interpreted a model of overwhelming temptation.

comparative statics, and then a comparison of the two algorithms. Section 4 discusses and concludes.

2 The Models

The "individual" is the subject of the experiment and is the only decision maker.

There are three periods: 0, 1, and 2. In period 0 the individual may have the opportunity to make commitment choices about consuming c_1 in period 1 or c_2 in period 2. When there is a price for this commitment to be enforced she will pay it in period 0. I assume that the individual has some w, in period 0 so that she can afford to pay this price[12]. If her period 0 commitment choice is not binding or if she does not make a commitment decision then she will make her consumption decisions in period 1.

The $\beta\delta$ model describes an individual with time inconsistent preferences who is particularly impatient for consumption now at the expense of consumption later, but less impatient between any two dates in the future. This is modeled with a utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ that is increasing, continuous, and concave. The individual's utility in period 0 is

$$\mathcal{U}_0 = \beta\delta\left[u(c_1) + \delta u(c_2)\right].$$

Her utility in period 1 is

$$\mathcal{U}_1 = u(c_1) + \beta\delta u(c_2).$$

$\beta, \delta \in [0, 1]$. It is the additional impatience, β, of the individual's future self over her current self that will lead to a preference for commitment. The magnitude of her present-bias, β, is a measure of how much the individual's period 0 and period 1 preferences misalign. For instance, if she can commit to a particular period 1,2 consumption path in period 0 she will

[12] w would most likely be a show-up fee paid to the individual.

maximize $\mathcal{U}_0$, while if she cannot commit she will maximize $\mathcal{U}_1$ in period 1. Because $u(c)$ is increasing and continuous, in period 2 she will consume everything that is available. A specific observation of an individual's present bias will be denoted by $\beta^{\star}$.

The Self-Control (SC) model is a generalization of the models presented in Gul and Pesendorfer (2001) and Krusell et al. (2010). It describes an individual with time-consistent preferences that exhibit temptation and costly self-control. $U : \mathbb{R}^2_+ \to \mathbb{R}$ is her time-separable *normative* utility. $U(c_1, c_2) = u(c_1) + \delta u(c_2)$, where $u : \mathbb{R} \to \mathbb{R}$ is continuous, concave, and increasing. $\delta \in [0, 1]$.

$V : \mathbb{R}^2_+ \to \mathbb{R}$ is her *temptation* utility. $V(c_1, c_2) = v(c_1) + \beta\delta v(c_2)$, and $v : \mathbb{R} \to \mathbb{R}$ is assumed to be continuous and increasing. $\beta \in [0, 1]$. A temptation utility that depends only on the immediate period's consumption, $\beta = 0$, is a model of myopic temptation. When $\beta > 0$ then temptation is farsighted.

$\gamma\left(\max_{\tilde{c}_1,\tilde{c}_2} V(\tilde{c}_1, \tilde{c}_2) - V(c_1, c_2)\right)$ is the *cost* of self-control and is always positive unless $c_i = \tilde{c}_i$, in which case it is zero. The temptation utilities are evaluated at the choice made by the individual, $\{c_1, c_2\}$, and the most tempting option available, $\{\tilde{c}_1, \tilde{c}_2\} = \arg\max_{\tilde{c}_1,\tilde{c}_2} V(\tilde{c}_1, \tilde{c}_2)$.

In period 0 she chooses the set of options, C, available to her in period 1. This is her commitment decision. Contingent on the available options her period 1 utility will be:

$$\begin{aligned}
\mathcal{U}_1 &= U(c_1, c_2) - \gamma\left(\max_{\{\tilde{c}_1,\tilde{c}_2\}\in C} V(\tilde{c}_1, \tilde{c}_2) - V(c_1, c_2)\right). \\
&= u(c_1) + \delta u(c_2) - \gamma\left(\left[\max_{\{\tilde{c}_1,\tilde{c}_2\}\in C} v(\tilde{c}_1) + \beta\delta v(\tilde{c}_2)\right] - v(c_1) + \beta\delta v(c_2)\right)
\end{aligned}$$

such that $\{c_1, c_2\} \in C$ and $\{\tilde{c}_1, \tilde{c}_2\} \in C$. If commitment is completely binding then $C = \{c_1, c_2\}$ and $\mathcal{U}_1 = u(c_1) + \delta u(c_2)$.

In period 1 the individual will take her most tempting option[13], $\{\tilde{c}_1, \tilde{c}_2\}$, as given and

[13] Technically the Gul and Pesendorfer (2005) model only suggests that the individual behaves in a particular way in period 1. However, in this paper I assume that this is how the individual actually behaves.

maximize the combination of normative and temptation utilities:

$$\max_{\{c_1,c_2\}\in C} \mathcal{U}_1 = \max_{\{c_1,c_2\}\in C} [U(c_1, c_2) + \gamma V(c_1, c_2)]$$

As γ goes to zero consumption is determined by the commitment utility. As γ increases towards infinity the the individual loses her ability to control herself in periods 1 and 2 and the temptation utility determines consumption decisions. γ can be interpreted as the effectiveness of self-control or the strength of temptation.

A preference for commitment is driven by the disagreement between the optimal levels of consumption for the normative and temptation utilities and the resulting cost of self-control. Commitment can eliminate tempting options. For instance, as shown above, if the individual commits to a particular consumption flow, $\{c_1, c_2\}$, then this reduces the cost of self-control to zero and $\mathcal{U}_1 = u(c_1) + \delta u(c_2)$.

In all the identification algorithms presented in this paper the individual will be choosing between consuming c in period 1 or Rc in period 2. The rate of return for which normative utility is indifferent between c and Rc will be denoted R^{fb}, where the superscript fb refers to "first best", and is defined by $u(c) = \delta u(R^{fb}c)$. The most tempting rate of return, $\tilde{R}$, is defined by $v(c) = \beta\delta v(\tilde{R}c)$. If $\beta = 0$ then $\tilde{R} = R_{max}$. The rate of return the individual will choose in period 1 with no prior commitment, $\overline{R}$, is defined by $u(c) + \gamma v(c) = \delta u(\overline{R}c) + \beta\delta\gamma v(\overline{R}c)$. As $\gamma \to \infty$ then $\overline{R} \to \tilde{R}$, and as $\gamma \to 0$ then $\overline{R} \to R^{fb}$. These equations are derived explicitly in the following section.

The individual is assumed to have full information whenever she is presented with any of the algorithms presented in this paper.

3 Identification

In the first subsection I use the simplifying assumption of linear utility to illustrate the identification problem. First I present the original identification algorithm for the $\beta\delta$ model and then the new SC identification algorithm. The subsequent section introduces the additional steps that are necessary so that the SC identification algorithm can estimate nonlinear utilities.

3.1 Linear Utility

When utility is linear the $\beta\delta$ model reduces to

$$\begin{aligned}\mathcal{U}_0 &= \beta\left[c_1 + \delta c_2\right] \\ \mathcal{U}_1 &= c_1 + \beta\delta c_2.\end{aligned}$$

In order to identify the $\beta\delta$ model with linear utility the individual makes decisions defined by the following algorithm, where $\mathcal{F}\left[R_{min}, R_{max}\right]$ denotes any continuous distribution with a lower bound of R_{min} and an upper bound of R_{max}.

Algorithm 1.

1. Period 0: choose the R^{fb} for which you are indifferent between a reward c in period 1 and reward $R^{fb}c$ in period 2.

   (a) After R^{fb} is chosen a random $R^{\star} \sim \mathcal{F}\left[R_{min}, R_{max}\right]$ is drawn.

   (b) If $R^{\star} \geq R^{fb}$ then the individual receives $R^{\star}c$ in period 2, otherwise she receives c in period 1.

2. Period 1: choose the $\overline{R}$ for which you are indifferent between a reward c now and reward $\overline{R}c$ in period 1.

(a) After $\overline{R}$ is chosen another $R^{\star} \sim \mathcal{F}\left[R_{min}, R^{max}\right]$ is drawn. If $R^{\star} \geq \overline{R}$ then the individual receives $R^{\star}c$ in period 2, otherwise she receives c in period 1.

Algorithm 1 presents the individual with two choices. First, in period 0 the individual must choose the R^{fb} that maximizes her expected utility:

$$\max_{R^{fb}} c \int_{R_{min}}^{R^{fb}} f(R)dR + \delta c \int_{R^{fb}}^{R_{max}} Rf(R)dR$$

This is the commitment choice. Then in period 1 she chooses the $\overline{R}$ that maximizes her period 1 expected utility:

$$\max_{\overline{R}} c \int_{R_{min}}^{\overline{R}} f(R)dR + \beta\delta c \int_{\overline{R}}^{R_{max}} Rf(R)dR$$

These two maximization problems result in two different linear first order conditions which identify the two unknowns: β and δ.

Period 0:			Period 1 :		
βc	$=$	$\beta\delta R^{fb}c$	c	$=$	$\beta\delta\overline{R}c$
$\Rightarrow \delta$	$=$	$\frac{1}{R^{fb}}$	$\Rightarrow \beta^{\star}$	$=$	$\frac{R^{fb}}{\overline{R}}$

For the linear SC model, in period 0 the individual chooses the R^{fb} that maximizes her expected utility. Given that choosing R^{fb} fully restricts her choice in period 1 her cost of self-control will be zero.

$$\max_{R^{fb}} \mathcal{U}_1 = \max_{R^{fb}} \int_{R_{min}}^{R^{fb}} cf(R)dR + \delta \int_{R^{fb}}^{R_{max}} Rcf(R)dR$$

If she does not commit in period 0 and must choose the rate of return in period 1, then her choice is subject to a cost of self-control:

$$\begin{aligned}\max_{\overline{R}} \mathcal{U}_1 &= \max_{\overline{R}} \left\{ (1+\gamma) \int_{R_{min}}^{\overline{R}} cf(R)dR + \delta(1+\gamma\beta) \int_{\overline{R}}^{R_{max}} Rcf(R)dR \right. \\ &- \left. \gamma \left[\int_{R_{min}}^{\tilde{R}} cf(R)dR + \beta\delta \int_{\tilde{R}}^{R_{max}} Rcf(R)dR \right] \right\}\end{aligned}$$

If $\gamma > 0$ and $\beta < 1$ then $\overline{R} > R^{fb}$. Costly self-control introduces a third variable into the mix: the strength of temptation, γ. Identification is not a problem if temptation is myopic, since in this case $\beta = 0$ and there are just two parameters to identify: δ and γ. However, in the case of farsighted temptation $\beta > 0$ a third equation is necessary for identification.

Theorem 1. *The SC model with farsighted temptation,*

$$\begin{aligned} U(c_1, c_2) &= c_1 + \delta c_2 \\ V(c_1, c_2) &= \gamma \left(c_1 + \beta\delta c_2\right) \end{aligned}$$

cannot be fully identified using Algorithm 1: $\delta = \frac{1}{R^{fb}}$, $\beta \in \left[0, \frac{R^{fb}}{\overline{R}}\right)$, $\gamma \in \left[\frac{\overline{R}-R^{fb}}{R^{fb}}, \infty\right)$.

When temptation is farsighted the myopia of the individual's temptation, β, can be traded off against the effectiveness of her self-control, γ, to keep her observed present bias, $\beta^\star$, constant. Therefore the algorithm needs at least one additional step to disentangle these two parameters. Algorithm 2 includes a step to elicit the maximum price the individual is willing to pay for her commitment choice in period 0 to be enforced in period 1, instead of waiting until period 1 arrives and making the decision then. Theorem 2 proves that this is sufficient to fully identify the SC model with farsighted temptation.

Algorithm 2.

1. Period 0:

(a) Choose the R^{fb} for which you are indifferent between a reward c at period 1 and reward $R^{fb}c$ at period 2.

(b) Choose the maximum price, p, that you are willing to pay to now to have R^{fb} enforced instead of $\overline{R}$.

(c) After R^{fb} and p are chosen a random $R^{\star} \sim \mathcal{F}\left[R_{min}, R_{max}\right]$ and $p^{\star} \sim \mathcal{P}\left[0, p_{max}\right]$ are drawn.

i. If $p^{\star} \leq p$ and $R^{\star} \geq R^{fb}$ the individual receives $R^{\star}c$ in period 2.

ii. If $p^{\star} \leq p$ and $R^{\star} < R^{fb}$ the individual receives c in period 1.

iii. If $p^{\star} > p$ the individual continues on to period 1 with no commitment enforced.

2. Period 1:

(a) Choose the $\overline{R}$ for which you are indifferent between a reward c now and reward $\overline{R}c$ at period 2.

i. After $\overline{R}$ is chosen another $R^{\star} \sim \mathcal{F}\left[R_{min}, R^{max}\right]$ is drawn. If $R^{\star} \geq \overline{R}$ the individual receives $R^{\star}c$ in period 2, else she receives c in period 1.

Theorem 2. *Algorithm 2 fully identifies the farsighted SC model.*

- *Step 1(b) is necessary only if* $\beta > 0$.

As temptation becomes more myopic the individual must use more self-control to obtain a particular level of present bias. Commitment allows her to avoid exerting costly self-control, so given a particular level of observed present-bias, the value of commitment is increasing with the level of the myopia of temptation. All else equal, if the researcher observes a higher willingness to pay for commitment then she will be able to conclude that individual's temptation is more myopic, and her self-control is more effective, than an individual with a lower willingness to pay for commitment.

3.1.1 Linear comparative statics

In this section I assume a uniform distribution is used for the reward multiplier R so that comparative statics with respect to the reward and the range of the multiplier can be found and simulated. I use the uniform distribution because it is straightforward to derive the comparative statics, and it also seems likely to be the least confusing and most intuitive distribution for subjects. I also assume without loss of generality that $R_{min} = 0$. Given these assumptions the researcher can modulate R_{max} and c to affect the prices individuals are willing to pay for commitment.

$$p = \frac{c\delta}{R_{max} - R_{min}} \left[\int_{R^{fb}}^{\overline{R}} [\delta R - 1]\, dR + \gamma \int_{\overline{R}}^{\tilde{R}} [1 - \beta\delta R]\, dR \right] + w \tag{1}$$

The first integral in equation 1 is the cost from making a suboptimal choice, while the second integral is the net cost of self-control without commitment. These are the two costs that are incurred when the individual is not able to commit to R^{fb}, and so the maximum price she is willing to pay to commit is just the addition of these two costs. This equation is derived in full in the appendix.

The observed level of present bias is $\beta^{\star} \equiv \frac{1}{\delta\overline{R}}$.

Theorem 3. *The the price for commitment is increasing in the maximum possible reward multiplier, R_{max}, when temptation is myopic, $\beta = 0$, but is decreasing when $\beta \to \beta^{\star}$. The price for commitment is increasing in the reward, c.*

Proof. Define $\zeta \equiv p(\beta = 0) - \lim_{\beta \to \beta^\star} p(\beta)$.

$$
\begin{aligned}
\zeta &= \gamma c \delta \frac{R_{max} - \overline{R}}{R_{max} - R_{min}} \\
\frac{\partial \zeta}{\partial c} &= \gamma \delta \frac{R_{max} - \overline{R}}{R_{max} - R_{min}} > 0 \\
\frac{\partial \zeta}{\partial R_{max}} &= \gamma c \delta \frac{R_{max}}{R_{max} - R_{min}} - \gamma c \delta \frac{R_{max} - \overline{R}}{(R_{max} - R_{min})^2} > 0
\end{aligned}
$$

□

As one can see from equation 1, c works as a linear scaling factor. Changes in R_{max} can have both positive and negative effects on any particular $p(\beta)$ depending on the myopia of temptation. This is illustrated in Figure 1. If $R_{max} > \frac{1}{\beta^\star \delta}$ there will exist a β' such that for $\beta \leq \beta'(R_{max})$ the most tempting option will be to choose R_{max}, and for more farsighted temptation the most tempting option will be to choose a rate of return less than R_{max}. Increasing R_{max} will increase the range in which self-control must be exercised for all $\beta \leq \beta'$, making commitment more valuable to the individual. Additionally, increasing R_{max} spreads the distribution, effectively putting more weight on large extreme values for R, and less on the small extreme values. The combination of these two effects mean that for $\beta \leq \beta'$ increasing R_{max} increases the price the individual is willing to pay, while for more farsighted temptation it will decrease the value of commitment since it decreases the likelihood a suboptimal choice is realized and reduces the weight on the area in which self-control must be exercised: between R_{min} and $\tilde{R}$.

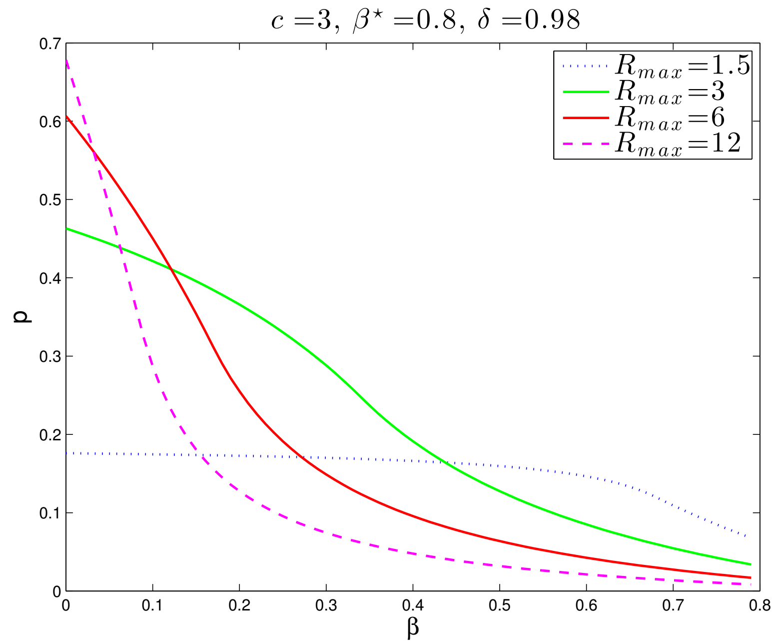

Figure 1: p as a function of β and R_{max}. $\beta = 0.8$, $\delta = 0.99$.

If the researcher just wants to test the hypothesis $\beta = 0$ then it is best to make R_{max} and c as large as possible. On the other hand, if the researcher would like to estimate β more precisely then an intermediate value for R_{max} would be best, again maximizing c.

3.2 Nonlinear Utility

Concave normative utility introduces the possibility of risk aversion and decreasing marginal utility, and the nonlinearity will affect the estimates of the parameter values if not taken into account. Nonlinear temptation utility can affect both what is tempting and how tempting it is depending on the payoffs. When temptation utility is convex the individual is always most tempted to consume everything available in a single period, and as the payoff in that period increases so does the strength of this temptation. Concave temptation utility results in a decreasing strength of temptation as payoffs increase, and it also allows for temptation that is more mild, in which the individual is not necessarily tempted to consume everything in just one period. One consequence of this difference is that when temptation is concave and farsighted the individual may find different options more or less tempting depending on the state of the world. For instance, if the individual receives a negative income shock she may be tempted to borrow, while if she receives a positive income shock she may be tempted to save (though not as much as her normative preferences would like). This means that state dependent contracts can be desirable when temptation is concave and farsighted. However, if temptation is convex and/or myopic the individual is always tempted to consume everything available, so state dependent contracts will not be useful in reducing the amount of self-control that the individual exerts.

When the normative and temptation utilities are nonlinear, observations for several different levels of reward are necessary to measure the curvature of the function. However, if the individual only makes fully binding commitment decisions then only the normative utility function will be estimated. This is because fully binding commitment restricts the individual's choice set, reducing her menu of options to a subset in which the most tempting options have been eliminated: her choice will be from the boundary of the set so no self-control is necessary. Therefore, to observe the curvature of both the normative and temptation utilities there should be no commitment in place during the decision. Furthermore, if the normative utility function is CRRA and the temptation utility function is an affine transformation of this function then intertemporal decisions, even without commitment and at a variety of reward levels, will not be able to identify the curvature. Her intertemporal trade-off decision when her utilities are CRRA is to choose the $\overline{R}$ such that

$$(1+\gamma)\frac{c^{1-\alpha}}{1-\alpha} = \delta(1+\gamma\beta)\frac{(\overline{R}c)^{1-\alpha}}{1-\alpha}$$

$\overline{R}$ is not going to be a function of c, so changing c will not allow the researcher to estimate α. Eliciting the individual's certainty equivalent for a lottery, or lotteries, all over immediate payoffs avoids this problem. The same continuous price list procedure used in the algorithms above can be used to elicit her certainty equivalent. Then her decision will be to choose the $\overline{x}$ such that

$$\max_{\overline{x}} \int_{x_{min}}^{\overline{x}} \int_{z_{min}}^{z_{max}} [u(z) + \gamma v(z)]\, f(z) dz f(x) dx + \int_{\overline{x}}^{x_{max}} [u(x) + \gamma v(x)]\, f(x) dx$$
$$-\gamma \left[\max_{\tilde{x}} \int_{x_{min}}^{\tilde{x}} \int_{z_{min}}^{z_{max}} v(z) f(z) dz f(x) dx + \int_{\tilde{x}}^{x_{max}} v(x) f(x) dx\right]$$

Which results in the following equality:

$$u(\overline{x}) + \gamma v(\overline{x}) = \int_{z_{min}}^{z_{max}} [u(z) + \gamma v(z)]\, f(z) dz$$

This will allow the researcher to estimate the utility functions $u(\cdot)$ and $v(\cdot)$. Algorithm 3 puts all of these steps together.

Algorithm 3.

1. Period 0:

   (a) ** Choose the payoff, $\overline{x}$, and the gamble, $\mathcal{Z}\,[z_{min}, z_{max}]$, that you are indifferent between.

      i. $x^\star \sim \mathcal{X}\,[x_{min}, x_{max}]$ is drawn. The individual receives $x^\star$ if $x^\star \geq \overline{x}$, else she receives $z \sim \mathcal{Z}\,[z_{min}, z_{max}]$. Either reward is received immediately.

   (b) Given c, choose the R^{fb} for which you are indifferent between a reward c in period 1 and reward $R^{fb}c$ in period 2.

   (c) * Choose the maximum price p you are willing to pay to have R^{fb} implemented.

   (d) After R^{fb} and p are chosen a random $R^\star \sim \mathcal{F}\,[R_{min}, R_{max}]$ and $p^\star \sim \mathcal{P}\,[0, p_{max}]$ are drawn.

      i. If $p^\star \leq p$ and $R^\star \geq R^{fb}$ the individual receives $R^\star c$ in period 2.

ii. If $p^\star \leq p$ and $R^\star < R^{fb}$ the individual receives c in period 1.

iii. If $p^\star > p$ the individual continues on to period 1 with no commitment enforced.

2. Period 1:

   (a) Choose the $\overline{R}$ for which you are indifferent between a reward c now and reward $\overline{R}c$ in period 2.

      i. After $\overline{R}$ is chosen $R^\star \sim \mathcal{F}\left[R_{min}, R^{max}\right]$ is drawn. If $R^\star \geq \overline{R}$ then the individual receives $R^\star c$ in period 2, else she receives c in period 1.

The following theorem proves that this algorithm is sufficient to identify the SC model.

Theorem 4. *Algorithm 1 identifies the SC model.*

- *Step 1(c) is necessary only if*

$$\begin{aligned} U(c, Rc) &= u(c) + \delta u(Rc) \\ V(c, Rc) &= \gamma\left[u(c) + \beta\delta u(Rc)\right] \end{aligned}$$

As in the linear setting, the price an individual is willing to pay for commitment has a monotonic relationship with respect to β and γ. Coupled with the two intertemporal choices this will allow the researcher to identify the three parameters β, γ, and δ.

Initially, the version of the SC model where $V(c, Rc) = \gamma\left[u(c) + \beta\delta u(Rc)\right]$ may seem uninteresting. However, it is the only version of the SC model that can approximate the $\beta\delta$ model to any level of precision (approaching it exactly in the limit as $\gamma \to \infty$). Because of this attribute, and the fact that it has only one nonlinear function, it has been used in a number of theoretical and empirical papers[14]. Finally, again because of these characteristics, it is a good first candidate to use to measure the departure of behavior from the popular $\beta\delta$ model.

3.2.1 Nonlinear comparative statics

As in the linear setting, I assume here a uniform distribution for R, with $R_{min} = 0$. In addition I use a CRRA utility function because it is commonly used and has a single parameter that determines the function's curvature. This results in the following equation for the price for commitment:

$$p = c\left[\frac{\delta}{R_{max} - R_{min}}\left[\int_{R^{fb}}^{\overline{R}} \delta\left(R^{1-\alpha} - 1\right) dR + \gamma \int_{\overline{R}}^{\tilde{R}} \left(1 - \beta\delta R^{1-\alpha}\right) dR\right]\right]^{\frac{1}{1-\alpha}} + w$$

The first integral is the cost of making a suboptimal choice, due to temptation. The second integral is the cost of self-control. It is these two costs that commitment allows the individual to avoid, and what makes paying for it worthwhile. The comparative statics with respect to c and R_{max} are the same as in the linear case.

Theorem 5. *The the price for commitment is increasing in the maximum possible reward multiplier, R_{max}, when temptation is myopic, $\beta = 0$, but is decreasing when $\beta \to \beta^{\star}$. The price for commitment is increasing in the reward, c.*

That p is increasing with c is easy to see. When $\beta = 0$ the individual is tempted to always take c immediately, no matter what R could possibly be, so $\tilde{R} = R_{max}$. This means that the cost of self-control is increasing with R_{max}. However, increasing R_{max} also decreases the weight of both costs from not committing. The increase in the cost of self-control is greater

[14]Krusell et al. (2010) introduced this formulation of the SC model in their paper on optimal taxation. Amador et al. (2006) use the model to analyze optimal savings contracts. Bucciol (2012) uses a completely myopic version to estimate the strength of self-control.

in magnitude than the decrease in the weight. As $\beta \to \beta^\star$ the cost of self-control goes to zero, so increasing R_{max} no longer has the first effect. Therefore $\frac{\partial p(\beta^\star)}{\partial R_{max}} < 0$. This means that the difference between the prices at the two extremes, ζ, is increasing with R_{max}, as it was in the linear case.

As shown in the following theorem, the comparative static with respect to the curvature of the utility function is more complex.

Theorem 6. *The change in the price for commitment, p, with respect to the curvature of the utility function, α, can be positive or negative.*

Increasing the curvature has a number of effects that can move in opposite directions with varying magnitudes. First, changing the curvature of the utility function can have a negative effect on the marginal utility from p if $\frac{p-w}{c} < 1$ and a positive one if $\frac{p-w}{c} > 1$[15]. Increasing the curvature also decreases the utility from the future return. This too can have ambiguous consequences since it will decrease the loss from choosing $\overline{R}$ instead of R^{fb}, but it will also increase the cost of self-control. The overall change in p will depend on the magnitudes of all of these different effects combined. Figure 2 illustrates two examples in which changes in the curvature, α, results in very different changes in p.

[15]This comes from the fact that the derivative of $\left(\frac{p-w}{c}\right)^{1-\alpha}$ with respect to α is positive if $\frac{p-w}{c} < 1$ and negative if $\frac{p-w}{c} > 1$. For more details see the proof in the appendix.

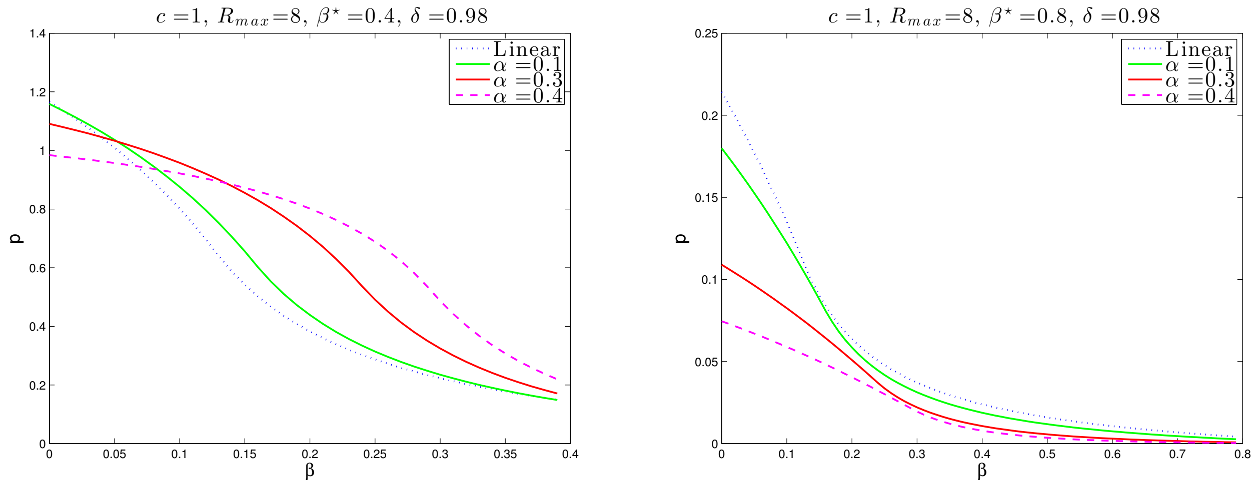

Figure 2: $p(\beta)$ with different utility curvatures.

3.3 Linear vs Nonlinear Estimates

The upshot of Theorem 6 is that if the true utility function is nonlinear a linear approximation may lead to an overestimation *or* an underestimation of β and γ. This is illustrated in Figure 2 where only $\beta^\star$ is varied, between 0.4 and 0.8. For instance, imagine that a linear approximation is used to estimate β given a true CRRA utility function with $\alpha = 0.4$. In the graph on the left, one can see that the lines for the linear approximation and the true utility cross at about $\beta = 0.065$ and $p = 0.95$. This means for any observed $p > 0.95$ the linear approximation will overestimate β, while for any $p < 0.95$ it will underestimate β. On the other hand, as shown in the graph on the right, if $\beta^\star = 0.8$ then the linear approximation will consistently overestimate β. Similar examples can be constructed with other types of utility functions as well.

This is not the only source of ambiguous misestimation. As shown in the following theorem the discount factor and the function $\frac{1+\gamma\beta}{1+\gamma}$ can also be either underestimated or overestimated when using linear utility instead of the concave utility function. A ramification of this is that even if there is no temptation or when there is temptation but no self-control, as in the $\beta\delta$ model, the linear estimation of the discount factor (and the present-bias in the second case) does not consistently give either an upper or a lower bound on the true value(s).

Theorem 7. *When the true utility function is strictly concave and continuous a linear approximation may result in over or underestimates for* δ *and/or* $\frac{1+\gamma\beta}{1+\gamma}$. *If utility is CRRA the linear approximation will always result in underestimates for* δ *and* $\frac{1+\gamma\beta}{1+\gamma}$.

Proof. The true value for the discount factor comes from the indifference condition $u(c) - \delta u(R^{fb}c) = 0$. The linear estimate of δ is $\delta_L = \frac{1}{R^{fb}}$.

$$\begin{aligned} \delta_L \text{ is an overestimate if } \frac{u(c)}{u(R^{fb}c)} &< \frac{1}{R^{fb}} \\ \text{and it is an underestimate if } \frac{u(c)}{u(R^{fb}c)} &> \frac{1}{R^{fb}} \end{aligned}$$

These are utility values, not marginal utilities, so an overestimation of δ is possible. For example, if $u(c) = \ln(c)$ then $R^{fb} = c^{\frac{1-\delta}{\delta}}$. If $c = 2$, and $\delta > 0.5$ then $\frac{\ln(2)}{\ln\left(2R^{fb}\right)} < \frac{1}{R^{fb}}$, while if $\delta < 0.5$ then $\frac{\ln(2)}{\ln\left(2R^{fb}\right)} > \frac{1}{R^{fb}}$. Similarly for the linear estimate for $\frac{\delta(1+\gamma\beta)}{1+\gamma} = \frac{1}{\overline{R}}$. Because $\overline{R} \geq R^{fb}$ and $u(Rc)$ is a concave function, it is possible that while δ is overestimated $\frac{\delta(1+\gamma\beta)}{1+\gamma}$ is underestimated.

When the utility function is CRRA linear estimates of δ and $\frac{\delta(1+\gamma\beta)}{1+\gamma}$ will always be low:

$$\begin{aligned} \delta = \frac{u(c)}{u(Rc)} &= \frac{c^{1-\alpha}}{(Rc)^{1-\alpha}} \\ &= \frac{1}{R^{1-\alpha}} > \frac{1}{R} \end{aligned}$$

Since $R > 1$ and $\alpha < 1$. □

However, there is still the problem of ambiguous misestimation of β and γ via p. Therefore it can be quite important to use the nonlinear algorithm, especially if one expects sufficiently curved utility functions.

4 Conclusion

The identification algorithms presented in this paper will provide sufficient data to identify and disentangle the effectiveness of self-control (strength of temptation) and the myopia of temptation. These two properties define the characteristics of commitment mechanisms that can be used to influence people's behavior and also determine whether or not these mechanisms are desirable from an individual's perspective.

If it is found that $\gamma \to \infty$ consistently for at least a subpopulation then this suggests that a time inconsistent model without costly self-control may be more robust than a model with costly self-control. However, note that $\gamma \to \infty$ does not mean that self-control does not exist, only that it is not directly costly to use. If this is the case then another algorithm will need to be derived to identify what an individual finds tempting.

The algorithms above use continuous price lists which present the individual with a convex range of values to choose from, resulting in point estimates for parameter values, as opposed to multiple price lists in which there are a finite number of values for the individual to choose from and results in set estimates. A continuous price list should be just as easy to implement in both lab and field experiments using either computers or pencil and paper. The shortcoming of both types of price lists is that the estimates for δ, γ, and β can be affected by affine translations of the utility functions. However, repeating the algorithms for multiple levels of reward, c, will identify the magnitude of the utility translation parameter. Alternatively, instead of repeating the continuous price list structure used in the algorithms above one could use a Convex Time Budget (CTB)[16]. In this procedure the individual is given a budget and decides the fraction that she will allocate between periods 1 and 2 given some interest rate r. This will result in a comparison of marginal utilities and the cardinality of the functions will not be an issue when estimating the parameters. Unfortunately the CTB

[16] Andreoni and Sprenger (2012)

procedure is not able to give very precise estimates unless the utility functions are sufficiently concave because the researcher will observe corner solutions for a range of parameter values.

These algorithms can be modified to avoid logistical issues and maximize data generated when running the experiments. However, the researcher needs to keep in mind how subjects' incentives can change if the timing of decisions and payments change. For instance, if subjects are not required to return to the lab for a second session given that they have paid for commitment, then it will be difficult to know if the participants are paying to commit or to avoid the hassle of having to return to the lab at a future date.

It still needs to be shown whether the algorithms presented in this paper generate enough data to identify models with nonlinear self-control costs, in particular convex cost as in Noor and Takeoka (2010); Fudenberg and Levine (2006, 2012). This is the subject of current research.

Further research should also be conducted on the effects of background wealth on behavior within experiments and on how to properly incorporate background wealth into the identification procedure. When utility is linear background wealth is not an issue, but it can be quite complicated when utility is nonlinear and the individual integrates her experimental rewards into her background wealth. The difficulty arrises from the fact that a change in one of the parameters, like the discount rate, can have opposite first order effects on savings today and tomorrow, which makes it difficult to identify. A first empirical stab at this was taken by Andersen et al. (2008), but not in the context of self-control and temptation. Quah and Strulovici (2013) study the effects of changes in the discount rate on stochastic continuous-time control and stopping problems and on individual's valuations. The background wealth problem may be able to be reformulated into one of their models. Alternatively, if the individual lives off of a very regular paycheck and has no unusual expenses during the course of the experiment, her background savings and consumption problem could be modeled as a reoccurring cake eating problem that is identical between periods of the experiment, with only the experimental rewards being different.

5 Appendix

Proof of Theorem 1

Proof. Linear utilities result normative and temptation utilities of the following form:

$$\begin{aligned} U(c_1, c_2) &= c_1 + \delta c_2 \\ V(c_1, c_2) &= \gamma \left(c_1 + \beta \delta c_2\right) \end{aligned}$$

Algorithm 1 results in the following two equations:

$$\begin{aligned} \text{Period 0 :} & \\ & \max_{R^{fb}} c \int\limits_{R_{min}}^{R^{fb}} f(R)dR + \delta c \int\limits_{R^{fb}}^{R_{max}} Rf(R)dR \\ \Rightarrow 0 &= cf(R^{fb}) - \delta c R^{fb} f(R^{fb}) \\ R^{fb} &= \frac{1}{\delta} \end{aligned}$$

Period 1 :

$$\max_{\overline{R}} c(1+\gamma) \int_{R_{min}}^{\overline{R}} f(R)dR + \delta(1+\gamma\beta)c \int_{\overline{R}}^{R_{max}} Rf(R)dR$$

$$-\gamma \max_{\tilde{R}} \left[cF(\tilde{R}) + \beta\delta c \int_{\tilde{R}}^{R_{max}} Rf(R)dR \right]$$

$$\begin{aligned} \Rightarrow 0 &= c\left(1+\gamma\right) f(\overline{R}) - \delta\left(1+\gamma\beta\right) c\overline{R} f(\overline{R}) \\ \frac{\overline{R}}{R^{fb}} &= \frac{1+\gamma}{(1+\gamma\beta)} \end{aligned} \tag{2}$$

Also, $\tilde{R} = \frac{1}{\beta\delta}$. Unless the individual is overcome by temptation, in which case $\tilde{R} = \overline{R}$, or her temptation is completely myopic, $\tilde{R} = R_{max}$, the researcher never observes $\tilde{R}$, and Algorithm 1 only supplies two equations for three unknowns. From equation 2 one can see that $\frac{\partial\beta}{\partial\gamma} > 0$, with a minimum of $\beta = 0$ when $\gamma = \frac{\overline{R} - R^{fb}}{R^{fb}}$, and a limit of $\beta \to \frac{R^{fb}}{\overline{R}}$ as $\gamma \to \infty$. □

Proof of Theorem 2

Proof. This augmented identification algorithm produces the following three equations.

Period $0.a$:

$$\max_{R^{fb}} c \int_{R_{min}}^{R^{fb}} f(R)dR + \delta c \int_{R^{fb}}^{R_{max}} Rf(R)dR$$

$$\begin{aligned} \Rightarrow \text{FC:}0 &= cf(R^{fb}) - \delta c R^{fb} f(R^{fb}) \\ R^{fb} &= \frac{1}{\delta} \end{aligned}$$

$$\text{Period } 0.b:$$

$$\begin{aligned}
\delta c \int_{R_{min}}^{R^{fb}} f(R)dR + \delta^2 c \int_{R^{fb}}^{R_{max}} Rf(R)dR - p + w &= c(1+\gamma)\delta \int_{R_{min}}^{\overline{R}} f(R)dR + \delta^2(1+\gamma\beta)c \int_{\overline{R}}^{R_{max}} Rf(R)dR \\
&\quad -\gamma \max_{\tilde{R}} \left[\delta c F(\tilde{R}) + \beta\delta^2 c \int_{\tilde{R}}^{R_{max}} Rf(R)dR \right]
\end{aligned}$$

$$\text{Period } 1:$$

$$\begin{aligned}
& \max_{\overline{R}} c(1+\gamma) \int_{R_{min}}^{\overline{R}} f(R)dR + \delta(1+\gamma\beta)c \int_{\overline{R}}^{R_{max}} Rf(R)dR \\
& -\gamma \max_{\tilde{R}} \left[cF(\tilde{R}) + \beta\delta c \int_{\tilde{R}}^{R_{max}} Rf(R)dR \right] \\
\Rightarrow \text{FC:} 0 &= c\,(1+\gamma)\,f(\overline{R}) - \delta\,(1+\gamma\beta)\,c\overline{R}f(\overline{R}) \\
\overline{R} &= \frac{1+\gamma}{\delta\,(1+\gamma\beta)}
\end{aligned}$$

The maximum price that the individual will be willing to pay will be the difference between her expected utility with and without commitment (R^{fb} vs $\overline{R}$ with the cost of self-control).

$$
\begin{aligned}
p &= w + \underbrace{\delta \int_{R_{min}}^{R^{fb}} cf(R)dR + \delta^2 \int_{R^{fb}}^{R_{max}} cRf(R)dR}_{\text{Expected utility with commitment}} \\
&\quad \underbrace{- \delta \left[\overbrace{(1+\gamma) \int_{R_{min}}^{\overline{R}} cf(R)dR + \delta(1+\gamma\beta) \int_{\overline{R}}^{R_{max}} cRf(R)dR}^{\text{normative + temptation utility}} \overbrace{- \gamma \left[\int_{R_{min}}^{\tilde{R}} cf(R)dR + \beta\delta \int_{\tilde{R}}^{R_{max}} cRf(R)dR \right]}^{\text{cost of self-control}} \right]}_{\text{Expected utility with no commitment}} \\
&= \underbrace{c\delta \int_{R^{fb}}^{\overline{R}} [\delta R - 1] f(R)dR}_{\substack{\text{cost from} \\ \text{suboptimal choice}}} + \underbrace{c\gamma\delta \left[\int_{\overline{R}}^{\tilde{R}} \left[1 - \beta\delta^2 R\right] f(R)dR \right]}_{\substack{\text{net cost of self-control} \\ \text{without commtiment}}} + w
\end{aligned}
$$

As can be seen from this last equation, commitment serves two purposes: it forces behavior in the direction of the normative optimum, and it decreases the net cost of self-control. All else equal, increasing γ will increase the expected cost from a suboptimal choice, since the individual succumbs a bit more to temptation. It will have two opposing effects on the expected net cost of self-control. On the one hand it will decrease the expected net cost since the individual will succumb some more to temptation ($\frac{\partial \overline{R}}{\partial \gamma} > 0$) and control herself less. On the other, it increases the expected cost of any self-control that is used. The first effect is of identical magnitude and opposite in sign as the increase in the expected cost from a suboptimal choice, so these wash out. The second does not cancel with anything else, so $\frac{\partial p}{\partial \gamma} > 0$.

Decreasing β has three effects on the expected net cost of self-control. First, as with an increase in γ, it also causes the expected cost from a suboptimal choice to increase, and by the same magnitude as the decrease in the expected net cost of self-control due to increasing $\overline{R}$, so these cancel out. Second, the decrease in β results in an increase in the range that self-control is used ($\frac{\partial \tilde{R}}{\partial \beta} < 0$). An increase in $\tilde{R}$ shifts a marginal amount of the reward from t_1 to period 0, however this shift will have no effect since it is along the margin at which the individual's temptation utility is indifferent between consumption today and consumption at t_1. Finally, decreasing β will result in a decrease in the temptation value of consumption tomorrow, thereby increasing the temptation to shift consumption from period t_1 to today. This last effect is the only one that does not wash out, so $\frac{\partial p}{\partial \beta} < 0$, all else equal.

For the model to be fully identified by Algorithm 2 p must change monotonically with either $\gamma(\beta)$ or $\beta(\gamma)$ as defined by $\overline{R} = \frac{1+\gamma}{\delta(1+\gamma\beta)}$. Either the effect of γ or the effect of β will have to dominate in order for p to change monotonically in this situation. Substitute $\beta = \frac{1+\gamma-\delta\overline{R}}{\gamma\delta\overline{R}}$ and take the derivative of p with respect to γ:

$$
\begin{aligned}
p &= c\delta \left[\int_{R^{fb}}^{\overline{R}} [\delta R - 1] f(R) dR + \int_{\overline{R}}^{\tilde{R}} \left[\gamma - \frac{1+\gamma-\delta\overline{R}}{\overline{R}} R \right] f(R) dR \right] + w \\
\frac{\partial p}{\partial \gamma} &= c\delta \int_{\overline{R}}^{\tilde{R}} \left[1 - \frac{R}{\overline{R}} \right] f(R) dR < 0
\end{aligned}
$$

Therefore the moderating effect of increasing β dominates the aggravating effect of an increasing γ. When $\overline{R}$ is constant the price for commitment is larger when temptation is more myopic (smaller β) than when it is more farsighted (larger β) because the individual must exert a greater amount of self-control.

$$
\begin{aligned}
p\left(\gamma = \frac{\overline{R} - R^{fb}}{R^{fb}}\right) &= c\delta\left[\int\limits_{R^{fb}}^{\overline{R}} [\delta R - 1]\, f(R)dR + \gamma\left[F(R_{max}) - F(\overline{R})\right]\right] + w > 0 \\
\lim_{\gamma\to\infty} p(\gamma) &= c\delta \int\limits_{R^{fb}}^{\tilde{R}} [\delta R - 1]\, f(R)dR + w > 0
\end{aligned}
$$

Define $\zeta \equiv p\left(\gamma = \frac{\overline{R}-R^{fb}}{R^{fb}}\right) - \lim_{\gamma\to\infty} p(\gamma)$.

$$
\zeta = c\delta\left[\int\limits_{\tilde{R}}^{\overline{R}} [\delta R - 1]\, f(R)dR + \gamma\left[F(R_{max}) - F(\overline{R})\right]\right] > 0
$$

Given that $\frac{\partial p}{\partial \gamma} < 0$ and $\zeta > 0$ any observed p will correspond to at most one value of γ. Therefore the model is fully identified.

□

Proof for Theorem 4

Proof. The individual's choice of R^{fb} in period 0 maximizes her expected utility in period 1. Her expected utility with commitment is

$$
\frac{1}{\delta}\mathcal{U}_0(\{R^{fb}\}) = \int\limits_{R_{min}}^{R^{fb}} u(c)f(R)dR + \delta \int\limits_{R^{fb}}^{R_{max}} u(Rc)f(R)dR.
$$

The first order conditions result in R^{fb} being defined by

$$
\begin{aligned}
u(c) - \delta u(R^{fb}c) &= 0 \\
\Rightarrow \delta &= \frac{u(c)}{u(R^{fb}c)}
\end{aligned}
$$

Expected utility with no commitment is

$$
\begin{aligned}
\frac{1}{\delta}\mathcal{U}_0(\mathcal{R}) &= \int\limits_{R_{min}}^{\overline{R}} u(c)f(R)dR + \delta \int\limits_{\overline{R}}^{R_{max}} u(Rc)f(R)dR \\
&\quad -\gamma \left(\int\limits_{R_{min}}^{\tilde{R}} v(c)f(R)dR + \beta\delta \int\limits_{\tilde{R}}^{R_{max}} v(Rc)f(R)dR - \int\limits_{R_{min}}^{\overline{R}} v(c)f(R)dR - \beta\delta \int\limits_{\overline{R}}^{R_{max}} v(Rc)f(R)dR \right)
\end{aligned}
$$

$\overline{R}$ is defined by the equality

$$
u(c) + \gamma v(c) - \delta \left[u(\overline{R}c) + \beta\gamma v(\overline{R}c) \right] = 0 \tag{3}
$$

The curvature of many utility functions could be measured by varying the reward, c. The Implicit Function theorem gives us

$$
\begin{aligned}
\frac{\partial \overline{R}}{\partial c} &= \frac{u'(c) + \gamma v'(c) - \delta \overline{R} \left[u'(\overline{R}c) + \gamma\beta v'(\overline{R}c) \right]}{c\delta \left[u'(\overline{R}c) + \gamma\beta v'(\overline{R}c) \right]} \\
&= \frac{u'(c) + \gamma v'(c)}{c\delta \left[u'(\overline{R}c) + \gamma\beta v'(\overline{R}c) \right]} - \frac{\overline{R}}{c}
\end{aligned}
$$

which is unique up to linear transformations of the form $\sigma \left[U(c_1, c_2) + V(c_1, c_2) \right]$. In the special case $V(c, Rc) = \gamma \left[u(c) + \beta\delta u(Rc) \right]$ then Equation (3) becomes

$$
\frac{\delta(1 + \gamma\beta)}{1 + \gamma} = \frac{u(c)}{u(\overline{R}c)}
$$

This can hold for a continuum of values for γ and β. If in addition, $u(\cdot)$ is CRRA then

$$\overline{R} = \left[\frac{1+\gamma}{\delta(1+\gamma\beta)}\right]^{\frac{1}{1-\alpha}}$$

which does not depend on c.

An alternative step is needed to measure the shape of $u(\cdot)$ if it is CRRA. The certainty equivalent is given by

$$u(\overline{x}) + \gamma v(\overline{x}) = \int\limits_{z_{min}}^{z_{max}} \left[u(z) + \gamma v(z)\right] f(z)dz$$

The combined utility function $u(c) + \gamma v(c)$ is a vNM utility function, so the ranking of lotteries will be unique up to affine transformations. If there is no measurement error, only one certainty equivalent is necessary for a CRRA utility function.

One more equation is needed to identify the parameters. The maximum price, p, that the individual is willing to pay for commitment gives us this additional necessary equation.

$$\delta \int\limits_{R_{min}}^{R^{fb}} u(c)f(R)dR + \delta^2 \int\limits_{R^{fb}}^{R_{max}} u(Rc)f(R)dR + u(w-p) = \begin{array}{c} \delta \int_{R_{min}}^{\overline{R}} u(c)f(R)dR + \delta^2 \int_{\overline{R}}^{R_{max}} u(Rc)f(R)dR \\ +\gamma\delta \int_{\overline{R}}^{\tilde{R}} \left[\beta\delta u(Rc) - u(c)\right] f(R)dR \end{array}$$

Substituting in $\beta = \frac{u(c)(1+\gamma)-\delta u(\overline{R}c)}{\gamma\delta u(\overline{R}c)}$:

$$\begin{aligned} u(w-p) &= \begin{array}{c} \delta \int_{R^{fb}}^{\overline{R}} u(c)f(R)dR - \delta^2 \int_{R^{fb}}^{\overline{R}} u(Rc)f(R)dR \\ +\gamma\delta \int_{\overline{R}}^{\tilde{R}} \left[\beta\delta u(Rc) - u(c)\right] f(R)dR \end{array} \\ &= \delta \int\limits_{R^{fb}}^{\overline{R}} u(c)f(R)dR - \delta^2 \int\limits_{R^{fb}}^{\overline{R}} u(Rc)f(R)dR + \delta \int\limits_{\overline{R}}^{\tilde{R}} \left[\frac{u(c)(1+\gamma) - \delta u(\overline{R}c)}{u(\overline{R}c)} u(Rc) - \gamma u(c)\right] f(R)dR \end{aligned}$$

Now use the implicit function theorem to find $\frac{\partial p}{\partial \gamma}$:

$$\frac{\partial p}{\partial \gamma} = \frac{u(c) \int_{\overline{R}}^{\tilde{R}} \left[1 - \frac{u(Rc)}{u(\overline{R}c)}\right] f(R) dR}{u'(w-p)} < 0$$

Therefore $p(\gamma)$ is unique with an upper bound when $\gamma = \frac{u(\overline{R}c) - u(R^{fb}c)}{u(R^{fb}c)}$ and a lower bound as $\gamma \to \infty$. With this observation we can identify all parameters and functional forms.

□

Relations for CRRA utility and KKS farsighted temptation that are useful for the following two proofs:

$$\begin{aligned} R^{fb} &= \left[\frac{1}{\delta}\right]^{\frac{1}{1-\alpha}} \\ \overline{R} &= \left[\frac{1+\gamma}{\delta(1+\gamma\beta)}\right]^{\frac{1}{1-\alpha}} \\ \tilde{R} &= \left[\frac{1}{\beta\delta}\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

Proof for Theorem 5

Proof. First the derivative of p with respect to R_{max}, when $\beta = 0$:

$$
\begin{aligned}
p(0) &= c\left[\frac{\delta}{R_{max}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR+\gamma\left(R_{max}-\overline{R}\right)\right]\right]^{\frac{1}{1-\alpha}}+w \\
\frac{\partial p(0)}{\partial R_{max}} &= \frac{c}{1-\alpha}\left[\frac{\delta}{R_{max}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR+\gamma\left(R_{max}-\overline{R}\right)\right]\right]^{\frac{\alpha}{1-\alpha}} \\
&\quad \frac{\delta^{t}}{R_{max}^{2}}\left\{\gamma R_{max}-\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR+\gamma\left(R_{max}-\overline{R}\right)\right]\right\} \\
&= \frac{c}{1-\alpha}\left(\frac{p}{c}\right)^{\alpha}\frac{\delta}{R_{max}^{2}}\left\{\gamma\overline{R}+\left(\overline{R}-R^{fb}\right)-\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR\right\} \\
&= \frac{c}{1-\alpha}\left(\frac{p}{c}\right)^{\alpha}\frac{\delta}{R_{max}^{2}}\left\{(1+\gamma)\overline{R}-R^{fb}-\frac{1}{2-\alpha}\left[\left(\frac{\overline{R}}{R^{fb}}\right)^{1-\alpha}\overline{R}-R^{fb}\right]\right\} \\
&= \frac{c}{1-\alpha}\left(\frac{p}{c}\right)^{\alpha}\frac{\delta}{R_{max}^{2}}\left\{(1+\gamma)\overline{R}-\frac{1-\alpha}{2-\alpha}R^{fb}-\frac{1+\gamma}{2-\alpha}\overline{R}\right\} \\
&= \frac{c}{1-\alpha}\left(\frac{p}{c}\right)^{\alpha}\frac{\delta}{R_{max}^{2}}\frac{1-\alpha}{2-\alpha}\left\{(1+\gamma)\overline{R}-R^{fb}\right\}>0
\end{aligned}
$$

The same derivative, but when $\beta \to \beta^{\star}$.

$$
\begin{aligned}
p(\beta^{\star}) &= c\left[\frac{\delta}{R_{max}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR\right]\right]^{\frac{1}{1-\alpha}}+w \\
\frac{\partial p(\beta^{\star})}{\partial R_{max}} &= \frac{p^{\alpha}}{1-\alpha}\left[-\frac{c}{1-\alpha}\left[\frac{\delta}{R_{max}^{2}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}} R^{1-\alpha}dR\right]\right]^{\frac{\alpha}{1-\alpha}}\right]<0
\end{aligned}
$$

The derivative with respect to c is straightforward and omitted. □

Proof for Theorem 6

Proof. The equation that determines the price the individual is willing to pay for commitment when utilities are CRRA is:

$$
\begin{array}{c}
\frac{(w-p)^{1-\alpha}}{1-\alpha}+\delta\frac{R^{fb}-R_{min}}{R_{max}-R_{min}}\frac{c^{1-\alpha}}{1-\alpha} \\
\frac{\delta^2}{+R_{max}-R_{min}}\int_{R^{fb}}^{R_{max}}\frac{(Rc)^{1-\alpha}}{1-\alpha}dR
\end{array}
=
\begin{array}{c}
\delta\frac{\overline{R}-R_{min}}{R_{max}-R_{min}}\frac{c^{1-\alpha}}{1-\alpha}+\frac{\delta^2}{R_{max}-R_{min}}\int_{\overline{R}}^{R_{max}}\frac{(Rc)^{1-\alpha}}{1-\alpha}dR \\
+\frac{\gamma\delta}{R_{max}-R_{min}}\int_{\overline{R}}^{\tilde{R}}\left[\beta\delta\frac{(Rc)^{1-\alpha}}{1-\alpha}-\frac{c^{1-\alpha}}{1-\alpha}\right]dR
\end{array}
$$

$$
\begin{aligned}
-(w-p)^{1-\alpha} &= \begin{array}{c}
-\delta\frac{\overline{R}-R^{fb}}{R_{max}-R_{min}}c^{1-\alpha}+\frac{\delta^2}{R_{max}-R_{min}}\int_{R^{fb}}^{\overline{R}}(Rc)^{1-\alpha}dR \\
-\frac{\gamma\delta}{R_{max}-R_{min}}\int_{\overline{R}}^{\tilde{R}}\left[\beta\delta\frac{(Rc)^{1-\alpha}}{1-\alpha}-\frac{c^{1-\alpha}}{1-\alpha}\right]dR
\end{array} \\
\Rightarrow p &= c\left[\frac{\delta}{R_{max}-R_{min}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}dR+\gamma\int_{\overline{R}}^{\tilde{R}}\left[1-\beta\delta R^{1-\alpha}\right]dR\right]\right]^{\frac{1}{1-\alpha}}+w
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial p}{\partial\alpha} &= c\left[\frac{\delta}{R_{max}-R_{min}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}dR-\gamma\int_{\overline{R}}^{\tilde{R}}\left[\beta\delta R^{1-\alpha}-1\right]dR\right]\right]^{\frac{1}{1-\alpha}} \\
&\quad\left\{\frac{1}{(1-\alpha)^2}\ln\left(\frac{\delta}{R_{max}-R_{min}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}dR-\gamma\int_{\overline{R}}^{\tilde{R}}\left[\beta\delta R^{1-\alpha}-1\right]dR\right]\right)\right. \\
&\quad\left.+\frac{\frac{\delta}{R_{max}-R_{min}}\left[-\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}\ln(R)dR+\gamma\int_{\overline{R}}^{\tilde{R}}\beta\delta R^{1-\alpha}\ln(R)dR\right]}{(1-\alpha)\frac{\delta}{R_{max}-R_{min}}\left[-\left(\overline{R}-R^{fb}\right)+\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}dR-\gamma\int_{\overline{R}}^{\tilde{R}}\left[\beta\delta R^{1-\alpha}-1\right]dR\right]}\right\} \\
&= (p-w)\left\{\frac{1}{(1-\alpha)^2}\ln\left(\left[\frac{p-w}{c}\right]^{1-\alpha}\right)+\frac{1}{1-\alpha}\left[\frac{c}{p-w}\right]^{1-\alpha}\left[\frac{\delta}{R_{max}-R_{min}}\left[-\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}\ln(R)dR+\gamma\int_{\overline{R}}^{\tilde{R}}\beta\delta R^{1-\alpha}\ln(R)dR\right]\right]\right\} \\
&= \frac{p-w}{1-\alpha}\left\{\ln\left(\frac{p-w}{c}\right)+\left[\frac{c}{p-w}\right]^{1-\alpha}\left[\frac{\delta}{R_{max}-R_{min}}\left[-\delta\int_{R^{fb}}^{\overline{R}}R^{1-\alpha}\ln(R)dR+\gamma\int_{\overline{R}}^{\tilde{R}}\beta\delta R^{1-\alpha}\ln(R)dR\right]\right]\right\}
\end{aligned}
$$

The sign of $\frac{\partial p}{\partial\alpha}$ is ambiguous. For large enough R_{max} then $p-w>c$ and $\ln(\frac{p-w}{c})>0$. This is illustrated in the two graphs in Figure 1.

□

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